Linear and Nonlinear Response of a Rotating Tokamak Plasma to a Resonant Error-Field

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Normalized Linear Response Equations

$$\frac{dW}{dt} = \frac{W}{2}\left(-1 + \frac{W_v^2}{W^2}\cos\varphi\right),$$

$$\frac{d\varphi}{dt} - \omega = -\frac{W_v^2}{W^2}\sin\varphi.$$

- W - island width. W_v - error-field strength. φ - island helical phase. ω - plasma rotation. t - time.
- First equation governs time evolution of island width, second governs evolution of island phase.
- Linear equations only valid when $W \ll 1$: i.e., when island width less than linear layer width.

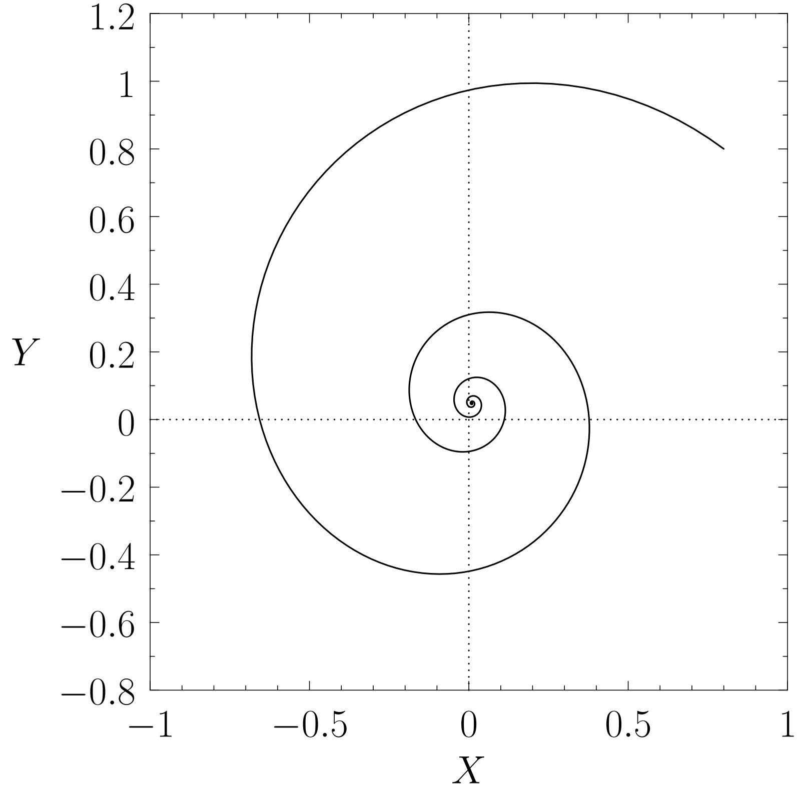
Normalized Nonlinear Response Equations

$$\frac{dW}{dt} = \frac{1}{2}\left(-1 + \frac{W_\nu^2}{W^2}\cos\varphi\right),$$

$$\frac{d\varphi}{dt} - \omega = 0.$$

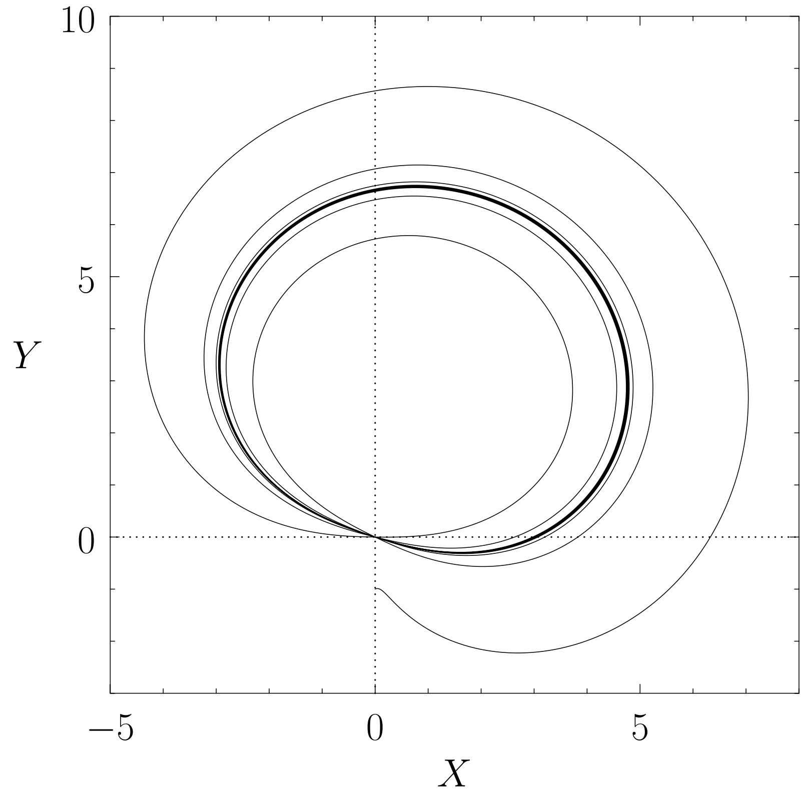
- Island width evolution equation surprisingly similar to corresponding linear equation.
- Island phase evolution equation, which encapsulates "no slip" constraint, quite different to corresponding linear equation.
- Nonlinear equations only valid when $W \gg 1$.

Linear Evolution of Reconnected Flux



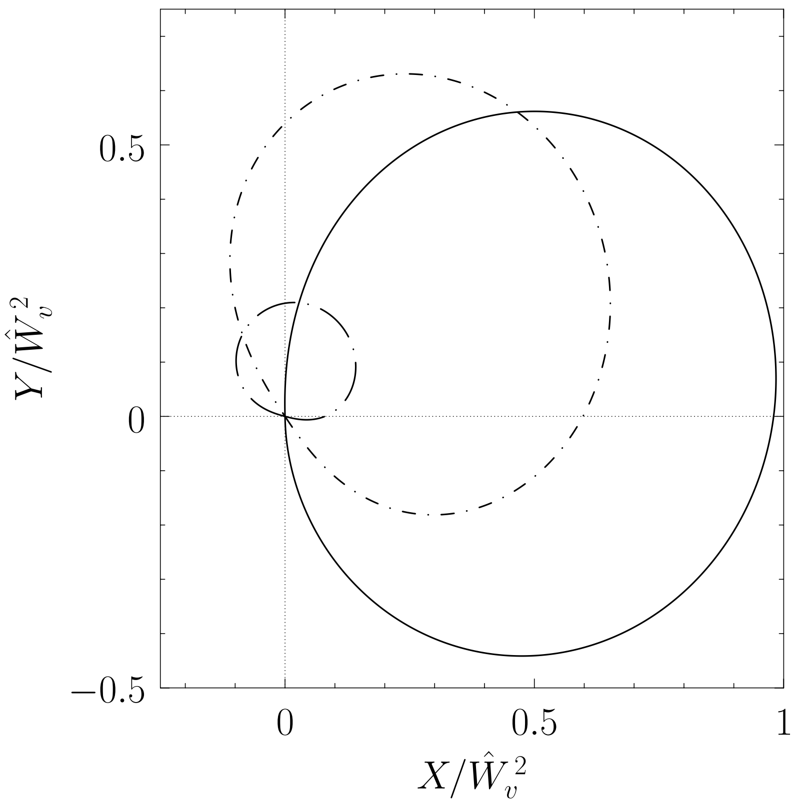
- $X = W \cos\varphi$, $Y = W \sin\varphi$. Evolution is to fixed point.

Nonlinear Evolution of Reconnected Flux

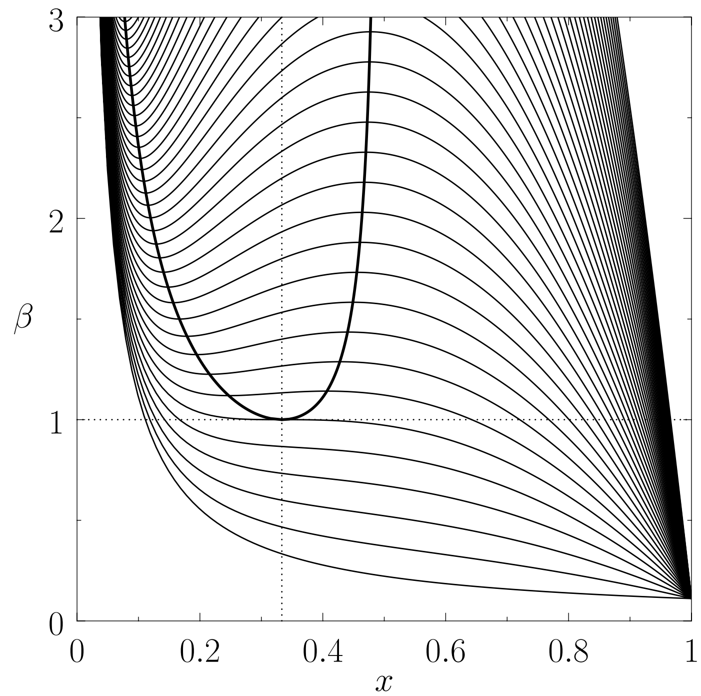


- Evolution is to limit cycle.

Limit Cycle Depends on Plasma Rotation Level

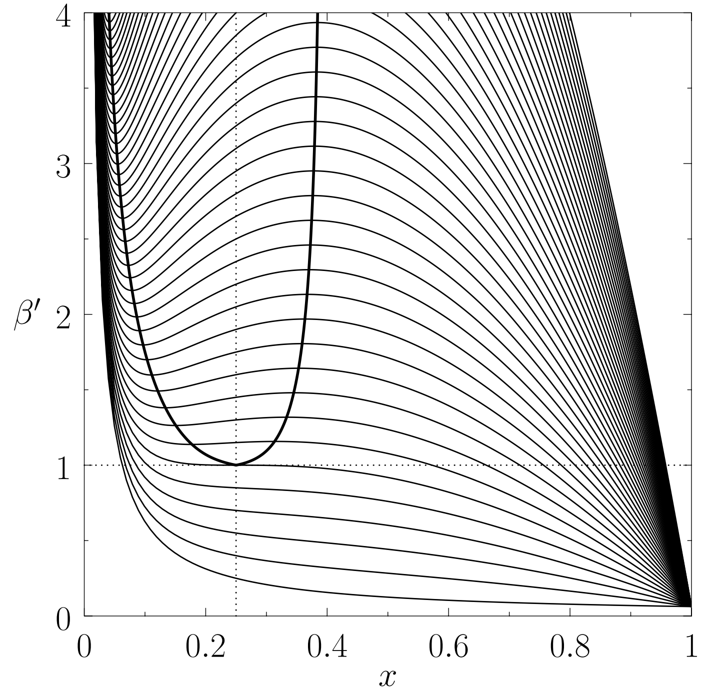


Torque-Balance Solutions in Linear Regime



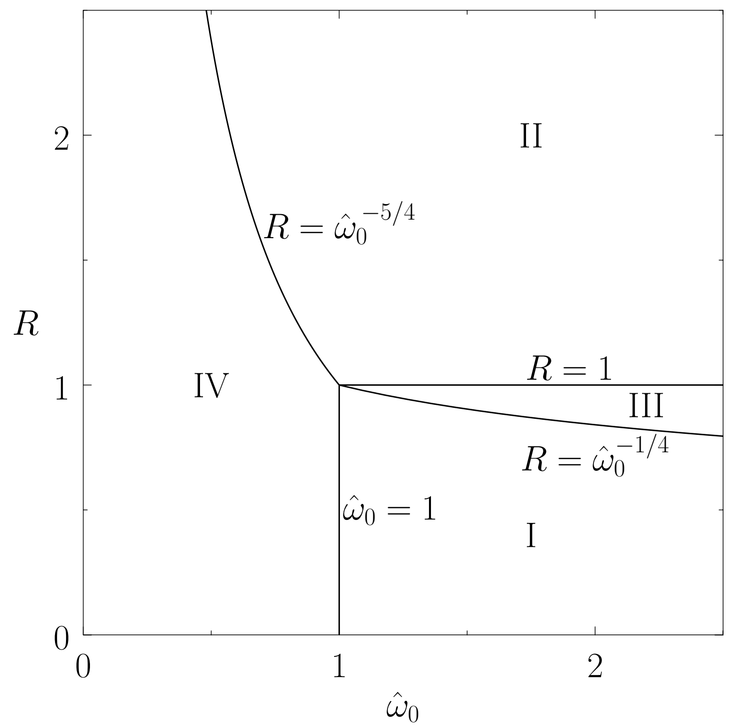
- x - plasma rotation. β - error-field amplitude.
- Solutions within thick curve are dynamically unstable.
- System exhibits bifurcations between high- and low-rotation states.

Torque-Balance Solutions in Nonlinear Regime



- x - plasma rotation. β' - error-field amplitude.
- Solutions within thick curve are dynamically unstable.
- System exhibits bifurcations between high- and low-rotation states.

Bifurcation Regimes



- $\hat{\omega}_0$ - natural plasma rotation. R - plasma resistivity.
- I: L $\leftrightarrow$ H bifurcations linear. II: L $\leftrightarrow$ H bifurcations nonlinear. III: L $\longrightarrow$ H bifurcation nonlinear. H $\longrightarrow$ L bifurcation linear. IV: No bifurcations.

Bifurcation Thresholds (no NC flow damping)

- $H \rightarrow L$ bifurcations in regimes I and III:

$$(b_r/B_T)_{\rm crit} \sim \zeta^{-13/3},$$

where $\zeta = B_T^{1/5} R_0^{1/4}$.

- $H \rightarrow L$ bifurcation in regime II:

$$(b_r/B_T)_{\rm crit} \sim \zeta^{-23/5}.$$

- $L \rightarrow H$ bifurcations in regimes I, II, and III:

$$(b_r/B_T)_{\rm crit} \sim \zeta^{-11/2}.$$

Bifurcation Thresholds (with NC flow damping)

- $H \longrightarrow L$ bifurcations in regimes I and III:

$$(b_r/B_T)_{crit} \sim \zeta^{-13/3}.$$

- $H \longrightarrow L$ bifurcation in regime II:

$$(b_r/B_T)_{crit} \sim \zeta^{-15/3}.$$

- $L \longrightarrow H$ bifurcations in regimes I, II, and III:

$$(b_r/B_T)_{crit} \sim \zeta^{-20/3}.$$

Summary

- General investigation of response of rotating (ohmically heated) tokamak plasma to resonant error-field.
- Both linear and nonlinear response regimes investigated.
- Neoclassical flow damping incorporated into analysis.
- Solutions exhibit bifurcations, but bifurcation thresholds have no dependence on plasma density.
- Calculation makes clear that observed linear scaling of error-field penetration threshold in ohmically heated plasmas can only be explained as a consequence of ion polarization current.