

Magnetic Island Theory - II

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Introduction: Non-MHD Effects

- This lecture discusses influence of non-MHD effects on Rutherford island equation. Effects considered are:
 - **Diamagnetic flow**. Causes ion and electron fluids to propagate at different speeds. Island propagates at intermediate speed. Leads to **stabilizing ion polarization term** in Rutherford eq.
 - **Ion parallel stress tensor**. Damps poloidal ion flow. Causes large parallel flow. Leads to **large enhancement** in polarization term in Rutherford equation.
 - **Electron parallel stress tensor**. Causes to Bootstrap current. Leads to **destabilizing Bootstrap term** in Rutherford equation.
 - **Finite parallel heat and particle transport**. Allows density and pressure to not be flux-surface functions at small island widths. Leads to **disappearance** of Bootstrap term in Rutherford eq.

Diamagnetic Effects

- According to two-fluid theory:

$$\vec{V}_{\perp i} \simeq \vec{V}_{\perp \text{MHD}},$$

$$\vec{V}_{\perp e} \simeq \vec{V}_{\perp \text{MHD}} + \vec{V}_{\perp *}$$

- So-called **diamagnetic velocity** takes form:

$$\vec{V}_{\perp *} \simeq -\frac{dP/dr}{n e B_{\phi}} \vec{e}_{\theta},$$

where P is plasma pressure, n electron number density, B_{ϕ} toroidal field-strength.

Pressure Flattening - I ^a

- In immediate vicinity of magnetic island (rational surface radius: r_s), unperturbed pressure takes form:

$$P(r) \simeq P'_s (r - r_s).$$

- How is this profile modified in presence of island?
- Pressure profile determined by competition between parallel and perpendicular thermal/particle transport:

$$\chi_{\parallel} \nabla_{\parallel}^2 P + \chi_{\perp} \nabla_{\perp}^2 P \simeq 0.$$

Note that $\chi_{\parallel} \gg \chi_{\perp}$.

^aR. Fitzpatrick, Phys. Plasmas **2**, 825 (1995).

Pressure Flattening - II

- Parallel transport term attempts to make pressure a **flux-surface function**. Cannot have odd flux-surface function inside island separatrix. So, if $P = P(\psi)$ then pressure is **flattened** inside island.
- Perpendicular transport term attempts to relax pressure profile to unperturbed profile. Opposes pressure flattening.
- Pressure is flattened across island if parallel transport term dominates perpendicular transport term.

Pressure Flattening - III

- Have

$$\nabla_{\parallel} \simeq \frac{B_{\theta}}{B_{\phi}} \frac{k_{\theta} W}{l_s},$$

where l_s is typical radial variation length-scale of B_{θ} .

- Also

$$\nabla_{\perp} \simeq \frac{1}{W}.$$

- Hence, parallel transport dominates perpendicular transport when

$$W \gg W_c,$$

where

$$k_{\theta} W_c \simeq \left(\frac{\chi_{\perp}}{\chi_{\parallel}} \right)^{1/4} \left(k_{\theta} l_s \frac{B_{\phi}}{B_{\theta}} \right)^{1/2}.$$

Pressure Flattening - IV

- Critical island width fairly small (few cm) in tokamak plasmas.
- Hence, if island width exceeds critical width then pressure flattened inside separatrix.
- Implies that diamagnetic velocity **zero** inside separatrix.
- Hence, ion fluid, electron fluid, and island velocities **identical** inside separatrix.