Analytic Nonlinear Neoclassical Two-Fluid Theory of Response of Tokamak Plasma to Resonant Error-Field

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Error-Field Driven Magnetic Reconnection

- Magnetic island chain propagates at fixed frequency (in ion diamagnetic direction[a]) w.r.t. $\mathbf{E}\times\mathbf{B}$ frame at resonant surface.
- Steady-state island chain maintained in plasma by error-field must have fixed phase relation w.r.t. resonant component of error-field: i.e., island chain must be static.
- Implies that plasma rotation at resonant surface modified s.t. natural frequency of island chain (i.e., $\mathbf{E}\times\mathbf{B}$ frequency plus diamagnetic offset) is zero.
- Error-field exerts e.m. locking torque at resonant surface to effect change in plasma rotation. Rotation change resisted by plasma ion viscosity and flow damping.

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[a] R. Fitzpatrick, PoP **25**, 042503 (2018); R.J. La Haye, et al., PoP **10**, 3644 (2003); P. Buratti, et al., NF **56**, 076004 (2016).

Suppression of Error-Field Driven Reconnection

- Q: What happens if error-field cannot exert requisite e.m. locking torque at resonant surface?
- A: Driven magnetic reconnection at resonant surface suppressed by large inductive current sheet.[a]
- Current sheet also gives rise to e.m. slowing-down torque acting on resonant surface. Torque insufficient to reduce natural frequency of island chain to zero.

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[a] R. Fitzpatrick, NF **33**, 1049 (1993).

Linear Model of Suppressed State

- Can model suppressed state using linear layer theory.[a]
- According to linear theory (and unlike nonlinear theory), plasma flow does not necessarily convect reconnected magnetic flux at resonant surface.
- Reconnected flux phase-locked to resonant component of error-field: i.e., flux non-rotating.
- Plasma flow relative to flux gives rise to steady phase-shift of flux w.r.t. resonant component of error-field.
- Current in linear layer partially suppresses magnetic reconnection; also gives rise to slowing-down e.m. torque at resonant surface.

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[a] R. Fitzpatrick, NF **33**, 1049 (1993); R. Fitzpatrick, PoP **5**, 3225 (1998); A. Cole and RF, PoP **13**, 032503 (2006).

Problems with Linear Model of Suppressed State

- Linear layer theory invalidated as soon as island width (i.e., width of magnetic separatrix) exceeds layer width.
- Width of magnetic separatrix scales as square-root of reconnected magnetic flux: i.e., small amount of residual reconnection at resonant surface can produce wide island.
- Linear layer width scales as $S^{-1/4}$. Layer width minuscule in high temperature tokamak plasma.
- Totally implausible that driven reconnection suppressed to such extent that island width falls below linear layer width. Need nonlinear model of suppressed state.

Nonlinear Model of Suppressed State[a]

- Impossible for island chain to have fixed phase relation w.r.t. resonant component of error-field if natural frequency nonzero.
- Suppose phase instantaneously destabilizing. Island width grows.
- Nonzero natural frequency causes island chain to propagate. Chain eventually acquires stabilizing phase. Island width shrinks.
- If island width shrinks to zero, X-points $\leftrightarrow$ O-points; equivalent to 180° phase jump. Converts stabilizing phase relation into destabilizing one. Island width grows again.
- Error-field drives pulsating island chain whose width periodically falls to zero. Associated phase jumps allow average phase relation to remain destabilizing, despite propagation of chain.

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[a] R. Fitzpatrick, PoP **5**, 3225 (1998).

IFS Island Dynamics Model

- Over course of many years, IFS scientists have developed analytic, single-helicity, fully nonlinear, neoclassical, two-fluid model of magnetic island dynamics in quasi-cylindrical tokamak plasma.[a]
- Model solves self-consistently for island width, island phase, density/temperature profiles, plasma flow profiles, and current profile in vicinity of resonant surface. Island frequency w.r.t. $\mathbf{E} \times \mathbf{B}$ frame calculated, rather than assumed.
- Can use model to investigate nonlinear suppressed state.

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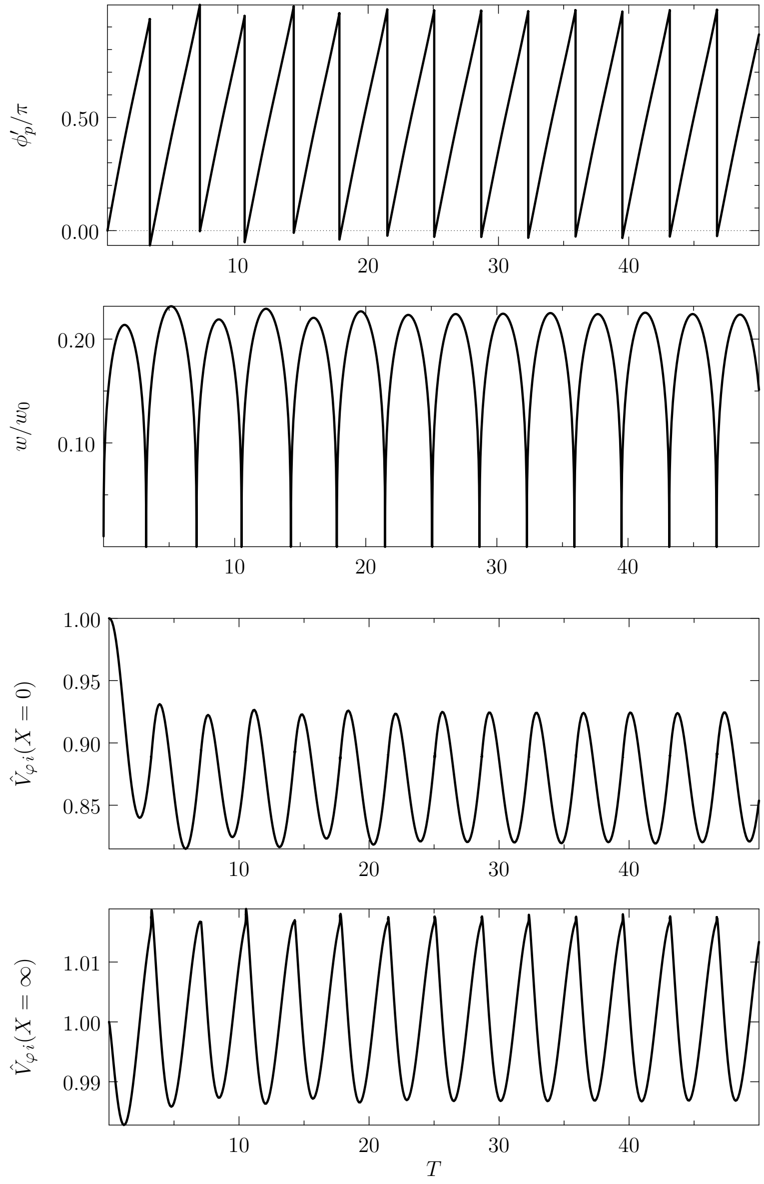
[a] R.D. Hazeltine, M. Kotschenruether, P.J. Morrison, PoF **28**, 2466 (1985); R. Fitzpatrick, F.L. Waelbroeck, PoP **12**, 022307 (2005); RF, FLW, PoP **16**, 072507 (2009); RF, PPCF **54**, 094002 (2012); RF, PoP **25**, 042503 (2018).

Static Error Field Below Penetration Threshold



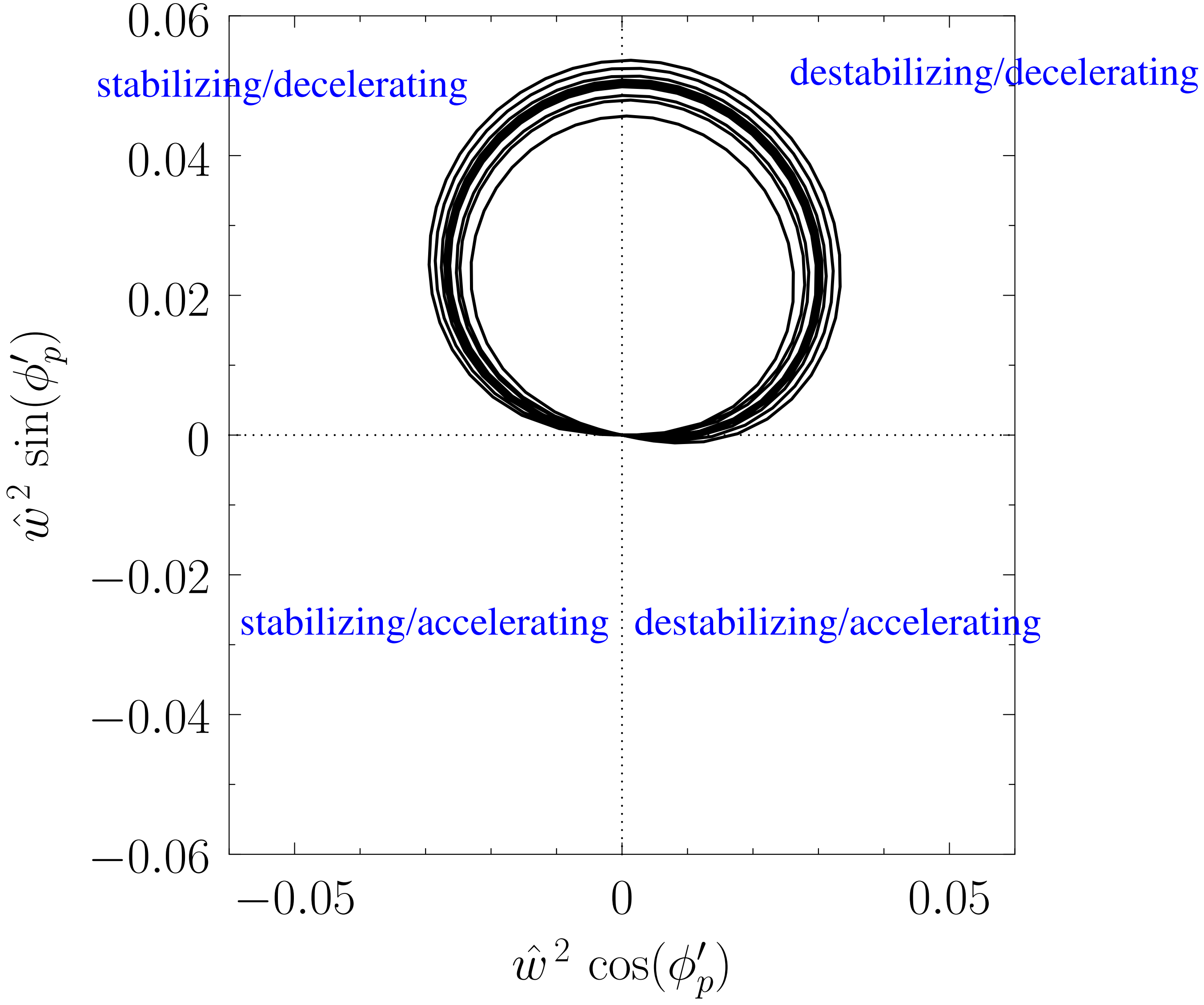
Helical phase

Normalized island width

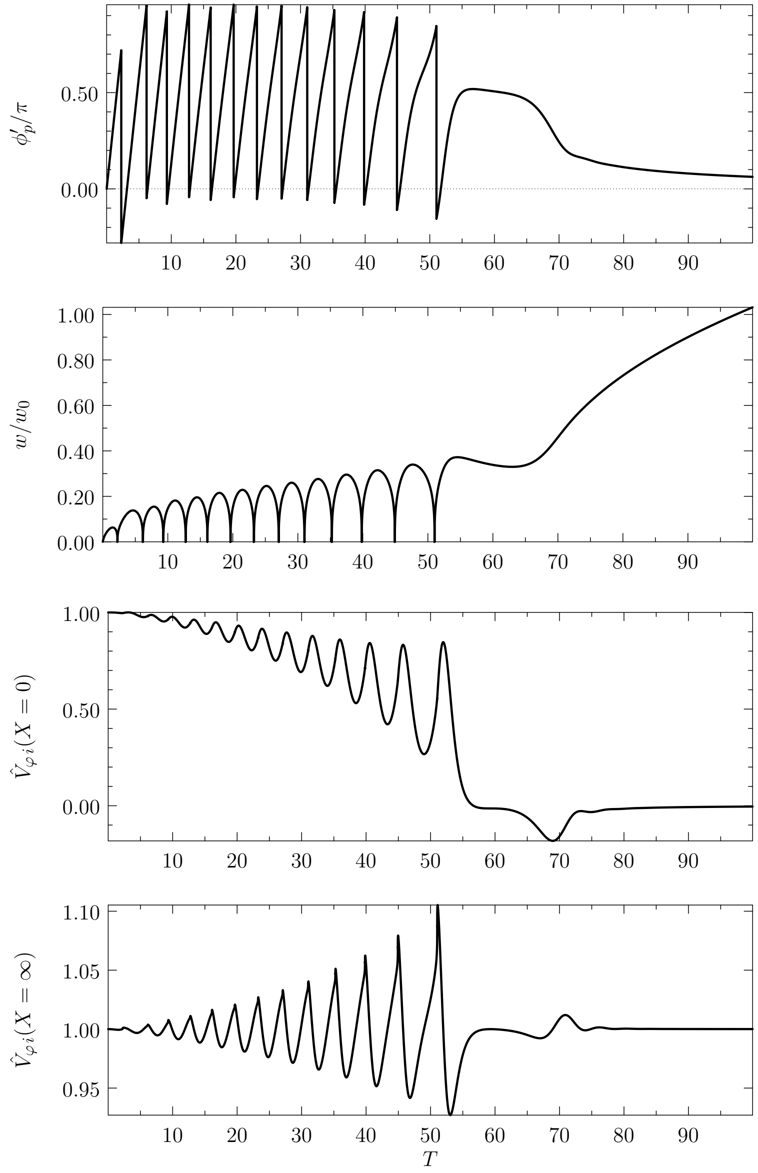
Toroidal ion velocity
inside separatrix

Toroidal ion velocity
many island widths
from separatrix

Phase−Space Trajectory



Error−Field Ramped Through Penetration Threshold



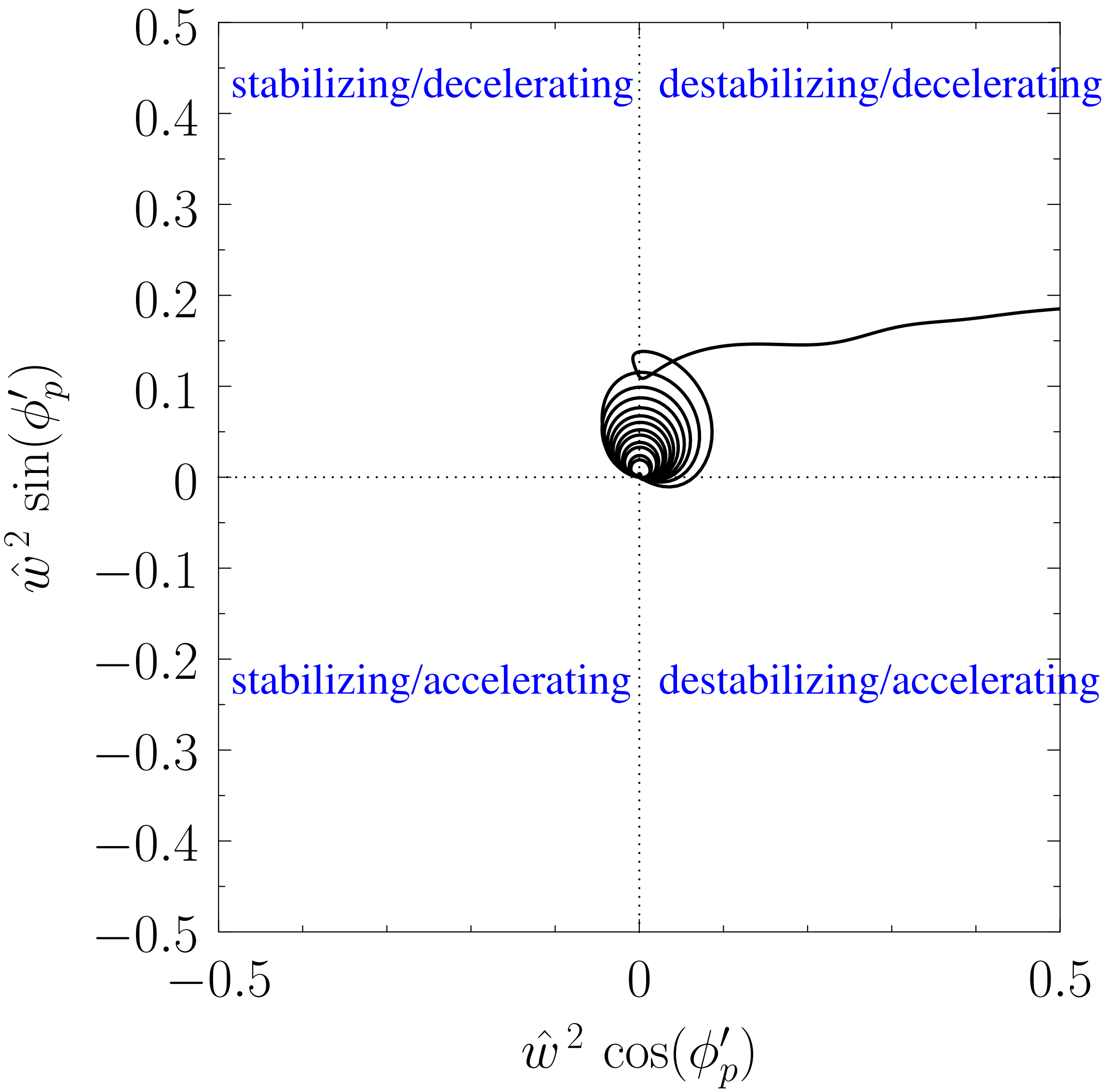
Helical phase

Normalized island width

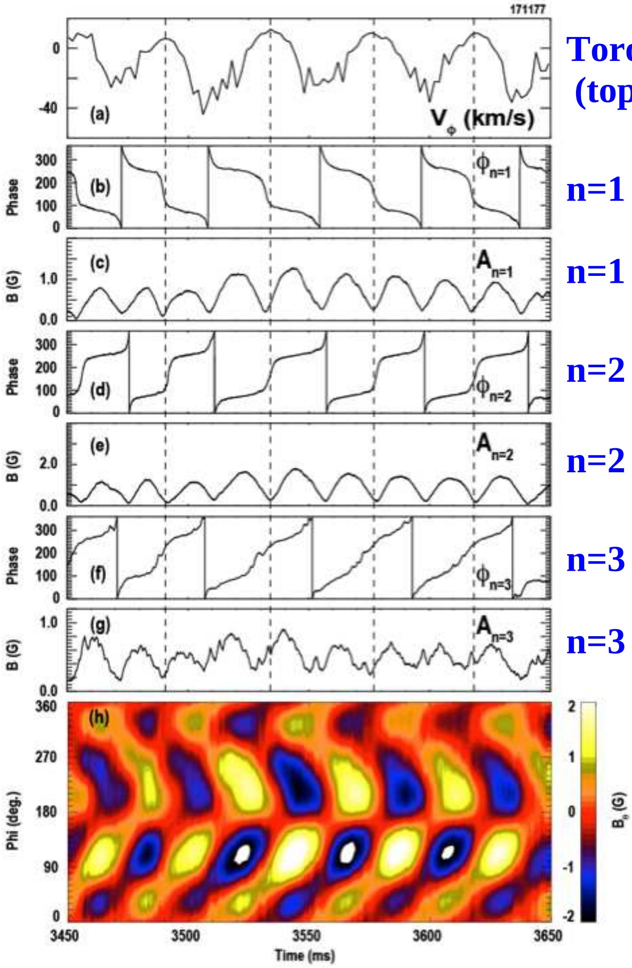
Toroidal ion velocity
inside separatrix

Toroidal ion velocity
many island widths
from separatrix

Phase−Space Trajectory



Recent DIII-D RMP ELM Suppression Data[a]



Toroidal ion velocity
(top of pedestal)

n=1 phase

n=1 amplitude

n=2 phase

n=2 amplitude

n=3 phase

n=3 amplitude

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[a] R. Nazikian, et al., to appear in Nuclear Fusion (2018).

Discussion

- DIII-D data can be interpreted as showing pulsating magnetic island chains driven at separate $n = 1$, $n = 2$, $n = 3$ resonant surfaces in outer regions of plasma.

- Island phase velocities and ion toroidal velocity modulate in sync with island widths in manner predicted by IFS model.

- $n = 1$ island chain rotates in electron diamagnetic direction relative to lab frame; $n = 2$, $n = 3$ chains rotate in ion diamagnetic direction.

- Due to strong ExB shear in edge region, expect driven island chains resonant just inside pedestal to rotate in electron diamagnetic direction; island chains resonant further towards edge rotate in ion diamagnetic direction.

Conclusions

- DIII-D data can only be explained on basis of nonlinear physics. According to linear physics, magnetic flux driven at resonant surface by static error-field cannot propagate, even in presence of plasma rotation.
- Minimum physics requirements for analysis of RMP ELM suppression data. Model must be resistive, rather than ideal; two-fluid, rather than single-fluid; nonlinear, rather than linear; must incorporate neoclassical viscosity (otherwise get wrong island propagation frequency); must determine island frequency self-consistently.