

**Determination of Non-Ideal Response of a High
Temperature Plasma to a Static External Magnetic
Perturbation via Asymptotic Matching**

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Error Fields

- Tokamak plasmas highly sensitive to externally generated, static, helical magnetic perturbations—a.k.a. “error fields” .
- Error fields drive magnetic reconnection in otherwise tearing stable plasmas, giving rise to formation of non-rotating magnetic island chains on rational magnetic surfaces—a.k.a. “locked modes” .
- Locked modes severely degrade plasma energy confinement, and often trigger disruptions.

Plasma Response to Error Fields

- Response very different to that predicted by naively superimposing vacuum perturbation onto equilibrium magnetic field.
- *Shielding currents* excited at rational surfaces by plasma rotation, and/or combination of pressure gradients and favorable average field-line curvature, act to suppress driven magnetic reconnection.
- *Distributed currents* generated by ideal response of realistic (i.e., highly elongated, moderate aspect-ratio, high- β) plasma equilibrium to perturbation profoundly modify perturbation structure. Consequently, cylindrical models, or models that rely on large aspect-ratio, low- β orderings, do very poor job of predicting plasma response.

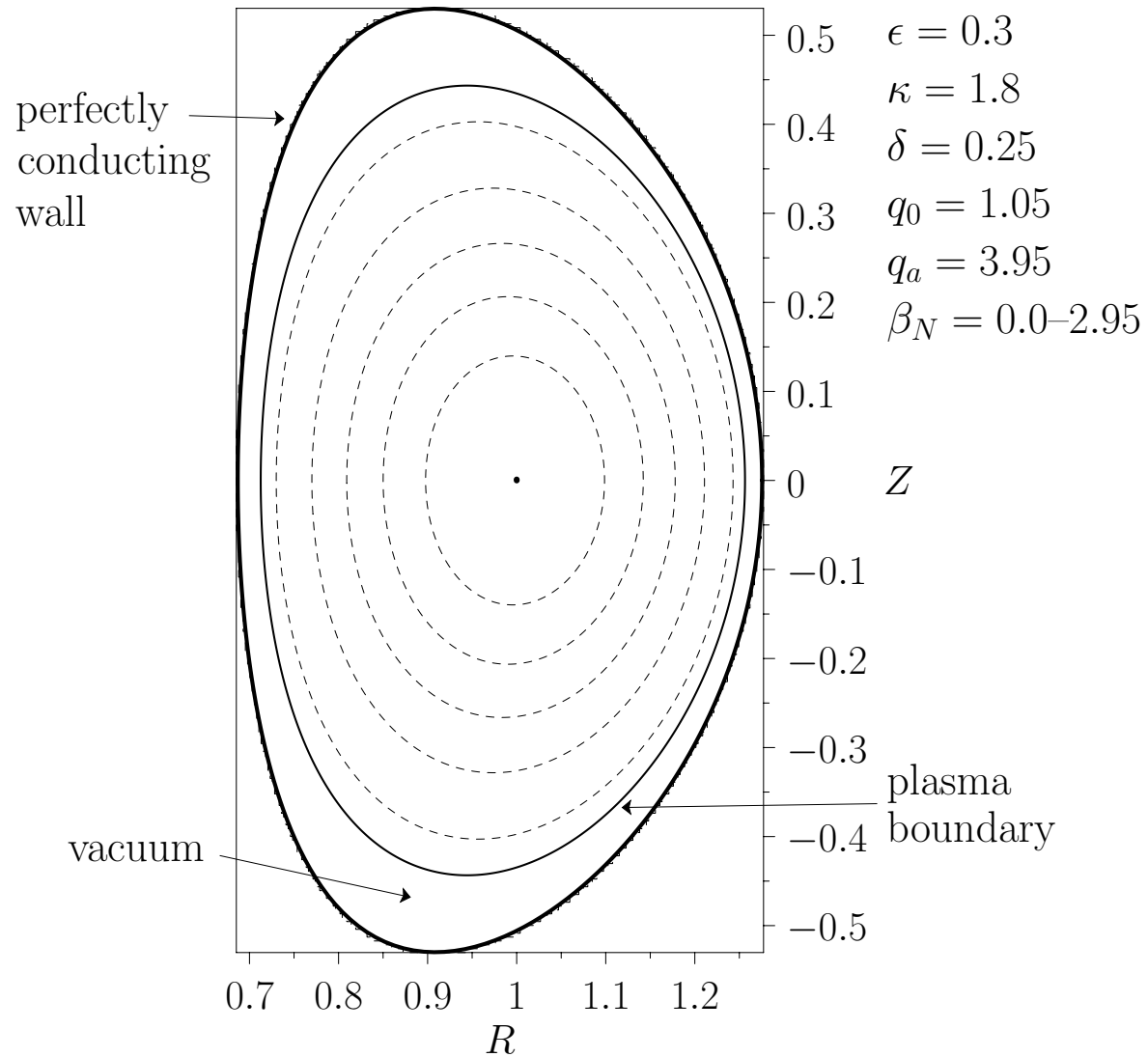
Approaches to Error Field Response Calculation

1. Solve resistive-MHD equations throughout whole plasma. Highly inefficient because resistivity and inertia only important close to rational surfaces. Plasma response elsewhere governed by much simpler equations of ideal-MHD.
2. Solve ideal-MHD equations throughout whole plasma, placing ideal current sheets at rational surfaces. Facilitates very rapid calculations. Correctly calculates distributed currents. Neglects non-ideal response of plasma at rational surfaces.
3. Solve ideal-MHD equations throughout bulk of plasma, and asymptotically match solutions to non-ideal layer solutions at rational surfaces. More efficient than approach 1. More accurate than approach 2.

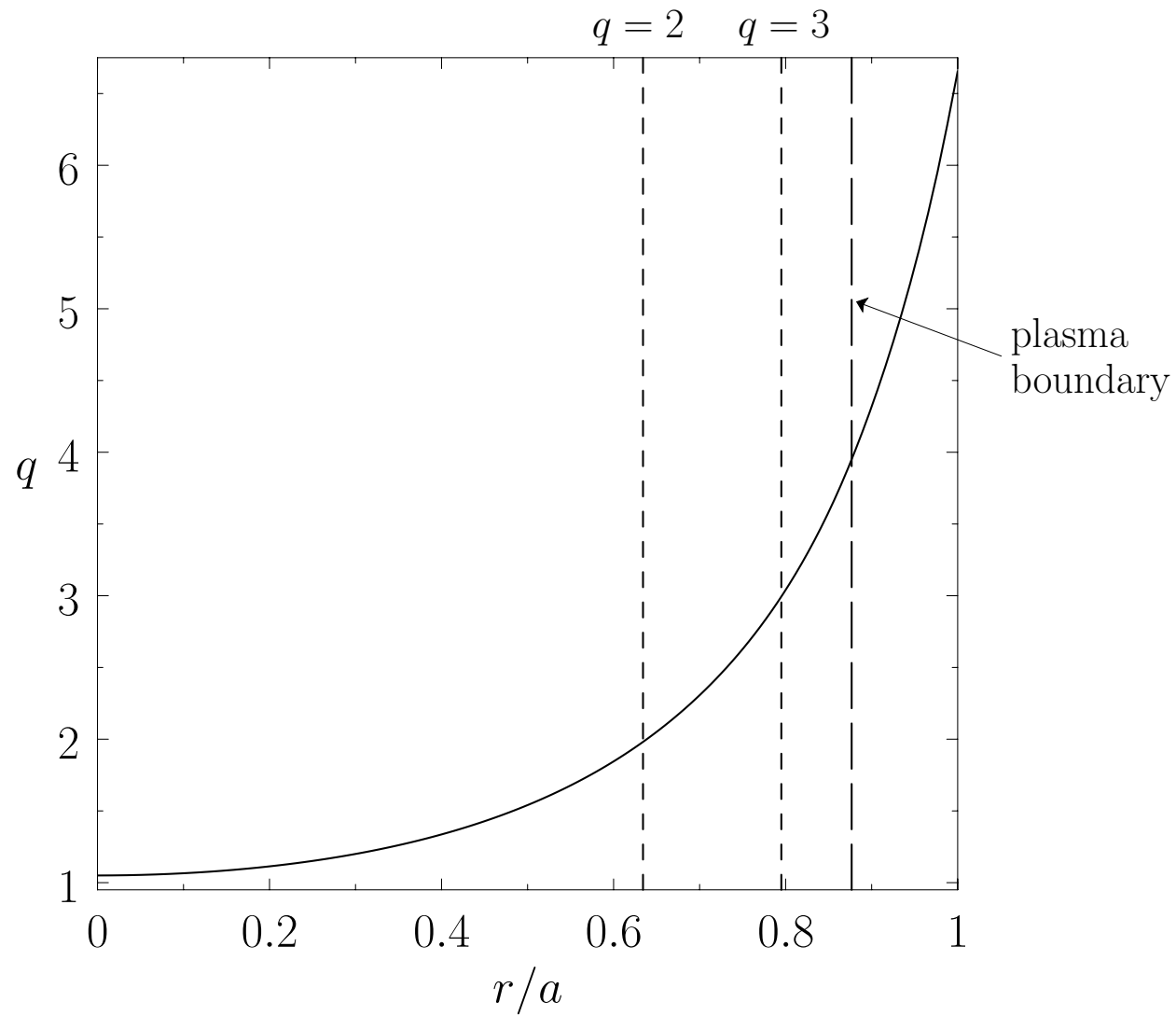
My Approach to Error Field Response Calculation

- Use recently developed TOMUHAWC code to calculate ideal-MHD response of realistic plasma equilibrium everywhere apart from immediate vicinity of rational surfaces.
- Asymptotically match ideal-MHD data obtained from TOMUHAWC to Glasser-Greene-Johnson linear layers at various rational surfaces. (GGJ linear layer response theory is simplest non-ideal response model that is consistent with realistic plasma equilibrium.)

Plasma Equilibrium



Safety Factor Profile



Coordinate System

- Adopt right-handed flux coordinate system: r, θ, ϕ .
 - r - flux-surface label
 - θ - “straight” poloidal angle
 - ϕ - geometric toroidal angle

- Jacobian:

$$(\nabla r \times \nabla \theta \cdot \nabla \phi)^{-1} = r R^2,$$

where R is major radius.

- Flux-surface average operator:

$$\langle \dots \rangle \equiv \frac{1}{2\pi} \oint (\dots) d\theta.$$

Perturbed Magnetic Field

- Let

$$\delta \mathbf{B} \cdot \nabla r = i \sum_j \frac{\psi_j(r)}{r R^2} \exp[i(m_j \theta - n \phi)].$$

- In immediate vicinity of k th rational surface [$q(r_k) = m_k/n$]

$$\psi_k(r) = \Psi_k F_k |\chi_k|^{\nu_{Lk}} + \Delta \Psi_k^\pm F_k \operatorname{sgn}(\chi_k) |\chi_k|^{\nu_{Sk}} + A_k \chi_k,$$

$$\chi_k = (r - r_k)/r_k,$$

$$F_k = \left(\frac{m_k^2 \langle |\nabla r|^2 \rangle + n^2 r^2}{2\sqrt{D_{Ik}}} \right)_{r_k}^{1/2},$$

$\nu_{Lk} = 1/2 - \sqrt{D_{Ik}}$, $\nu_{Sk} = 1/2 + \sqrt{D_{Ik}}$, and D_{Ik} is GGJ ideal stability index at k th rational surface.

Asymptotic Matching Parameters

- Ψ_k - measures tearing parity reconnected magnetic flux at kth rational surface.
- $\Delta\Psi_k \equiv \Delta\Psi_k^+ + \Delta\Psi_k^-$ - measures tearing parity shielding currents excited at kth rational surface.
- $\Delta_k \equiv \Delta\Psi_k/\Psi_k$ - determined by non-ideal tearing parity layer solution at kth rational surface.

Homogeneous Tearing Parity Dispersion Relation

- Dispersion relation:

$$\sum_{k'} (E_{kk'} - \delta_{kk'} \Delta_{k'}) \Psi_{k'} = 0.$$

- TOMUHAWC code calculates elements of hermitian \mathbf{E} -matrix, $E_{kk'}$. Diagonal elements of matrix are toroidal generalizations of Δ' parameter of Furth-Killeen-Rosenbluth theory.
- Toroidal electromagnetic torque at k th surface:

$$\delta T_k = 2 n \pi^2 \text{Im}(\Delta_k) |\Psi_k|^2.$$

Fact that \mathbf{E} -matrix is hermitian ensures that zero net torque exerted on plasma.

Error Field Generation

- Perfectly conducting wall subject to small amplitude, static, helical displacement:

$$\xi = \delta r \frac{|\nabla r|}{|\nabla r|^2},$$

where

$$\delta r(\theta, \phi) = \sum_j \Xi_j \exp[i(m_j \theta - n \phi)].$$

Inhomogeneous Tearing Parity Dispersion Relation

- Dispersion relation:

$$\sum_{k'} (\Xi_{kk'} - \delta_{kk'} \Delta_{k'}) \Psi_{k'} = \chi_k,$$

where

$$\chi_k = \sum_j \Xi_j \xi_{k,j}.$$

- TOMUHAWC code calculates $\xi_{k,j}$ parameters from homogeneous tearing eigenfunctions.

Shielding Factor

- Assume that k th rational surface locked to error field, whereas plasma at other rational surfaces rotating sufficiently rapidly to suppress driven reconnection.
- Reconnected flux driven at k th surface:

$$\Psi_k = \frac{\chi_k}{E_{kk}} \mathcal{S}_k,$$

where

$$\mathcal{S}_k = \frac{E_{kk}}{E_{kk} - \Delta_k}$$

is so-called *shielding factor*—i.e., factor by which driven reconnection at k th rational surface suppressed by shielding currents.

Optimum Wall Displacement

- Optimum wall displacement for driving reconnected flux at k th rational surface is

$$\Xi_j = \delta \hat{\xi}_{k,j},$$

where

$$\hat{\xi}_{k,j} = \frac{\xi_{k,j}}{\|\xi_k\|},$$

and

$$\|\xi_k\| = \sum_j |\xi_{k,j}|^2.$$

Here, δ is mean wall displacement (in r).

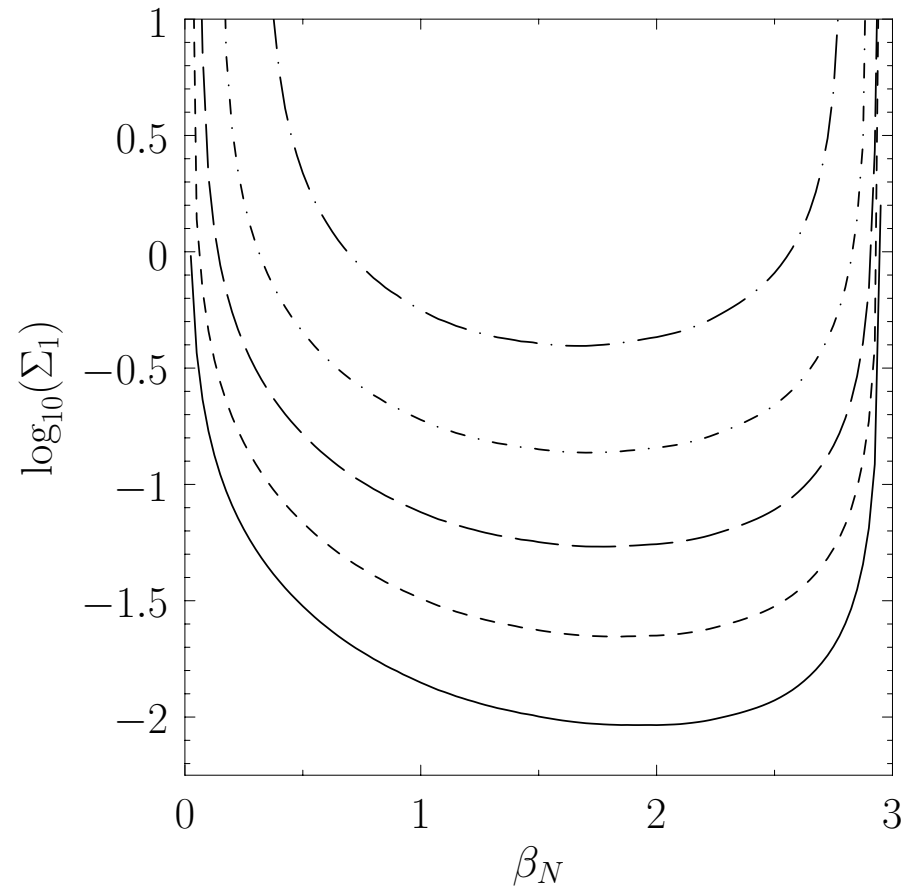
Overlap Factor

- Overlap factor

$$\alpha_{12} = \left| \sum_j \hat{\xi}_{1j} \hat{\xi}_{2j}^* \right|$$

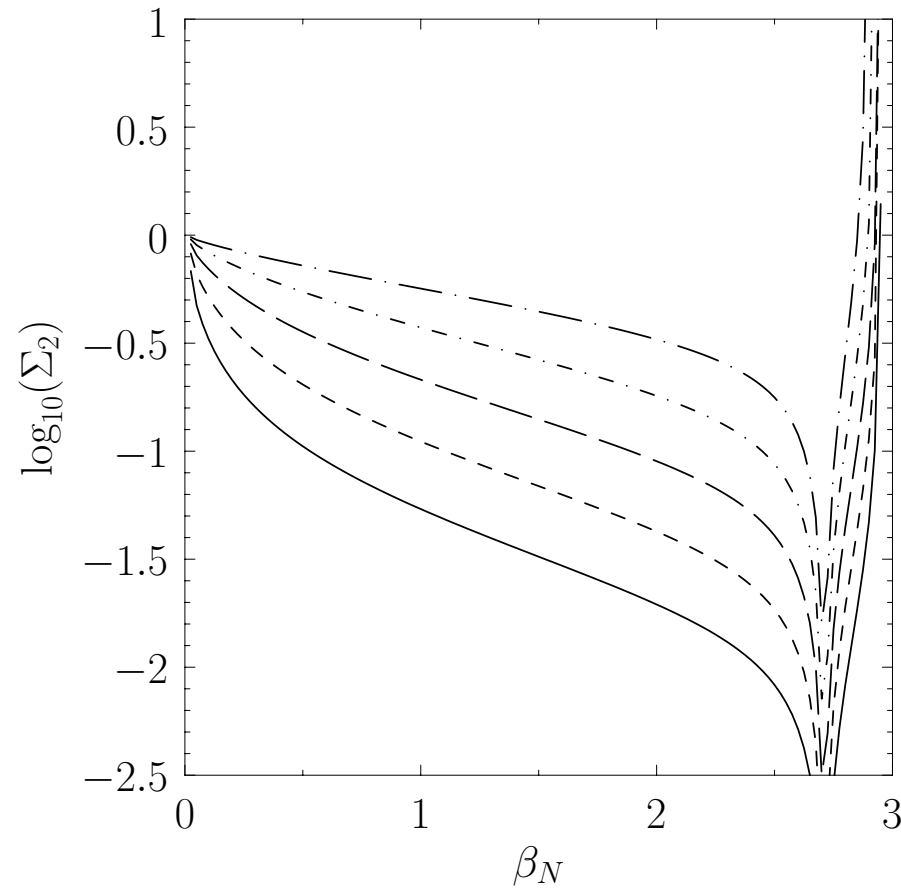
measures extent to which optimum wall displacement for driving reconnected flux at rational surface 1 is similar to optimal wall displacement for driving reconnected flux at rational surface 2. If $\alpha_{12} = 1$ then wall displacements are identical. If $\alpha_{12} = 0$ then wall displacements are independent.

Shielding Factor at $q = 2$ surface



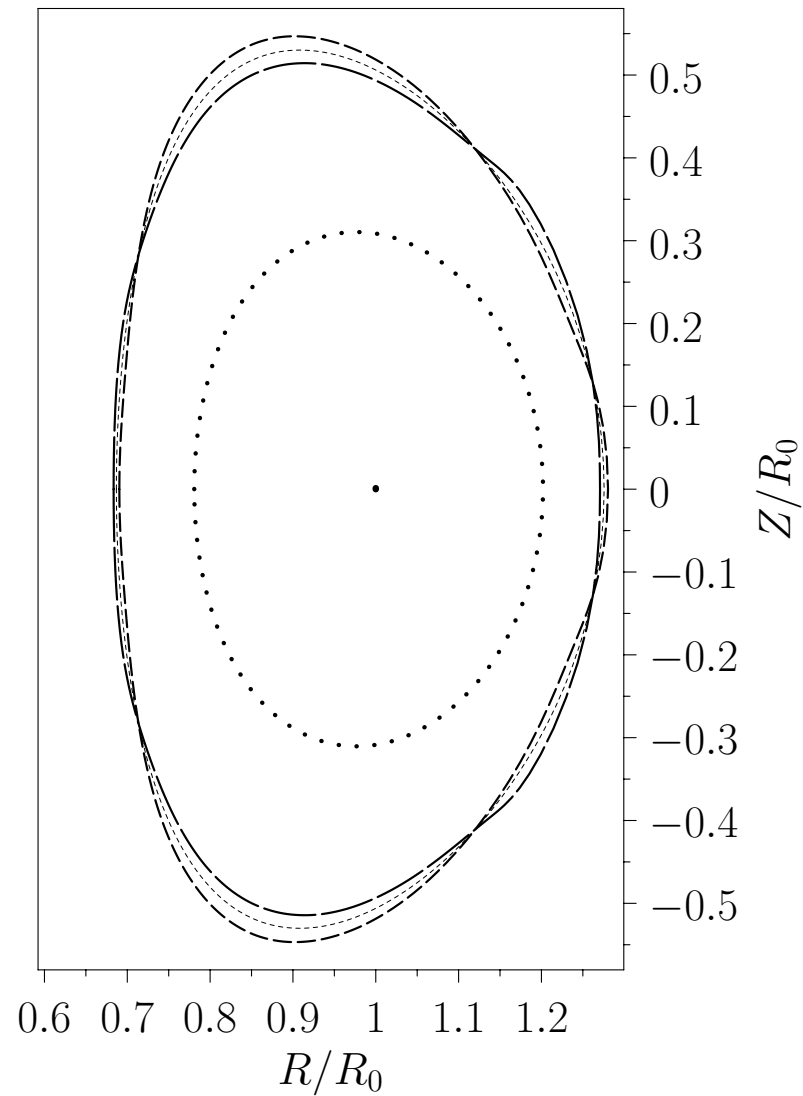
Top to bottom: $S_1 = 10^6$, $S_1 = 10^7$, $S_1 = 10^8$, $S_1 = 10^9$, $S_1 = 10^{10}$.

Shielding Factor at $q = 3$ surface

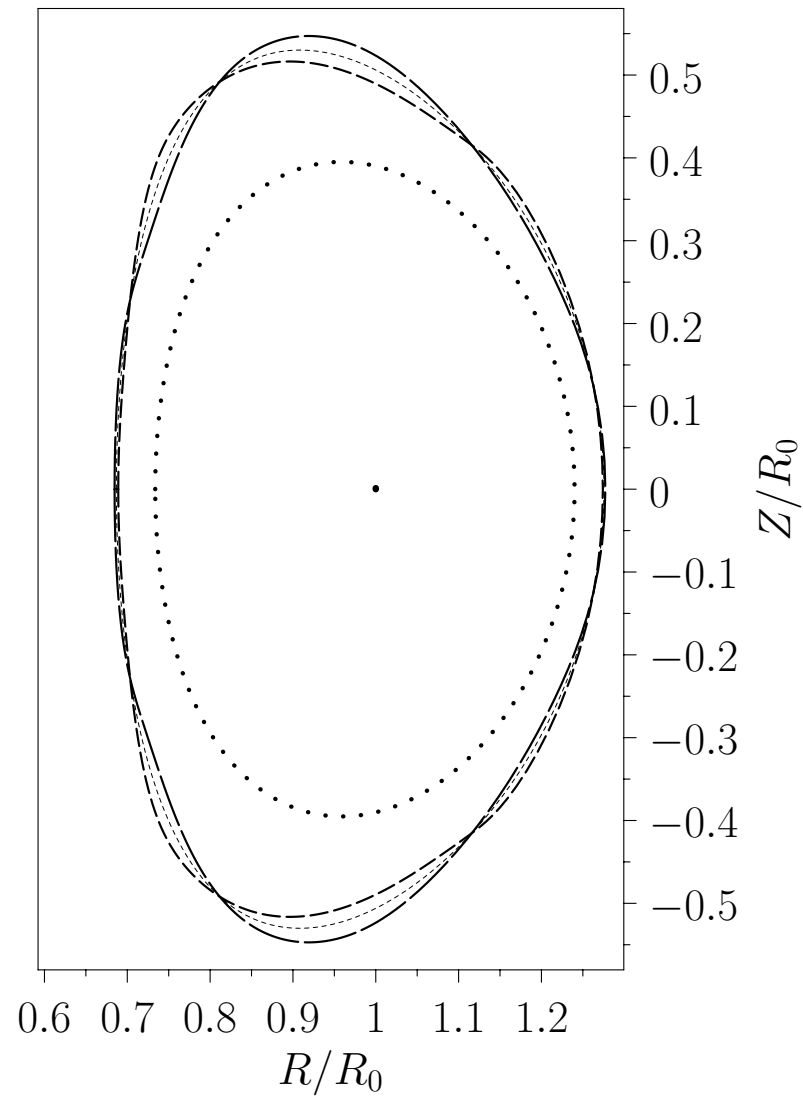


Top to bottom: $S_2 = 10^6$, $S_2 = 10^7$, $S_2 = 10^8$, $S_2 = 10^9$, $S_2 = 10^{10}$.

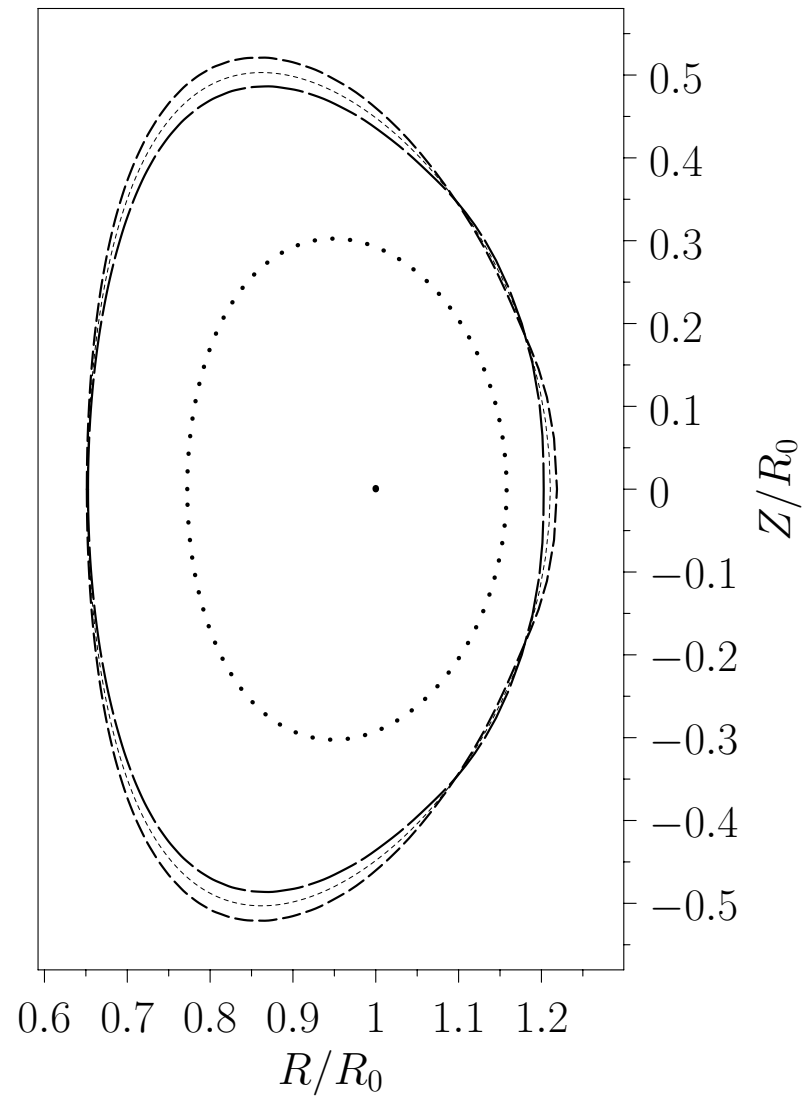
Optimum Wall Displacement: $q = 2$, Low- β



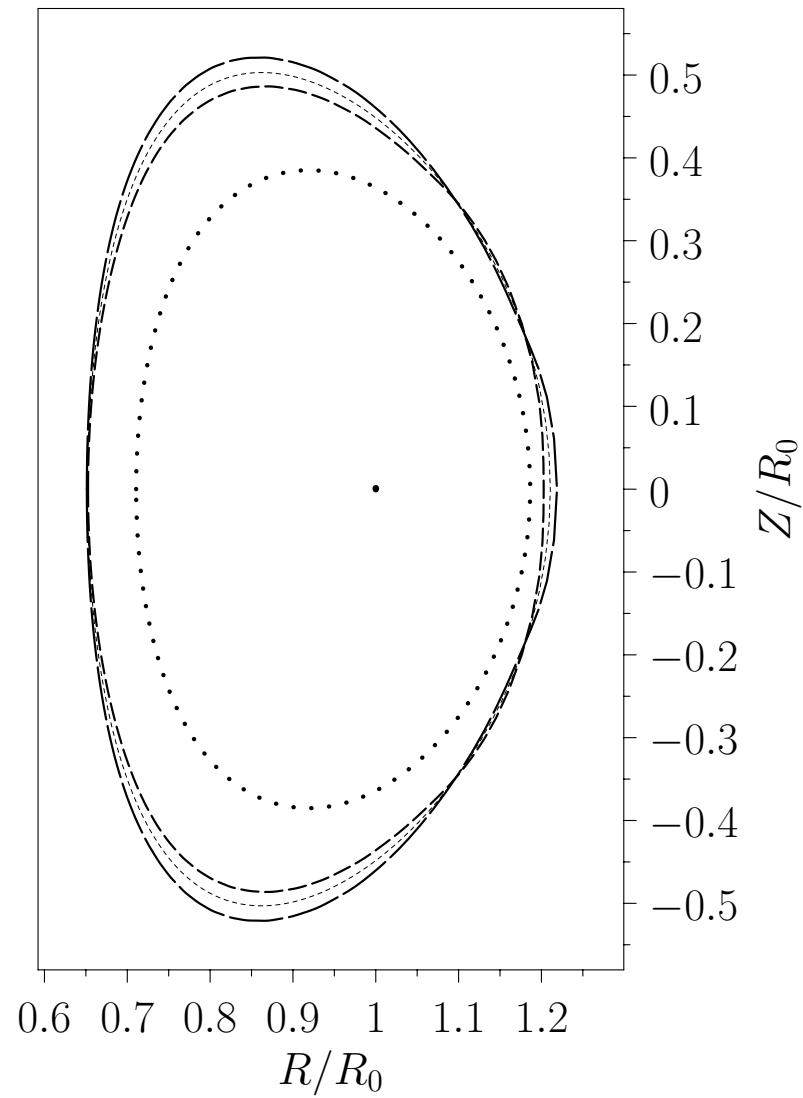
Optimum Wall Displacement: $q = 3$, Low- β



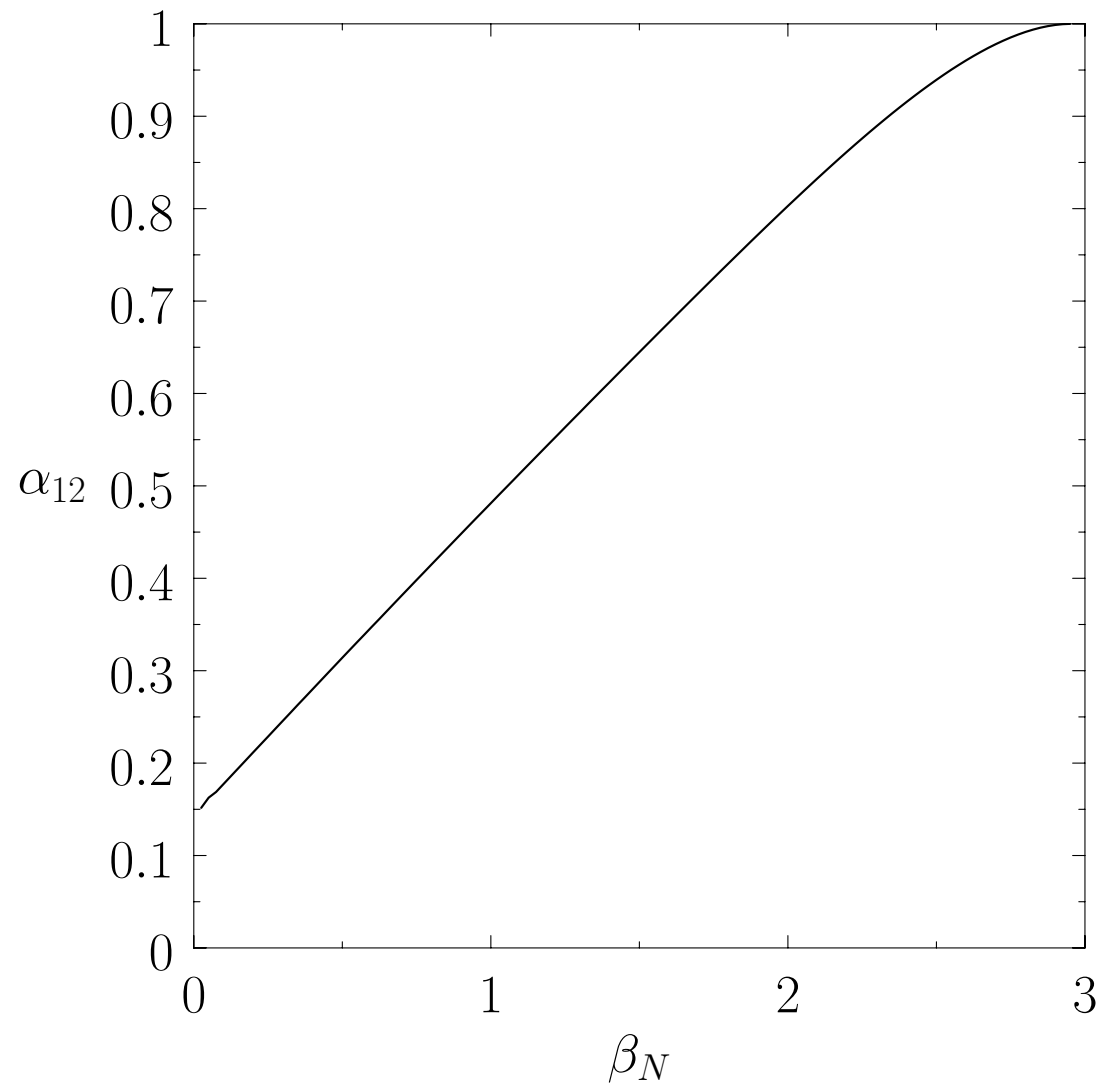
Optimum Wall Displacement: $q = 2$, High- β



Optimum Wall Displacement: $q = 3$, High- β



$q = 2/q = 3$ Overlap Parameter



Future Plans

- Include plasma rotation in linear GGJ layer response calculation.
- Generalize GGJ linear layer response model to equivalent nonlinear island response model, so as to determine threshold error-field strengths for penetration.
- Include two-fluid and low collisionality effects in linear layer response model.
- Use TOMUHAWC to calculate error-field induced drag torque due to neoclassical toroidal viscosity.