Asymptotic Matching Approach to Modeling Tearing Mode Dynamics in Tokamak Plasmas

Richard Fitzpatrick

Institute for Fusion Studies, Department of Physics, University of Texas at Austin

Peculiar Properties of Tokamak Plasmas

- The solar wind is typical naturally-occurring plasma: supersonic, super-Alfvénic outflow from Sun that is filled with shocks and other violent variations in plasma properties.
- Tokamak plasmas are, by design, extraordinarily quiescent. When tokamak is operating normally, plasma is subsonic, sub-Alfvénic, with no shocks or violent motions, just low-level turbulence and occasional slowly-growing tearing modes.
- Present-day tokamak discharges last a few seconds: equivalent to 10^8 Alfvén times. ITER discharges will last a few minutes: equivalent to 10^{11} Alfvén times. Clear that, unlike most naturally-occurring plasmas, Alfvén time is totally irrelevant timescale for tokamak plasmas.

Disruptions, Error-Fields, and RMPs

- Tokamak plasmas sometimes terminate prematurely in violent events known as disruptions.
- One class of disruption is triggered by crossing ideal-MHD stability boundaries. This class is (mostly) avoidable because location of ideal stability boundaries can be accurately calculated.
- Virtually all other types of disruptions are associated with tearing modes: especially, tearing modes that grow to (relatively) large amplitudes, and then lock to stationary error-fields (i.e., accidentally produced, non-axisymmetric, equilibrium magnetic fields).
- Error-fields sometimes deliberately applied to tokamaks, in which case they are called resonant magnetic perturbations (RMPs), so as to drive small-amplitude tearing modes in plasma in controlled manner (e.g., to suppress edge localized modes).

Tearing Modes in Tokamak Plasmas

- Tearing modes are plasma instabilities that reconnect magnetic flux at resonant surfaces (where $\mathbf{k}\cdot\mathbf{B} = 0$).
- Tearing modes degrade plasma confinement, and trigger disruptions when they attain (relatively) large amplitudes.
- Unlike magnetic reconnection in most naturally-occurring plasmas, tearing-mode-mediated reconnection in tokamak plasmas takes place extraordinarily slowly, due to intrinsic stability of tokamak plasmas.
- In short-lived linear regime, reconnection timescale is $S^{3/5}\,\tau_A$, where $S \ggg 1$ is Lundquist number. In nonlinear regime (when island width exceeds linear layer width), reconnection timescale is $S\,\tau_A$.
- Slow reconnection is problematic because, to simulate it, need to follow plasma evolution on timescales that exceed most basic plasma timescales by very many orders of magnitude.

MHD Codes

- What is appropriate vehicle for simulating interaction of tearing modes with error-fields and RMPs in tokamak plasmas?
- Conventional answer is that appropriate vehicle is toroidal, nonlinear, two-fluid, resistive-MHD code.
- Purpose of talk is to explain why this answer is incorrect, and to provide correct answer.

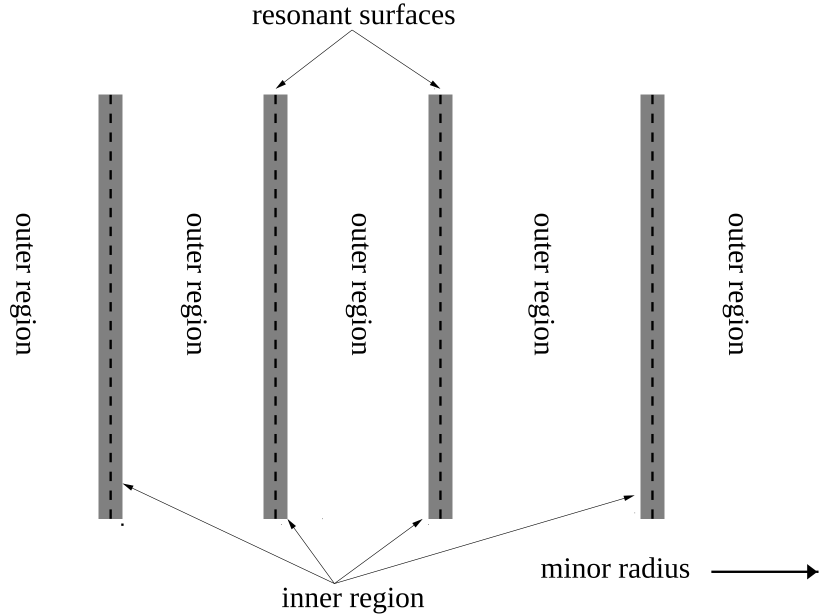
Alfvén Time

- By definition, most important timescale in an MHD code is Alfvén time.
- But, Alfvén time is completely irrelevant timescale in type of calculation under consideration.
- Explicit MHD codes forced to take timesteps that are less than Alfvén time by CFL condition. Would need to take 10^8 timesteps to simulate present-day tokamak plasmas, and 10^{11} timesteps to simulate ITER plasmas. Completely impossible!
- Implicit MHD codes can take longer timesteps, but have to solve large-dimension, ill-posed, computationally-expensive, matrix problem every timestep. Still impossible to simulate whole plasma discharge.

Asymptotic Matching - I

- Asymptotic matching approach to tearing mode evolution in high-temperature tokamak plasma recognizes that problem can be split into two halves.
- Outer region, which comprises virtually all of plasma, as well as surrounding vacuum, is governed by linear, marginally-stable, ideal-MHD.
- Inner region, which comprises series of thin layers centered on resonant surfaces, is governed by nonlinear, two-fluid, resistive-MHD.
- Can solve for plasma response in inner and outer regions independently, and then asymptotically match two sets of solutions to get overall solution.

Asymptotic Matching - II



Asymptotic Matching - III

- Important to realize that asymptotic matching approach only feasible because of intrinsic stability of tokamak plasmas.
- Radial width of given component of inner region is greater of linear layer width (very thin in high-S plasma) and magnetic island width.
- Tokamak plasmas are sufficiently stable that tearing modes saturate at relatively low amplitudes (i.e., such that associated island width is, at most, a few percent of minor radius). Hence, inner region occupies very small volume fraction of plasma.
- By contrast, in reversed field pinches, which are much less stable cousins of tokamaks, magnetic island widths grow to such an extent that inner region engulfs whole plasma. Asymptotic matching does not work under such circumstances.

Outer Region - I

- Outer region governed by so-called "linear, marginally-stable, ideal-MHD".
- Nomenclature is misleading. MHD necessarily involves plasma inertia. However, linear, marginally-stable, ideal-MHD completely neglects plasma inertia.
- In fact, linear, marginally-stable, ideal-MHD is not MHD at all: it is simply linearized force balance.
- Solution of outer region essentially boils down to solving a slightly non-axisymmetric plasma equilibrium problem.
- No sane person would use an MHD code to solve a conventional axisymmetric plasma equilibrium problem. It makes just as little sense to use an MHD code to solve a slightly non-axisymmetric plasma equilibrium problem. But this is precisely what MHD codes do every timestep.

Outer Region - II

- Once you have determined the positions of the resonant surfaces, and calculated the linearized tearing mode eigenfunctions, solution in outer region is completely described once amplitudes and phases of reconnected magnetic fluxes at resonant surfaces are specified.
- In other words, in order to evolve outer solution, you just need to evolve amplitudes and phases of reconnected fluxes.
- Obviously, if equilibrium changes significantly then you need to recalculate positions of resonant surfaces and tearing eigenfunctions. But, such changes typically take place on timescale of few ms (i.e., 10^4 Alfvén times).

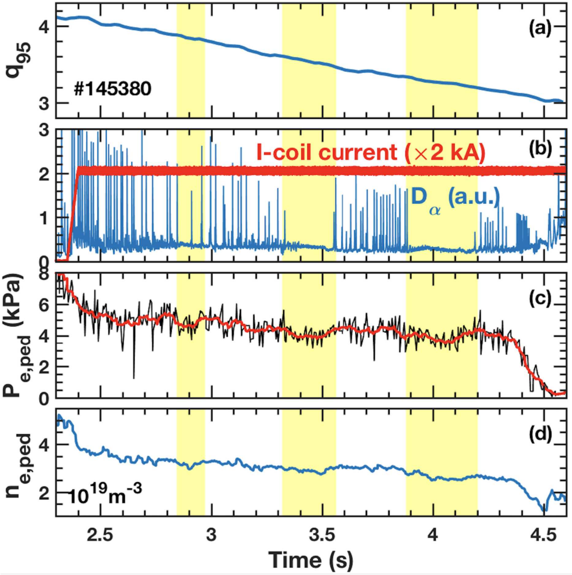
Outer Region - III

- MHD codes solve perturbed equilibrium problem in outer region by finite-difference/finite-element method every timestep. This is time-consuming calculation because it necessarily involves solution of Poisson's equation on large multi-dimensional grid.
- Such an approach is spectacularly inefficient, because solution is actually completely parameterized by relatively small number of variables: i.e., amplitudes and phases of reconnected fluxes.
- In other words, even if it were possible to simulate a whole tokamak plasma discharge with an MHD code, you would not want to, because useful information/total information ratio is minuscule.

EPEC Code

- The EPEC code has been developed to implement asymptotic matching approach to modeling tearing mode dynamics in tokamak plasmas.
- EPEC has been completely integrated into OMFIT framework. This facilitates access to experimental data, as well as incorporation of data from other codes such as GPEC (used to calculate ideal response of plasma to RMP), TRANSP (used to calculate diffusivity profiles), and SOLPS (used to calculate neutral particle profile).
- In order to describe how EPEC works, will concentrate on simulation of DIII-D discharge #145380, in which an $n = 3$ RMP was applied to a plasma whose edge safety-factor was gradually ramped down. Purpose of experiment was to suppress edge localized modes.

DIII-D Discharge #145380



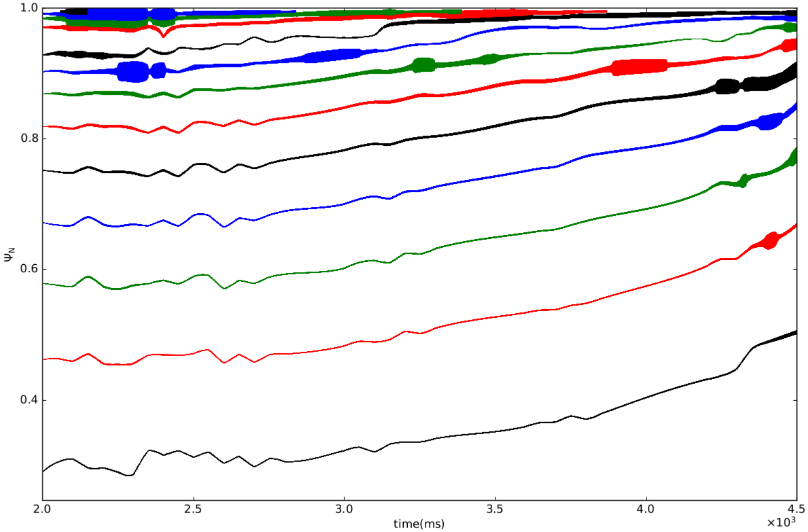
EPEC Simulation of DIII-D Discharge #145380 - I

- Experimental equilibrium data (in form of EQDSK gFiles), profile data (in form of Osbourne pFiles), and diffusivity data (output from TRANSP in form of so-called cFiles) available every 50 ms.
- gFiles are used as input for GPEC code to calculate ideal-MHD response of plasma equilibrium to lower RMP coil set (generating so-called lFiles), and upper RMP coil set (generating so-called uFiles), every 50 ms.
- EPEC interpolates gFiles, pFiles, and cFiles to produce requisite equilibrium data (generating so-called fFiles) and profile data (generating so-called nFiles) at each resonant surface in the plasma every 10 ms.

EPEC Simulation of DIII-D Discharge #145380 - II

- EPEC interpolates fFiles, nFiles, lFiles, and uFiles to simulate plasma in series of 1 ms (10^4 Alfvén times) chunks.
- Equilibrium, profiles, and tearing eigenfunctions effectively recalculated every 1 ms.
- Within 1 ms chunks, EPEC evolves amplitudes and phases of reconnected fluxes at every resonant surface in the plasma, as well as data that specifies modified velocity profiles at resonant surfaces. Typical timestep is between 10 and 100 Alfvén times. Calculation is feasible because only a relatively small number of variables (~ 1000) are being evolved in time.

EPEC Simulation of DIII-D Discharge #145380 - II



EPEC Code: Achievements and Further Plans

- EPEC code has been used to successfully model RMP-induced edge localized mode (ELM) suppression in DIII-D and KSTAR tokamaks.
- Preliminary calculations offer some insight as to why RMP-induced ELM suppression did not work in NSTX and MAST spherical tokamaks (devices too small and cold for plasma rotation to generate effective shielding of driven magnetic reconnection).
- Working to incorporate SOLPS data into code, in order to characterize neutral particle profiles in edge regions of plasma.
- EPEC incorporates accurate neoclassical model that can be used to simulate neoclassical tearing modes (NTMs). Intend to examine whether RPMs are likely to trigger NTMs.
- Intend to incorporate neoclassical toroidal viscosity into model (important for spherical tokamaks).