Investigation of Neoclassical Tearing Mode Detection by ECE Radiometry in ITER via Asymptotic Matching Techniques[1]

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Neoclassical Tearing Modes

- ITER needs to avoid disruptions.
- Neoclassical tearing modes (NTMs) are main cause of disruptions in ITER-relevant discharges in present-day tokamaks.
- NTMs can be stabilized by means of localized electron cyclotron current drive (ECCD) that is targeted at one of O-points of NTM island chain.
- Early detection of NTM island chain, combined with accurate location of O-points, is key to successful disruption avoidance.

Electron Cyclotron Emission

- Electron cyclotron emission (ECE) simultaneously detects NTM island chain, and accurately determines its location.
- In existing synthetic ECE diagnostics, a single-harmonic, radially-symmetric, NTM island chain, centered on rational surface, is crudely inserted into plasma equilibrium.
- In reality, an NTM consists of multiple helical harmonics.
- Harmonics with same toroidal mode number as NTM, but different poloidal mode numbers, are coupled linearly via toroidicity and flux-surface shaping throughout plasma.
- Harmonics whose poloidal and toroidal mode numbers are in same ratio as those of NTM are coupled nonlinearly in immediate vicinity of island chain.
- NTM island chains are radially asymmetric due to mean gradient in tearing eigenfunction at rational surface.

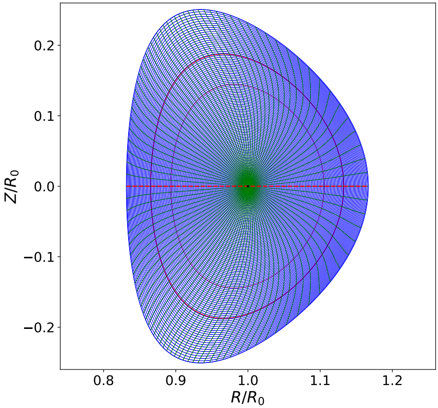
Determining Magnetic Structure of NTM

- In asymptotic matching approach, plasma divided into outer region, that comprises most of plasma, and inner region that is localized in vicinity of NTM rational surface.
- Tearing perturbation in outer region determined by solving linearized, marginally-stable, ideal-MHD equations in full toroidal geometry (using TJ toroidal tearing mode code).
- Tearing perturbation in inner region consists of nonlinear, radially-asymmetric, island equilibrium.
- Solutions in inner and outer regions asymptotically matched to one another at boundary between two regions to determine properties of island chain (e.g., radial asymmetry) in terms of tearing eigenfunction in outer region.

Determining Electron Temperature Perturbation

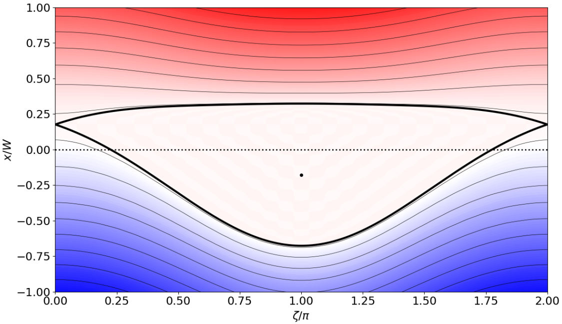
- No change in topology of magnetic flux-surfaces in outer region. Electron temperature assumed to be passively convected by plasma. Hence, perturbed temperature is minus product of radial plasma displacement and equilibrium temperature gradient.
- In inner region, electron temperature assumed to be constant on island magnetic flux-surfaces. Temperature profile determined by solution of $\langle\nabla^2 T_e\rangle = 0$, where $\langle\cdots\rangle$ denotes island flux-surface average.
- Global temperature profile determined by asymptotically matching temperatures in inner and outer region at boundary between two regions. Determines reduction in core temperature due to island chain.

Example Plasma Equilibrium



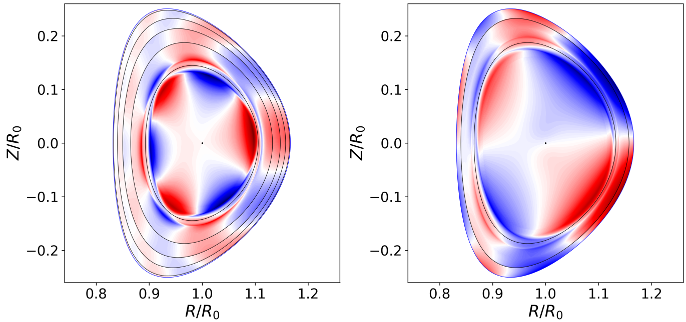
- Red curves show locations of $3, 2$ and $2, 1$ rational surfaces.
- Dashed line shows ECE measurement chord.

Asymmetric Magnetic Island



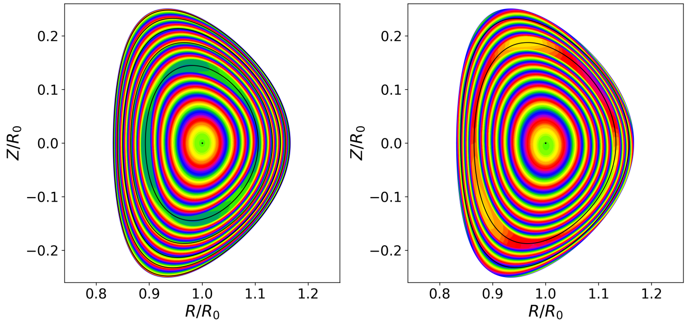
- $x = r - r_s$, $\zeta = m\,\theta - n\,\phi$, and W is island width. Thick curve: island separatrix. Dot: island O-point. Dashed line: rational surface. Contours show electron temperature distribution.
- Note that island O-point, which is true ECCD target, is shifted radially inward from rational surface.

Perturbed Electron Temperature



- Left/right panels: $3, 2/2, 1$ NTMs. Black curves: toroidally coupled rational surfaces. Black dot: magnetic axis.
- Realistic temperature perturbation associated with NTM is surprisingly complicated.

Total Electron Temperature



- Nevertheless, when perturbed electron temperature added to equilibrium temperature, result is appropriate helical flat-spot at NTM rational surface.

ECE Signal - I

- In optically thick plasma, intensity of ECE emission directly proportional to electron temperature.
- Angular frequency of jth harmonic ECE signal is

$$\omega = \frac{j\,\Omega_0\,R_0}{R}\left[1-\left(\frac{v}{c}\right)^2\right]^{1/2},$$

  where $\Omega_0 = e\,B_0/m_e$, B_0 is on-axis toroidal magnetic field-strength, R_0 is major radius of magnetic axis, and v is electron speed.
- Let

$$R_\omega(\omega) = \frac{j\,\Omega_0\,R_0}{\omega}$$

  be the major radius from which ECE of frequency ω is emitted in absence of relativistic mass increase. R is actual major radius from which signal emitted.

ECE Signal - II

- Distribution of electron speeds is

$$f(v) = A\,v^2 \exp\left(-\frac{1}{\theta_\omega}\left[1-\left(\frac{v}{c}\right)^2\right]^{-1/2}\right),$$

where $\theta_\omega(\omega) = T_e(R_\omega)/(m_e\,c^2) \ll 1$.

- Can define

$$F(R, R_\omega) = \left[1-\left(\frac{R}{R_\omega}\right)^2\right]\exp\left(-\frac{1}{\theta_\omega}\,\frac{R_\omega}{R}\right).$$

- Electron temperature measured by ECE diagnostic is convolution of actual signal, $T_e(R)$, and function $F(R, R_\omega)$:

$$T_e(R_\omega) = \frac{\int_{R_{\rm min}}^{R_\omega} T_e(R)\,F(R, R_\omega)\,dR}{\int_{R_{\rm min}}^{R_\omega} F(R, R_\omega)\,dR}.$$

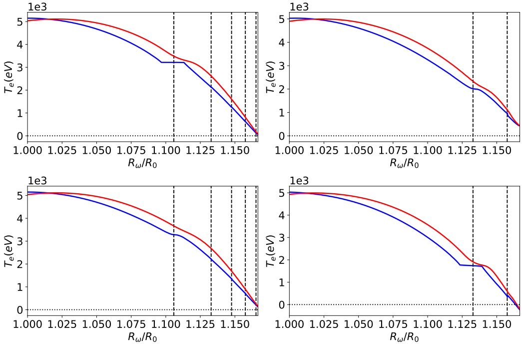
ECE Signal - III

- Convolution specifies distortion of ECE signal due to relativistic mass increase.
- Taylor expanding:

$$\begin{aligned} T_e(R_\omega) &= T_e(R_\omega) - 2\,\theta_\omega \left(1 - \frac{13}{2}\,\theta_\omega\right) R_\omega\,\frac{dT_e(R_\omega)}{dR} \\ &+3\,\theta_\omega^{\,2}\,R_\omega^{\,2}\,\frac{d^2\,T_e(R_\omega)}{dR^2} + \mathcal{O}(\theta_\omega^{\,3}). \end{aligned}$$

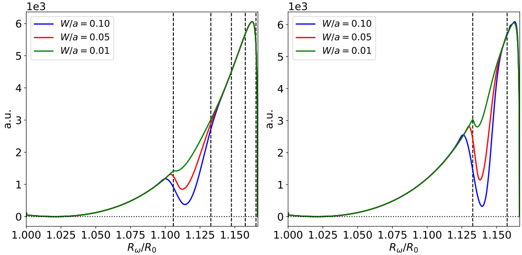
- To first order in θ_ω, measured temperature profile is $T_e[R_\omega\,(1-2\,\theta_\omega)]$: i.e., measured profile shifted outward in major radius distance $2\,\theta_\omega\,R_\omega$.
- To second order, measured profile smeared out in major radius.

Synthetic ECE Diagnostic



- Left/right-panels: 3,2/2,1 NTMs. Top/bottom panels: two different toroidal angles. Red/blue curves: distorted /undistorted ECE signals. Dashed lines: rational surfaces.
- Relativistic mass increase shifts inferred location of ECE signal outward in major radius, and also smears out signal.

Synthetic Berrino Algorithm



- Synthetic Berrino algorithm (radial gradient of ECE signal averaged over toroidal angle) gives clear signal for NTM islands whose widths are as small as 1% of minor radius.
- Minimum of Berrino signal (which is usually taken as target radius for ECCD) shifted outward in major radius due to relativistic mass increase.

Conclusions

- Asymptotic matching techniques permit rapid and realistic calculation of ECE signal generated by an NTM.
- Asymptotic matching techniques can calculate magnetic, temperature, and density perturbations associated with tearing mode in both linear and nonlinear regimes.
- As such, asymptotic matching techniques could be used to simulate any diagnostic used to study tearing modes.