Investigation of Neoclassical Tearing Mode Detection by ECE Radiometry in Tokamaks via Asymptotic Matching Techniques [1]

Richard Fitzpatrick

Institute of Fusion Studies, Department of Physics, University of Texas at Austin

[1] arXiv:2506.05553

Disruption Avoidance

- Next generation tokamaks, such as ITER, need to avoid disruptions (i.e., sudden catastrophic losses of plasma confinement) which could severely damage devices.
- Disruptions can be avoided by keeping net toroidal plasma current, mean plasma pressure, and mean plasma density, below well-known limiting values.[2]
- However, there is one type of disruption (i.e., that associated with neoclassical tearing modes) that cannot be passively avoided during normal operation.
- Instead, active measures are needed to prevent NTM-triggered disruptions.

[2] P.C. de Vries, et al. Nucl. Fusion **51**, 053018 (2011).

Neoclassical Tearing Modes

- Nor surprisingly, neoclassical tearing modes (NTMs) are observed to be main cause of disruptions in ITER-relevant discharges in present-day tokamaks.[3]
- NTMs are linearly stable, but are destabilized nonlinearly by loss of non-inductive bootstrap current, due to flattening of pressure profile within magnetic separatrix of associated island chain.
- NTMs are triggered in fairly random fashion by otherwise benign plasma instabilities such as sawtooth crashes.
- Once triggered, NTMs in ITER predicted to grow as rotating modes on timescale of about 3 s, eventually lock to vacuum vessel when $W/a \sim 0.08$, and then trigger disruption.[4]
- Remedial action must be taken prior to locking!

[3] R.J. La Haye, Phys. Plasmas **13**, 055501 (2006).
[4] R. Fitzpatrick. Phys. Plasmas **30**, 042514 (2023).

Electron Cyclotron Current Drive

- NTMs can be stabilized by means of localized electron cyclotron current drive (ECCD) that is targeted at one of O-points of NTM island chain.[5] Idea is to replace missing bootstrap current.
- ECCD driven by suitably phased and directed electron cyclotron waves injected into plasma.[6]
- Early detection of NTM island chain, together with accurate location of island O-points, is key to successful disruption avoidance by means of ECCD.

[5] G. Gantenbien, et al. Phys. Rev. Lett. **85**, 1242 (2000).
[6] R. Prater, Phys. Plasmas **13**, 055501 (2006).

Electron Cyclotron Emission

- Tokamak plasma spontaneously emit electron cyclotron waves.[7]
- Intensity of electron cyclotron emission (ECE) (almost exactly) proportion to electron temperature of emitting region.
- Frequency of electron cyclotron emission (ECE) depends mostly on magnetic field-strength, which is (almost exactly) inversely proportion to major radius of emitting region.
- Hence, ECE emission can be used to simultaneously detect local flattening of electron temperature due NTM island chain, and accurately determine major radius of chain.

[7] M. Bornatici, et al. Nucl. Fusion **23**, 1153 (1983).

Synthetic ECE Diagnostics

- Accurate modeling of expected ECE signal due to NTM is important component of design of ECCD suppression system.
- In existing synthetic ECE diagnostics, a single-harmonic, radially-symmetric, NTM island chain, centered on rational surface, is crudely inserted into plasma equilibrium.[8]
- In reality, an NTM consists of multiple helical harmonics.
- Harmonics with same toroidal mode number as NTM, but different poloidal mode numbers, are coupled linearly via toroidicity and flux-surface shaping throughout plasma.
- Harmonics whose poloidal and toroidal mode numbers are in same ratio as those of NTM are coupled nonlinearly in immediate vicinity of island chain.
- NTM island chains are radially asymmetric due to mean gradient in tearing eigenfunction at rational surface.

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[8] J.P. Ziegel, et al., Nucl. Fusion **64**, 126032 (2024).

Determining Magnetic Structure of NTM

- By far, most efficient method of determining stability and structure of NTM is via asymptotic matching.[9]
- In this approach, plasma divided into outer region, that comprises most of plasma, and inner region that is localized in vicinity of NTM rational surface.
- Tearing perturbation in outer region determined by solving linearized, marginally-stable, ideal-MHD equations in full toroidal geometry (e.g. using TJ code.[10])
- Tearing perturbation in inner region consists of nonlinear, radially-asymmetric, island equilibrium.
- Solutions in inner and outer regions asymptotically matched to one another at boundary between two regions to determine properties of island chain (e.g., radial asymmetry) in terms of tearing eigenfunction in outer region.

[9] H.P. Furth, J. Killeen and M.N. Rosenbluth, Phys. Fluids **6**, 459 (1963).
[10] R. Fitzpatrick, Phys. Plasmas **31**, 102507 (2024).

Determining Electron Temperature Perturbation

- No change in topology of magnetic flux-surfaces in outer region. Electron temperature assumed to be passively convected by plasma. Hence, perturbed temperature is minus product of radial plasma displacement and equilibrium temperature gradient.
- In inner region, electron temperature assumed to be constant on island magnetic flux-surfaces. Temperature profile determined by solution of $\langle\nabla^2 T_e\rangle = 0$, where $\langle\cdots\rangle$ denotes island flux-surface average.
- Global temperature profile determined by asymptotically matching temperatures in inner and outer region at boundary between two regions. Determines reduction in core temperature due to island chain.

Asymmetric Magnetic Island - I

- Let $x = r - r_s$, $X = x/W$, and $\zeta = m\,\theta - n\,\phi$, where W is full width of NTM island chain's magnetic separatrix.
- If island magnetic flux-surfaces are contours of $\Omega(X, \zeta)$ then equilibrium force-balance yields

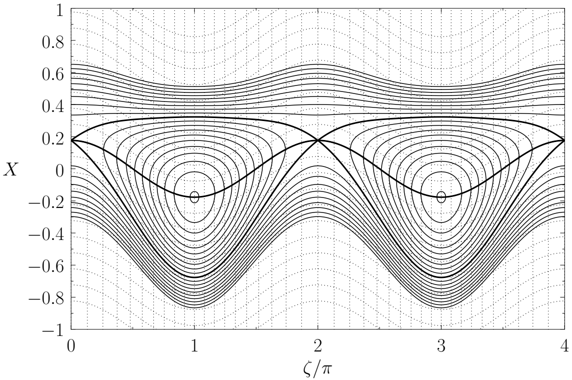
$$\left[\left.\frac{\partial^2\Omega}{\partial X^2}\right|_\zeta, \Omega\right] = 0,$$

where

$$[A, B] \equiv \left.\frac{\partial A}{\partial X}\right|_\zeta \left.\frac{\partial B}{\partial \zeta}\right|_X - \left.\frac{\partial B}{\partial X}\right|_\zeta \left.\frac{\partial A}{\partial \zeta}\right|_X.$$

- This requirement stipulates that the current density in the island region must be constant on magnetic flux-surfaces.

Asymmetric Magnetic Island - II

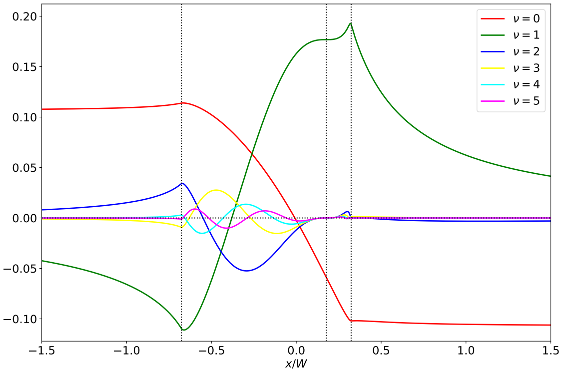


- Suitable solution is

$$\Omega(X,\zeta) = 8\,X^2 + \cos(\zeta - \delta^2\,\sin\zeta) - 2\sqrt{8}\,\delta\,X\,\cos\zeta + \delta^2\,\cos^2\zeta.$$

- Free parameter δ determines degree of radial asymmetry.

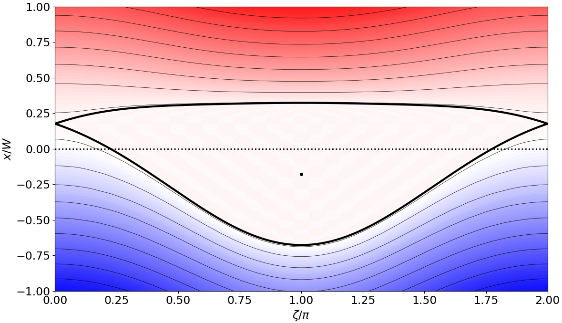
Asymmetric Magnetic Island - III



- Perturbed electron temperature in island region written

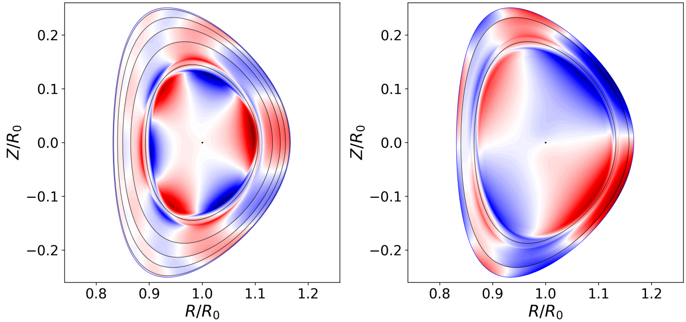
$$\delta T_e(X,\zeta) = \delta T_0(x) + \sum_{\nu>0} \delta T_\nu(X)\,\cos(\nu\,\zeta).$$

Asymmetric Magnetic Island - IV



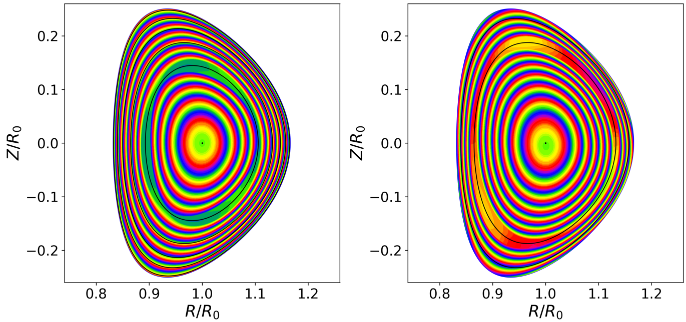
- Thick curve: island separatrix. Dot: island O-point. Dashed line: rational surface. Contours show $T_e(X, \zeta)$.
- Note that island O-point, which is true ECCD target, is shifted radially inward from rational surface.

Perturbed Electron Temperature



- Left/right panels: $3,2/2,1$ NTMs. Black curves: toroidally coupled rational surfaces. Black dot: magnetic axis.
- Realistic temperature perturbation associated with NTM is surprisingly complicated.

Total Electron Temperature



- Nevertheless, when perturbed electron temperature added to equilibrium temperature, result is appropriate helical flat-spot at NTM rational surface.

ECE Signal - I

- In optically thick plasma, intensity of ECE emission directly proportional to electron temperature.
- Angular frequency of jth harmonic ECE signal is

$$\omega = \frac{j\,\Omega_0\,R_0}{R}\left[1-\left(\frac{v}{c}\right)^2\right]^{1/2},$$

  where $\Omega_0 = e\,B_0/m_e$, B_0 is on-axis toroidal magnetic field-strength, R_0 is major radius of magnetic axis, and v is electron speed.
- Let

$$R_\omega(\omega) = \frac{j\,\Omega_0\,R_0}{\omega}$$

  be the major radius from which ECE of frequency ω is emitted in absence of relativistic mass increase. R is actual major radius from which signal emitted.

ECE Signal - II

- It follows that

$$\frac{v}{c} = \begin{cases} \left[1 - \left(\frac{R}{R_\omega}\right)^2\right]^{1/2} & R \leq R_\omega \\ 0 & R > R_\omega \end{cases} .$$

- Distribution of electron speeds is

$$f(v) = A\, v^2 \exp\left(-\frac{1}{\theta_\omega}\left[1 - \left(\frac{v}{c}\right)^2\right]^{-1/2}\right),$$

where $\theta_\omega(\omega) = T_e(R_\omega)/(m_e\, c^2) \ll 1$.

ECE Signal - III

- Previous two equations allow us to define

$$F(R, R_\omega) = \left[1 - \left(\frac{R}{R_\omega}\right)^2\right] \exp\left(-\frac{1}{\theta_\omega}\frac{R_\omega}{R}\right).$$

- Electron temperature measured by ECE diagnostic is convolution of actual signal, $T_e(R)$, and function $F(R, R_\omega)$:

$$T_e(R_\omega) = \frac{\int_{R_{\mathrm{min}}}^{R_\omega} T_e(R)\, F(R, R_\omega)\, dR}{\int_{R_{\mathrm{min}}}^{R_\omega} F(R, R_\omega)\, dR}.$$

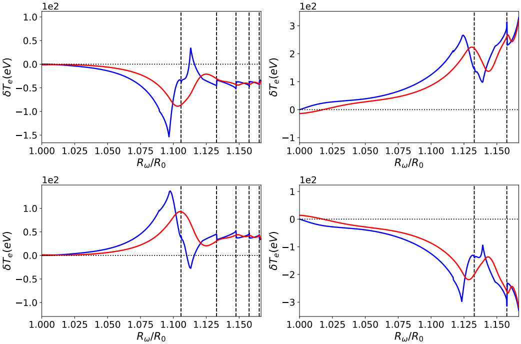
ECE Signal - IV

- Convolution specifies distortion of ECE signal due to relativistic mass increase.
- Taylor expanding:

$$T_e(R_\omega) = T_e(R_\omega) - 2\,\theta_\omega\left(1 - \frac{13}{2}\,\theta_\omega\right) R_\omega\,\frac{dT_e(R_\omega)}{dR} + 3\,\theta_\omega^2\,R_\omega^2\,\frac{d^2\,T_e(R_\omega)}{dR^2} + \mathcal{O}(\theta_\omega^3).$$

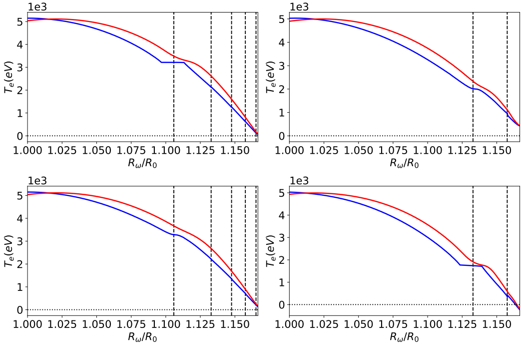
- To first order in θ_ω, measured temperature profile is $T_e[R_\omega\,(1 - 2\,\theta_\omega)]$: i.e., measured profile shifted outward in major radius distance $2\,\theta_\omega\,R_\omega$.
- To second order, measured profile smeared out in major radius.

Synthetic ECE Diagnostic - I



- Left/right-panels: 3,2/2,1 NTMs. Top/bottom panels: two different toroidal angles. Red/blue curves: distorted/ undistorted δT_e ECE signals. Dashed lines: rational surfaces.
- Relativistic mass increase shifts inferred location of ECE signal outward in major radius, and also smears out signal.

Synthetic ECE Diagnostic - II



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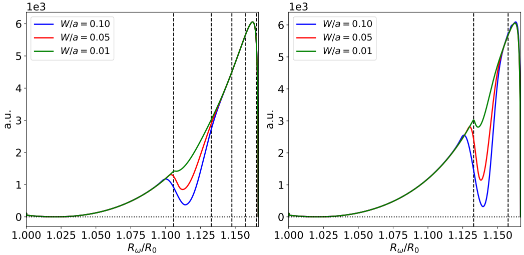
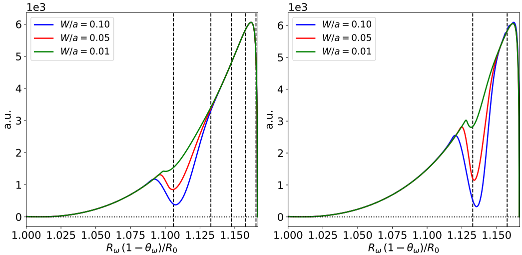
Synthetic Berrino Algorithm - I



- Synthetic Berrino algorithm[11] (radial gradient of ECE signal averaged over toroidal angle) gives clear signal for NTM islands whose widths are as small as 1% of minor radius.
- Minimum of Berrino signal (which is usually taken as target radius for ECCD) shifted outward in major radius due to relativistic mass increase.

[11] J. Berrino, et al, Nucl. Fusion **45**, 1350 (2005).

Synthetic Berrino Algorithm - II



- If inferred major radius of signal taken to be $R_\omega\,(1-\theta_\omega)$, instead of R_ω, then minimum in Berrino signal gives much better indication of location of rational surface.
- However, signal still need to be corrected for inward shift of island O-points.

Conclusions

- Asymptotic matching techniques permit rapid and realistic calculation of ECE signal generated by an NTM.
- Asymptotic matching techniques can calculate magnetic, temperature, and density perturbations associated with tearing mode in both linear and nonlinear regimes.
- As such, asymptotic matching techniques could be used to simulate any diagnostic used to study tearing modes.