IFS Analytic Island Dynamics Model

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Introduction

- Over course of many years, IFS scientists have developed analytic, single-helicity, fully nonlinear, neoclassical, two-fluid model of magnetic island dynamics in quasi-cylindrical tokamak plasma. [a]
- Purpose of model to understand interaction of magnetic island with externally generated magnetic perturbation (e.g., error-field, sawtooth, ELM).
- Model used to investigate problems of great importance to DOE FES research program: e.g., penetration threshold for error-field driven locked modes; triggering of neoclassical tearing modes; ELM suppression via resonant magnetic perturbations.

[a] R.D. Hazeltine, M. Kotschenruether, P.J. Morrison, Phys. Fluids **28**, 2466 (1985); R. Fitzpatrick, F.L. Waelbroeck, Phys. Plasmas **12**, 022307 (2005); R. Fitzpatrick, F.L. Waelbroeck, Phys. Plasmas **16**, 072507 (2009); R. Fitzpatrick, Plasma Phys. Control. Fusion **54**, 094002 (2012); R. Fitzpatrick, Phys. Plasmas **25**, 042503 (2018).

Four-Field Model

$$\partial_t (w^2 \psi)/(w^2) = [\phi + \tau N, \psi] + \eta J,$$

$$\partial_t (w N)/w = [\phi, N] - \rho [\alpha_n V + J, \psi] - \alpha_c \rho [\phi + \tau N, X],$$

$$\partial_t V = [\phi, V] - \alpha_n (1 + \tau) [N, \psi],$$

$$\epsilon \partial_t (w \partial_X^2 \phi)/w = \epsilon \partial_X [\phi - N, \partial_X \phi] + [J, \psi] + \alpha_c (1 + \tau) [N, X],$$

$$J = \beta^{-1} \left(\partial_X^2 \psi - 1\right).$$

- Core of IFS model is nonlinear, reduced, two-fluid, toroidal "four-field" model of Hazeltine, Kotschenreuther, and Morrison (1985). Four fields: poloidal flux, ψ; electron number density, N; parallel ion velocity, V; scalar potential, ϕ. Perturbed parallel current, J, is auxillary field.

Turbulent Perpendicular Transport

$$\partial_t (w\, N)/w = [\phi, N] - \rho\, [\alpha_n\, V + J, \psi] - \alpha_c\, \rho\, [\phi + \tau\, N, X]$$
$$+ D\, \partial_X^2 N,$$
$$\partial_t V = [\phi, V] - \alpha_n\, (1 + \tau)\, [N, \psi] + \mu\, \partial_X^2 V,$$
$$\epsilon\, \partial_t (w\, \partial_X^2 \phi)/w = \epsilon\, \partial_X [\phi - N, \partial_X \phi] + [J, \psi] + \alpha_c\, (1 + \tau)\, [N, X]$$
$$+ \epsilon\, \mu\, \partial_X^4 (\phi - N).$$

- IFS core model augmented by phenomenological terms representing turbulent perpendicular transport of density (temperature) and ion momentum.

Neoclassical Viscosity

$$\partial_t(w^2\,\psi)/(w^2) = [\phi + \tau\,N, \psi] + \eta\,J$$

$$+\,\alpha_n^{-1}\,\hat{\nu}_{\theta\,e}[\alpha_n^{-1}\,J + V - \partial_X(\phi + \tau\,\nu_{\theta\,e}\,N) - \nu_{\theta\,i} - \tau\,\nu_{\theta\,e}],$$

$$\partial_t(w\,N)/w = [\phi, N] - \rho\,[\alpha_n\,V + J, \psi] - \alpha_c\,\rho\,[\phi + \tau\,N, X]$$

$$+\,D\,\partial_X^2 N,$$

$$\partial_t V = [\phi, V] - \alpha_n\,(1+\tau)\,[N, \psi] + \mu\,\partial_X^2 V$$

$$-\,\hat{\nu}_{\theta\,i}[V - \partial_X(\phi - \nu_{\theta\,i}\,N)],$$

$$\epsilon\,\partial_t(w\,\partial_X^2\phi)/w = \epsilon\,\partial_X[\phi - N, \partial_X\phi] + [J, \psi] + \alpha_c\,(1+\tau)\,[N, X]$$

$$+\,\epsilon\,\mu\,\partial_X^4(\phi - N) + \hat{\nu}_{\theta\,i}\,\partial_X[V - \partial_X(\phi - \nu_{\theta\,i}\,N)]$$

$$+\,\hat{\nu}_{\perp\,i}\,\partial_X[-\partial_X(\phi - \nu\,N)].$$

- Model completed by neoclassical electron/ion viscosity terms: represent bootstrap current, ion poloidal/toroidal flow damping.

Spatial Boundary Conditions

$$\psi(X, \zeta, \hat{t}) \rightarrow \frac{1}{2} X^2 + \cos\zeta,$$

$$\partial_X N(X, \zeta, \hat{t}) \rightarrow -1,$$

$$\partial_X \phi(X, \zeta, \hat{t}) \rightarrow -v,$$

$$\partial_X V(X, \zeta, \hat{t}) \rightarrow 0,$$

as $|X| \rightarrow \infty$.

- Spatial boundary conditions: single-helicity, constant-ψ, radially symmetric, magnetic island embedded in equilibrium with uniform local density gradient, and neoclassically-relaxed ion flow profile.

Asymptotic Matching

$$0 = \Delta' \, r_s + 2\, m_\theta \left(\frac{w_v}{w}\right)^2 \cos\phi_p + J_c\, \beta\, \frac{r_s}{w},$$

$$0 = -2\, m_\theta \left(\frac{w_v}{w}\right)^2 \sin\phi_p + J_s\, \beta\, \frac{r_s}{w},$$

$$J_c = -2 \int_{-\infty}^{\infty} J\, \cos\zeta\, dX\, \frac{d\zeta}{2\pi},$$

$$J_s = -2 \int_{-\infty}^{\infty} J\, \sin\zeta\, dX\, \frac{d\zeta}{2\pi}.$$

- Terms involving w_v incorporate externally-generated resonant magnetic perturbation into problem. First matching condition yields modified Rutherford equation; second matching condition yields island phase evolution equation.

Method of Solution

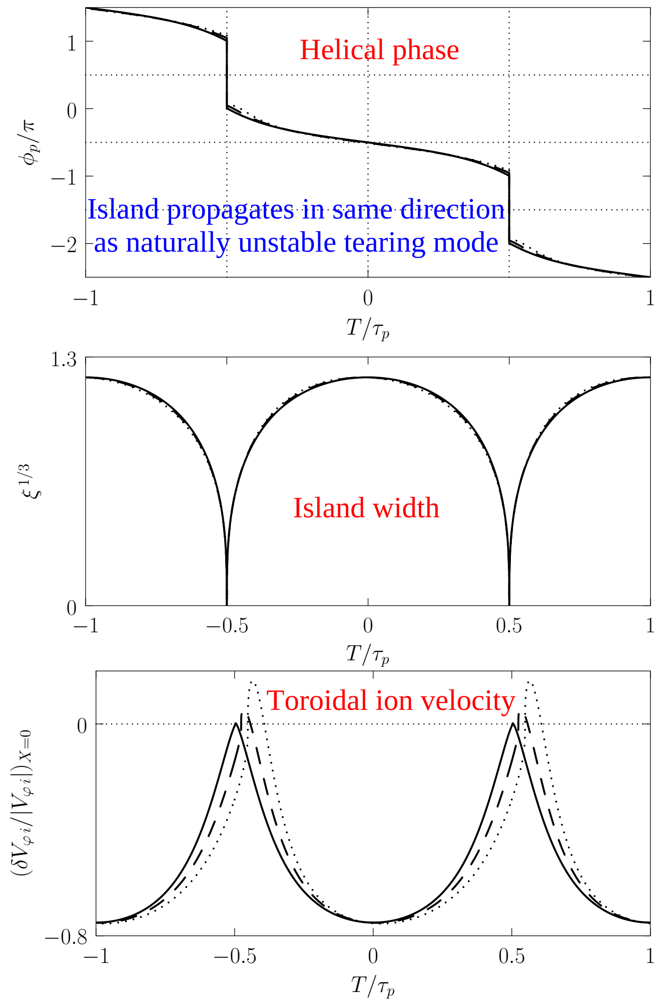
- Magnetic island is helical magnetic equilibrium that evolves on comparatively slow transport timescale.
- Method of solution analogous to that employed in solving global plasma equilibrium.
- Lowest-order force balance (ignore relatively small transport and neoclassical terms) reveals that $N = N(\psi)$, $\phi = \phi(\psi)$, $V = V(\psi)$, $J = J(\psi)$.
- In order to determine form of flux-surface functions must solve transport problem across island region: i.e., include relatively small transport and neoclassical terms; annihilate dominant force balance terms via suitable flux-surface averaging. (IFS model only analytic model that incorporates second step.)

Island Propagation Frequency

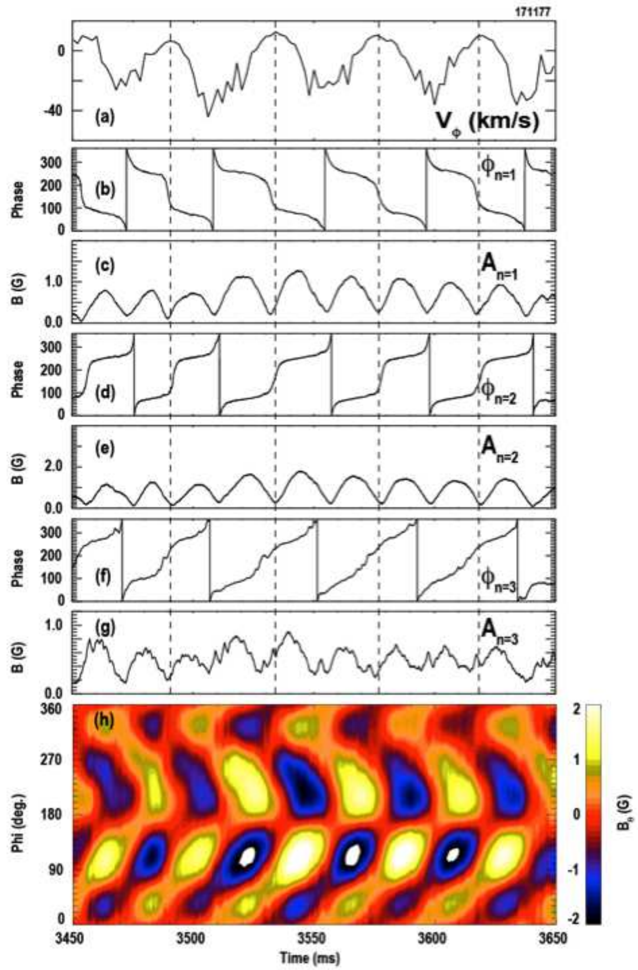
- Determination of $\phi(\psi)$ profile yields island propagation frequency relative to local ExB frame.
- Linear physics: tearing mode propagates in electron diamagnetic direction relative to local ExB frame.
- Extremely narrow linear layer widths in high temperature tokamak plasmas. Detectable tearing mode already in nonlinear regime.
- Nonlinear physics: island propagates in ion diamagnetic direction relative to local ExB frame.
- Experimental confirmation of ion diamagnetic rotation of magnetic islands.[a] US Fusion community do not seem to have got message.

[a] R.J. La Haye, et al., Phys. Plasmas **10**, 3644 (2003); P. Buratti, et al., Nucl. Fusion **56**, 076004 (2016).

Response of Tearing Stable Plasma to Static RMP



Recent DIII-D RMP ELM Suppression Data[a]



[a] R. Nazikian, et al., Submitted to Nuclear Fusion.

Discussion - I

- Data can be interpreted as showing pulsating magnetic island chains driven at separate $n = 1$, $n = 2$, $n = 3$ resonant surfaces in outer regions of plasma.

- Island phase velocities and ion toroidal velocity modulate in sync with island widths in manner predicted by IFS model.

- $n = 1$ island chain rotates in electron diamagnetic direction relative to lab frame; $n = 2$, $n = 3$ chains rotate in ion diamagnetic direction.

- Due to strong ExB shear in edge region, expect driven island chains resonant just inside pedestal to rotate in electron diamagnetic direction; island chains resonant further towards edge rotate in ion diamagnetic direction.

Discussion - II

- Data can only be explained on basis of nonlinear physics. Linear physics: magnetic flux driven at rational surface by static error-field cannot propagate, even in presence of plasma rotation.
- Minimum physics requirements for analysis of RMP ELM suppression data. Model must be resistive, rather than ideal; nonlinear, rather than linear; must incorporate neoclassical viscosity (otherwise get wrong island propagation frequency); must solve transport problem to determine island frequency self-consistently (otherwise cannot determine island frequency).

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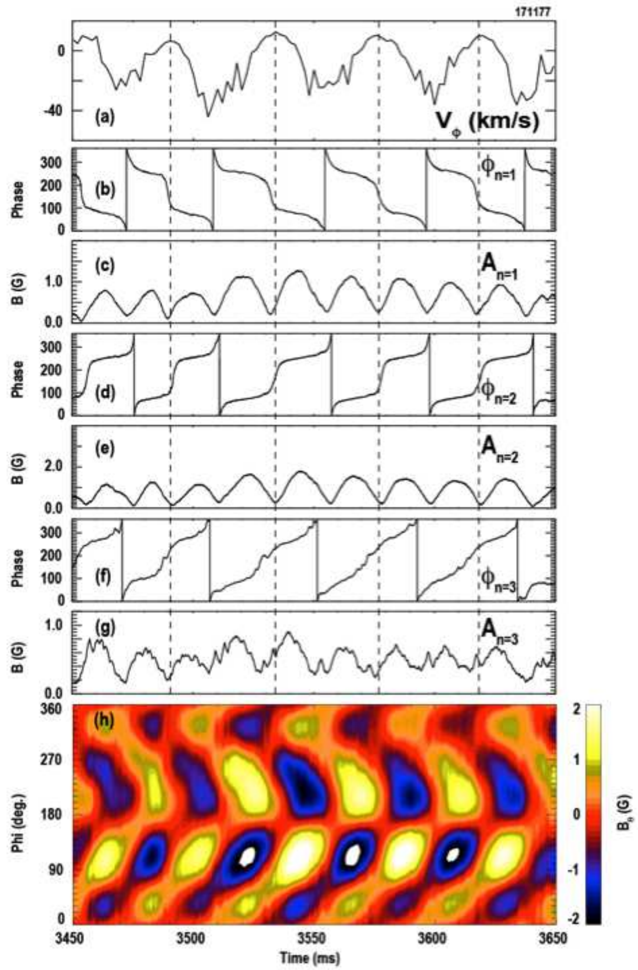

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