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Linear stability of low mode number tearing modes in the banana collisionality regime

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The semicollisional layer equations governing the linear stability of small mode number tearing modes in a low beta, large aspect ratio, tokamak equilibrium are derived from an expansion of the gyrokinetic equation. In this analysis only the cases where the ion Larmor radius is either much less than, or much greater than, the layer width are considered. Both the electrons and the ions are assumed to lie in the banana collisionality regime. One interesting feature of the derived layer equations, in the limit of small ion Larmor radius, is a substantial reduction in the effective collisionality of the system due to neoclassical ion dynamics. Next, using a shooting code, a dispersion relation is obtained from the layer equations in the limits of small ion Larmor radius and a vanishingly small fraction of trapped particles. As expected, strong semicollisional stabilization of the mode is found, but, in addition, a somewhat weaker destabilizing effect is obtained in the transition region between the collisional and semicollisional regimes.

I. INTRODUCTION

A. The physics of linear tearing modes

Tearing modes are generally thought to play a significant role in determining the confinement properties of tokamak plasmas. In this paper we shall consider the behavior of linear tearing modes, where the island widths are assumed to be miniscule. As is well known, such modes consist of a thin resistive layer, which is centered on a "rational surface," embedded in an essentially ideal magnetohydrodynamic (MHD) plasma. Flow of plasma across magnetic field lines within the resistive layer induces a large parallel component of the electric field which, in turn, causes a large parallel layer current to flow. Such a thin current sheet can be unstable to filamentation along the direction of the field which entails, among other things, the tearing and reconnection of magnetic field lines to eventually produce magnetic islands.

The edge of the resistive layer is conventionally defined as the point at which the perturbed parallel current becomes negligible. It turns out that there are three distinct physical mechanisms for cutting-off the parallel current, each corresponding to one of the collisionality regimes employed in tearing mode theory. These are as follows.

(i) The collisionless regime: In this regime, finite magnetic shear at the rational surface causes "off-center" electrons, which are thermally streaming along magnetic field lines, to experience an ac, rather than a dc, driving force from the perturbed parallel electric field. For points that lie far enough away from the center of the layer, the frequency of oscillation of this driving force is far too large for the electrons to make any effective response, leading to a cutoff in the perturbed parallel current. We typically have $\omega \sim k_{\parallel} v_{\rm th}$ at the edge of a collisional layer. Here, ω is the mode frequency, $k_{\parallel}$ is the parallel wave vector, and $v_{\rm th}$ is the electron thermal velocity.

(ii) The semicollisional regime: This regime is entirely analogous to the previous regime, except that the electron motion along field lines is not free-streaming but, instead, random-walk diffusion due to collisions with other particles. Typically, we have $\omega \sim (k_{\parallel} v_{\rm th})^2/\nu_c$ at the edge of a semicollisional layer, where ν_c is the electron collision frequency.

(iii) The collisional regime: In this regime, electron motion along field lines is essentially fluidlike. The off-center variation of the perturbed parallel electric field along field lines gives rise to bunching of electrons. This, in turn, sets up a "back potential" which, for points far enough away from the center of the layer, is strong enough to neutralize $E_{\parallel}$, and hence to cut off the parallel plasma current. We typically have $\omega \gg (k_{\parallel} v_{\rm th})^2/\nu_c$ at the edge of a collisional layer.

Unless the growth rate is exceptionally large, most tearing instabilities of physical interest in tokamaks are drift-tearing modes. These have $\omega \sim \omega_* + i\gamma$, with $|\gamma| \ll |\omega_*|$. Here, γ is the growth rate and ω_* is the electron diamagnetic frequency. The inclusion of diamagnetic effects greatly complicates the situation because the equations describing the stability of the layer become complex. It is interesting to note that most of the initial-value computer codes that have been developed, over the years, to deal with linear tearing modes cannot cope with this level of complexity, and consequently have to unphysically suppress diamagnetic effects. The neglect of diamagnetic effects is probably not such a serious drawback in those initial-value codes concerned with the nonlinear evolution of tearing modes.

For typical JET parameters (see, for instance, Gill *et al.*[1]), we find that $\omega_* \ll \nu_c$ for low poloidal mode numbers (i.e., $m_\theta \leqslant 2$), with $\omega_* \gg \nu_c$ for intermediate and high m_θ. When $\omega_* \ll \nu_c$, the electron dynamics are generally collisional, or semicollisional, and the layer physics is "fluidlike," i.e., the equations determining the stability of the mode are essentially fluid equations. On the other hand, when $\omega_* \gg \nu_c$ the electron dynamics are collisionless (apart from a small region of velocity space at the trapped/passing boundary); in this case, we must use a kinetic approach in order to calculate the stability of the layer. In this paper we shall only be considering the $\omega_* \ll \nu_c$ limit.

Another crucial factor in determining the stability properties of the layer is the ratio of the ion Larmor radius to the layer width. If this ratio is small, then the ion response is essentially that due to standard polarization drift. If the ratio is large, however, the ion response is greatly reduced. Under normal circumstances, the ion Larmor radius tends to be somewhat larger than the layer width for JET-like plasma parameters. Note, however, that there is growing evidence that the magnetic shear at the $q = 1$ surface is very small for the $m_\theta = 1$ modes associated with "sawtooth" collapses (cf. Gill *et al.*[1]). Under these circumstances, the ion Larmor radius can be smaller than the layer width for JET-like plasma parameters.

Finally, we must consider the effect of toroidicity. Now, for JET-like parameters both the electrons and the ions lie in the banana regime, leading to the existence of a population of trapped particles. The presence of trapped particles gives rise to effects such as an enhancement of the radial transport, a bootstrap current, and a Ware pinch, all of which can significantly modify the stability properties of the mode.

B. A brief survey of published research on linear tearing modes

The standard paper on cylindrical tearing modes is Drake and Lee,[2] in which the large ion Larmor radius limit for the collisionless and semicollisional regimes and the small ion Larmor radius limit for the collisional regime are considered. Drake *et al.*[3] look at the small ion Larmor radius limit in the semicollisional regime (this is appropriate if the ions are cold), finding strong semicollisional stabilization of the mode. Cowley *et al.*[4] consider the large ion Larmor radius limit, and calculate the first-order magnetization corrections in the collisionless and semicollisional regimes. They also obtain a strong semicollisional stabilization effect.

In toroidal geometry, the standard paper on high mode number, micro-tearing, instabilities is Catto and Rosenbluth.[5] Conner *et al.*[6] look at intermediate mode number instabilities, where the layer physics is not quite as collisionless as is the case in Catto and Rosenbluth. For the collisional regime, Qu and Callen[7] have done some pioneering work on the small ion Larmor radius limit, as has Hahm,[8] who covers essentially the same ground. It is found that neoclassical ion effects significantly weaken the standard tearing mode in this regime, while trapped particle effects simultaneously destabilize the lowest-order resistive g mode.

The only case not adequately covered by the above-mentioned papers is that of a semicollisional mode in toroidal geometry. Unfortunately, this is precisely the situation that has most relevance to modern tokamaks. It is the purpose of this paper to stop up this significant gap in the existing linear theory of tearing modes.

C. Synopsis

In Sec. II, we present a rigorous derivation of the fluid-like equations governing the linear stability of a low mode number tearing mode. Our calculation is valid for a low beta, large aspect ratio tokamak. We assume that the mode is either semicollisional or collisional (with both electrons and ions in the banana regime), and we allow the ion Larmor radius to be either much less than, or much greater than, the layer width. All neoclassical effects are systematically included in our analysis.

In Sec. III, we obtain a dispersion relation from a reduced version of the set of equations derived in Sec. II.

II. DERIVATION OF THE LAYER EQUATIONS

A. Introduction

The starting point for our analysis is a low beta, large aspect ratio tokamak equilibrium, which we shall refer to using a standard right-handed set of toroidal coordinates r, θ, and φ. We shall neglect equilibrium poloidal currents, so it follows that we can write the equilibrium magnetic field as

$$\mathbf{B} = (B_0 R_0 / R)[\hat{\boldsymbol{\varphi}} + (\epsilon/q)\,\hat{\boldsymbol{\theta}}], \quad (1)$$

where $R = R_0 + r\cos\theta$ is the major radius with R_0 the major radius to the magnetic axis, $\epsilon = r/R$ is the inverse aspect ratio (which is assumed to be small), and $q = rB_\varphi / RB_\theta$ is the safety factor. Note that $B \propto R^{-1}$ to $O(\epsilon^2)$. Note also that the following analysis is strictly only valid for equilibria with circular cross sections. However, we have no reason to suppose that the presence of shaped cross sections will *qualitatively* modify our results; all that will probably happen is that the coefficients in our equations associated with particle trapping will be modified.

Consider a linear perturbation to the above equilibrium in which the most general perturbed quantities vary like

$$a(r,\theta,\varphi,t,\dots) \equiv a(r,\theta,\dots)\exp[i(m_\theta\theta - l_\varphi\varphi - \omega t)], \quad (2)$$

where m_θ and l_φ are poloidal and toroidal mode numbers, respectively. Note the residual, nonsinusoidal, θ dependence that must be retained in perturbed quantities in order to take into account the variation of the equilibrium magnetic field strength around flux surfaces. The parallel component of the wave vector $\mathbf{k}$ is given by $k_\parallel = (m_\theta - l_\varphi q)/qR$, so the mode is resonant (i.e., $k_\parallel = 0$) on the "rational" surface, where $m_\theta = l_\varphi q$. The poloidal component of the wave vector k_θ takes the value m_θ / r.

Let $f(r,\theta,\mathbf{v})$ be the perturbed particle distribution function, where $\mathbf{v}$ is the particle velocity. Following Catto and Rosenbluth,[5] we can write

$$\langle f\rangle_{\rm gyro} = -i\frac{c}{B_\theta}\frac{F_{\rm M}}{\omega l_n}\left[1 + \eta\left(\frac{v^2}{v_{\rm th}^2} - \frac{3}{2}\right)\right]E_\varphi + G, \quad (3)$$

where $\langle\cdots\rangle_{\rm gyro}$ denotes a gyroaverage. In the above equation,

$$F_{\rm M} = (n/\pi^{3/2}v_{\rm th}^3)\exp(-v^2/v_{\rm th}^2) \quad (4)$$

denotes the equilibrium Maxwellian particle distribution function, where $n(r)$ is the equilibrium number density, $T(r)$ is the equilibrium temperature, $v_{\rm th}^2 = 2T/m$ is the equilibrium thermal velocity, $\eta = d(\ln T)/d(\ln n)$, and $l_n = -[d(\ln n)/dr]^{-1}$ is the equilibrium density scale length. The constants c, e, m, and Z denote the velocity of light, the magnitude of the charge on an electron, particle mass, and particle charge number, respectively. The perturbed electric field $\mathbf{E}(r,\theta)$ is given by

$$\mathbf{E} = i[(\omega/c)\mathbf{A} - \mathbf{k}\Phi], \quad (5)$$

where $\mathbf{A}(r,\theta)$ and $\Phi(r,\theta)$ are the perturbed vector and scalar potentials, respectively. [In writing (5), we have anticipated the fact that $\Phi(r,\theta)$ will be found to be θ independent to lowest order.] Finally, the function $G(r,\theta,\mathbf{v})$ is defined

$$G = g - (Ze/T)F_* \Phi, \quad (6)$$

where $g(r,\theta,\mathbf{v})$ is the "nonadiabatic" portion of the gyroaveraged perturbed distribution function (i.e., the part that cannot be absorbed into a gyroaveraged Maxwellian function of energy and canonical angular momentum—cf. Rutherford[9]),

$$F_* = F_M(1 - (\omega_*/\omega)[1 + \eta(v^2/v_{th}^2 - \tfrac{3}{2})]), \quad (7)$$

and

$$\omega_* = -ck_\theta T/ZeBl_n \quad (8)$$

is the equilibrium electron diamagnetic frequency.

In the following quantities with no species subscript or with the subscript e refer to electrons, whereas quantities with the subscript i refer to an ion species with charge number Z.

B. Electron gyrokinetics

1. Introduction

The electron gyrokinetic equation can be written (cf. Ref. 5)

$$\left(\omega - k_\parallel v_\parallel - \mathbf{k}_\perp\cdot\mathbf{v}_d - iC + i\frac{v_\parallel}{qR}\frac{\partial}{\partial\theta}\right)g = -\omega\frac{e}{T}F_*\left(\Phi - \frac{v_\parallel}{c}A_\parallel\right), \quad (9)$$

where $C = C_{ei} + C_{ee}$ is the electron collision operator (made up of the sum of the electron–ion and electron–electron collision operators),

$$\mathbf{k}_\perp \simeq -i\hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\boldsymbol{\theta}}\frac{m_\theta}{r} \quad (10)$$

is the perpendicular wave vector, and

$$\mathbf{v}_d = [(v_\parallel^2 + \tfrac{1}{2}v_\perp^2)/\Omega]\hat{\mathbf{n}}\wedge\nabla(\ln B) \equiv -v_\parallel\hat{\mathbf{n}}\wedge\nabla(v_\parallel/\Omega) \quad (11)$$

is the electron cross-field drift velocity. In Eq. (11), Ω is the electron gyrofrequency (which is assumed to be much greater than any other electron frequency) and $\hat{\mathbf{n}} = \mathbf{B}/B$. In writing Eq. (9) we have neglected compressional magnetic perturbations (this is reasonable at low beta), implying that

$$\mathbf{A} = A_\parallel\hat{\mathbf{n}}. \quad (12)$$

We have also, justifiably, assumed that the electron Larmor radius is much less than the layer width and have, in addition, neglected electron finite Larmor radius (FLR) effects with respect to ion FLR effects.

2. The hierarchy of electron frequencies

We shall expand the electron gyrokinetic equation using the following hierarchy of frequencies:

$$\nu_{trans} \gg \nu_c, \mathbf{k}_r\cdot\mathbf{v}_d \gg k_\parallel v_\parallel \gg \omega, \mathbf{k}_\theta\cdot\mathbf{v}_d, \quad (13)$$

valid for low m_θ (i.e., $m_\theta < 2$) modes in JET-like tokamaks. In the above, $\nu_{trans} = v_{th}/qR$ is the electron transit frequency and ν_c the electron collision frequency. It is assumed that the mode frequency is of the order of the electron diamagnetic frequency, and also that the layer width is of the order of the semicollisional layer width, so that $k_\parallel v_{th} \sim (\omega_* \nu_c)^{1/2}$.

3. The expansion of the electron gyrokinetic equation

The electron gyrokinetic equation is most conveniently written

$$(\omega - k_\parallel v_\parallel - iC)G + \left[B_0\frac{v_\parallel}{B}\frac{\partial}{\partial r}\left(\frac{v_\parallel}{\Omega}\right)k_\theta + i\frac{B_0}{r}\frac{v_\parallel}{B}\frac{\partial}{\partial\theta}\left(\frac{v_\parallel}{\Omega}\right)\frac{\partial}{\partial r} + i\frac{v_\parallel}{qR}\frac{\partial}{\partial\theta}\right]g = -i(e/T)F_* v_\parallel E_\parallel, \quad (14)$$

where we have expanded $\mathbf{k}_\perp\cdot\mathbf{v}_d$. We have also made use of the standard result $C(F_M) = C(v^2F_M) = 0$.

Note that the expansion procedure that follows is similar in form to that employed by Connor and Chen.[10] In the following, the superscript (n) refers to an nth-order quantity.

The zeroth-order expansion of Eq. (14) yields

$$i\frac{v_\parallel}{qR}\frac{\partial g^{(0)}}{\partial\theta} = 0, \quad (15)$$

implying that $g^{(0)}$ is θ independent. We shall assume that $\Phi^{(0)}$ and $A_\parallel^{(0)}$ are θ independent (to be justified in Sec. II D), and thus (15) also implies that $G^{(0)}$ is θ independent.

The first-order expansion of (14) gives

$$-iC(G^{(0)}) + i\frac{B_0}{r}\frac{v_\parallel}{B}\frac{\partial}{\partial\theta}\left(\frac{v_\parallel}{\Omega}\right)\frac{\partial g^{(0)}}{\partial r} + i\frac{v_\parallel}{qR}\frac{\partial g^{(1)}}{\partial\theta} = 0. \quad (16)$$

Consider now the (unnormalized) bounce/transit average operator (cf. Ref. 5)

$$\tilde{Q} \equiv \begin{cases} \displaystyle\int_{-\pi}^{\pi}\frac{Q\,qR\,d\theta}{v_\parallel}, & \text{for passing particles,} \\ \displaystyle\sum_\sigma\int_{\theta_{b(1)}}^{\theta_{b(2)}}\frac{Q\,qR\,d\theta}{|v_\parallel|}, & \text{for trapped particles,} \end{cases} \quad (17)$$

where $\theta_{b(1)}$ and $\theta_{b(2)}$ are the angular coordinates of the bounce points. Note that $v_\parallel = \sigma(v^2 - 2\mu B)^{1/2}$ in velocity coordinates, where σ can take the values ± 1, and μ is the magnetic moment. Clearly, $\tilde{Q}$ annihilates any function Q whose θ dependence can be written in the form

$$Q(\theta) = \frac{v_\parallel}{qR}\frac{\partial}{\partial\theta}P(\theta),$$

where $P(\theta)$ is general. It follows that if we operate on (16) with the bounce/transit average operator, we annihilate both the radial-drift and transit terms, leaving

$$- i\tilde{C}(G^{(0)}) = 0. \tag{18}$$

The only nontrivial solution of (18) is $C(G^{(0)}) = 0$, implying that $G^{(0)}$ is Maxwellian. In fact, the most general expression for $G^{(0)}(r,v)$ is

$$G^{(0)} = \frac{\delta n}{n} F_M + \frac{\delta T}{T} F_M \left(\frac{v^2}{v_{th}^2} - \frac{3}{2}\right), \tag{19}$$

where $\delta n(r)$ and $\delta T(r)$ denote the specifically "semicollisional" contributions to the perturbed electron density and temperature, respectively. Equation (16) also yields

$$g^{(1)} = \hat{g}^{(1)} - \frac{B_0}{\Omega_p}\left(\frac{v_\parallel}{B}\right)\frac{\partial g^{(0)}}{\partial r}, \tag{20}$$

where $\hat{g}^{(1)}(r)$ is θ independent and $\Omega_p = -eB_\theta/mc$ is the electron poloidal gyrofrequency.

The second-order expansion of (14) gives

$$\begin{aligned} &- k_\parallel v_\parallel G^{(0)} - iC(G^{(1)}) \\ &\quad + i\frac{B_0}{r}\frac{v_\parallel}{B}\frac{\partial}{\partial\theta}\left(\frac{v_\parallel}{\Omega}\right)\frac{\partial g^{(1)}}{\partial r} + i\frac{v_\parallel}{qR}\frac{\partial g^{(2)}}{\partial\theta} \\ &\quad = -i(e/T)F_* v_\parallel E_\parallel^{(0)}. \end{aligned} \tag{21}$$

If we operate on the above with the bounce/transit average operator, making use of Eq. (20), we again annihilate the radial-drift and transit terms, leaving

$$\tilde{C}(g^{(1)}) = \tilde{v}_\parallel [ik_\parallel G^{(0)} + (e/T)F_* E_\parallel^{(0)}]. \tag{22}$$

Equation (22), which we shall denote the "neoclassical equation," determines $\hat{g}^{(1)}$ and hence the parallel and radial electron currents.

If we operate on $(-e)\times(14)$ with $\langle\int(\cdots)d^3v\rangle$, where

$$\langle Q\rangle = \frac{1}{2\pi}\oint Q\,d\theta \tag{23}$$

is the flux-surface average operator, we obtain the following charge-conservation equation:

$$\omega(-e)\delta n - k_\parallel (j_\parallel)_e - \left(-i\frac{\partial}{\partial r}\right)(j_r)_e - k_\theta (j_c)_e = 0, \tag{24}$$

where

$$(j_\parallel)_e = (-e)\left\langle\int v_\parallel g^{(1)}\,d^3v\right\rangle, \tag{25}$$

$$(j_r)_e = (-e)\frac{B_0}{r}\left\langle\int \frac{v_\parallel}{B}\frac{\partial}{\partial\theta}\left(\frac{v_\parallel}{\Omega}\right)g^{(2)}\,d^3v\right\rangle, \tag{26}$$

$$(j_c)_e = -(-e)B_0\left\langle\int \frac{v_\parallel}{B}\frac{\partial}{\partial r}\left(\frac{v_\parallel}{\Omega}\right)g^{(0)}\,d^3v\right\rangle. \tag{27}$$

In deriving Eqs. (24)–(27) we have made use of the number conserving properties of the collision operator, as well as the identity

$$\left\langle\int Q\,d^3v\right\rangle \equiv \sum_{\substack{\sigma \\ \text{(passing} \\ \text{particles)}}} \int_0^\infty \int_0^{v^2/2B(\theta=0)} \frac{B}{qR}\tilde{Q}\,v\,dv\,d\mu. \tag{28}$$

We have also neglected the slow r dependence of equilibrium quantities with respect to the fast r dependence of perturbed quantities across the layer.

If we operate on $(\frac{1}{2}mv^2)\times(14)$ with $\langle\int(\cdots)d^3v\rangle$, we obtain the following energy-conservation equation:

$$\begin{aligned} &\frac{3}{2}\omega\left(\frac{\delta n}{n} + \frac{\delta T}{T}\right)nT \\ &\quad - k_\parallel (Q_\parallel)_e - \left(-i\frac{\partial}{\partial r}\right)(Q_r)_e - k_\theta (Q_c)_e = 0, \end{aligned} \tag{29}$$

where

$$(Q_\parallel)_e = \frac{1}{2}m\left\langle\int v_\parallel v^2 g^{(1)}\,d^3v\right\rangle, \tag{30}$$

$$(Q_r)_e = \frac{1}{2}m\frac{B_0}{r}\left\langle\int \frac{v_\parallel}{B}\frac{\partial}{\partial\theta}\left(\frac{v_\parallel}{\Omega}\right)v^2 g^{(2)}\,d^3v\right\rangle, \tag{31}$$

$$(Q_c)_e = -\frac{1}{2}mB_0\left\langle\int \frac{v_\parallel}{B}\frac{\partial}{\partial r}\left(\frac{v_\parallel}{\Omega}\right)v^2 g^{(0)}\,d^3v\right\rangle. \tag{32}$$

In deriving Eq. (29) we have neglected collisional energy exchange between the ions and electrons.

Equations (20), (25), and (30) yield

$$\begin{aligned} (j_\parallel)_e &= (-e)\left\langle\int v_\parallel \hat{g}^{(1)}\,d^3v\right\rangle \\ &\quad - \frac{c}{B_\theta}\left\langle\frac{B_0}{B}\right\rangle\frac{\partial}{\partial r}\left[\left(\frac{\delta n}{n} + \frac{\delta T}{T}\right)\right. \\ &\quad \left. - \frac{e}{T}\left(1 - \frac{\omega_*}{\omega}(1+\eta)\right)\Phi^{(0)}\right]nT \end{aligned} \tag{33}$$

and

$$\begin{aligned} (Q_\parallel)_e &= T\left\langle\int v_\parallel\left(\frac{v}{v_{th}}\right)^2 \hat{g}^{(1)}\,d^3v\right\rangle \\ &\quad - \frac{5}{2}\frac{c}{B_\theta}\frac{T}{(-e)}\left\langle\frac{B_0}{B}\right\rangle\frac{\partial}{\partial r}\left[\left(\frac{\delta n}{n} + 2\frac{\delta T}{T}\right)\right. \\ &\quad \left. - \frac{e}{T}\left(1 - \frac{\omega_*}{\omega}(1+2\eta)\right)\Phi^{(0)}\right]nT. \end{aligned} \tag{34}$$

Integrating (26) by parts in θ and then substituting from Eq. (21), we obtain

$$\begin{aligned} (j_r)_e &= -\frac{c}{B_\theta}\left\langle\frac{B_0}{B}\int mv_\parallel C(g^{(1)})d^3v\right\rangle \\ &\quad + \frac{c}{B_\theta}\left\langle\frac{B_0}{B}\right\rangle\left[ik_\parallel\left(\frac{\delta n}{n} + \frac{\delta T}{T}\right)\right. \\ &\quad \left. + \frac{e}{T}\left(1 - \frac{\omega_*}{\omega}(1+\eta)\right)E_\parallel^{(0)}\right]nT. \end{aligned} \tag{35}$$

A similar calculation yields

$$\begin{aligned} (Q_r)_e &= -\frac{c}{B_\theta}\frac{T}{(-e)}\left\langle\frac{B_0}{B}\int mv_\parallel\left(\frac{v}{v_{th}}\right)^2 C(g^{(1)})d^3v\right\rangle \\ &\quad + \frac{5}{2}\frac{c}{B_\theta}\frac{T}{(-e)}\left\langle\frac{B_0}{B}\right\rangle\left[ik_\parallel\left(\frac{\delta n}{n} + 2\frac{\delta T}{T}\right)\right. \\ &\quad \left. + \frac{e}{T}\left(1 - \frac{\omega_*}{\omega}(1+2\eta)\right)E_\parallel^{(0)}\right]nT. \end{aligned} \tag{36}$$

Finally, Eqs. (27) and (32) imply that

$$(j_c)_e = -\frac{c}{B_0}\left\langle\frac{\partial}{\partial r}\left(\frac{B_0}{B}\right)^2\right\rangle\left[\left(\frac{\delta n}{n}+\frac{\delta T}{T}\right) - \frac{e}{T}\left(1-\frac{\omega_*}{\omega}(1+\eta)\right)\Phi^{(0)}\right]n \qquad (37)$$

and

$$(Q_c)_e = \frac{5}{2}\frac{c}{B_0}\frac{T}{(-e)}\left\langle\frac{\partial}{\partial r}\left(\frac{B_0}{B}\right)^2\right\rangle\left[\left(\frac{\delta n}{n}+2\frac{\delta T}{T}\right) - \frac{e}{T}\left(1-\frac{\omega_*}{\omega}(1+2\eta)\right)\Phi^{(0)}\right]nT. \qquad (38)$$

4. A model collision operator

In order to facilitate the solution of the electron (and ion) neoclassical equation(s) we shall make use of the following model collision operator (cf. Rutherford *et al.*[11] and Connor[12]):

$$C_{jk}(f) = \nu_{jk}\left[\frac{v_\parallel}{B}\frac{\partial}{\partial\mu}\left(\mu v_\parallel\frac{\partial f_j}{\partial\mu}\right) + v_\parallel (F_M)_j\frac{(v_{th})_k^2}{(v_{th})_j^2}\frac{\int d^3v\,\nu_{kj}v_\parallel f_k}{\int d^3v\,\nu_{kj}v_\parallel^2(F_M)_k}\right], \qquad (39)$$

with

$$\nu_{ei} = \frac{3\pi^{1/2}}{4}\left(\frac{v_{th}}{v}\right)^3\nu_c = \bar{\nu}_{ei}\nu_c,$$

$$\nu_{ee} = \frac{3\pi^{1/2}}{4}Z^{-1}h\left(\frac{v^2}{v_{th}^2}\right)\left(\frac{v_{th}}{v}\right)^3\nu_c = \bar{\nu}_{ee}\nu_c,$$

$$\nu_{ie} = Z\left(\frac{m}{m_i}\right)\nu_c = \bar{\nu}_{ie}\nu_c,$$

$$\nu_{ii} = \frac{3\pi^{1/2}}{4}Z^2\left(\frac{m}{m_i}\right)^{1/2}\left(\frac{T}{T_i}\right)^{3/2}h\left(\frac{v^2}{(v_{th})_i^2}\right)\left(\frac{(v_{th})_i}{v}\right)^3\nu_c = \bar{\nu}_{ii}\nu_c. \qquad (40)$$

Here,

$$\nu_c = (16\pi^{1/2}/3)Ze^4 n\ln\Lambda/m^2v_{th}^3 \qquad (41)$$

is the standard Braginskii electron collision frequency, and

$$h(x) = \left(1-\frac{1}{2x}\right)\eta(x) + \frac{d}{dx}\eta(x), \qquad (42)$$

where

$$\eta(x) = \frac{2}{\pi^{1/2}}\int_0^x t^{1/2}e^{-t}\,dt. \qquad (43)$$

The above operator conserves numbers, momentum, and energy; for most purposes it is an adequate approximation to the true collision operator.

5. Electron currents and energy fluxes

Using the model collision operator (39), we can invert the electron neoclassical equation (22) to obtain an expression for $\hat{g}^{(1)}$. Making use of this expression, Eqs. (33)–(36) reduce to

$$\begin{aligned}(j_\parallel)_e = {}& \sigma_\parallel(1-\zeta\epsilon^{1/2})\Big\{ik_\parallel\left(\alpha_n^\parallel(1-\zeta A_n\epsilon^{1/2})\frac{\delta n}{n} + \alpha_T^\parallel(1-\zeta A_T\epsilon^{1/2})\frac{\delta T}{T}\right)\frac{T}{e} \\ &+ \left[\alpha_n^\parallel(1-\zeta A_n\epsilon^{1/2})\left(1-\frac{\omega_*}{\omega}\right) + \alpha_T^\parallel(1-\zeta A_T\epsilon^{1/2})\left(-\eta\frac{\omega_*}{\omega}\right)\right]E_\parallel^{(0)}\Big\} \\ &- \zeta\epsilon^{1/2}\frac{c}{B_\theta}\frac{\partial}{\partial r}\left\{\left(\alpha_n^B\frac{\delta n}{n}+\alpha_T^B\frac{\delta T}{T}\right) - \frac{e}{T}\left[\alpha_n^B\left(1-\frac{\omega_*}{\omega}\right)+\alpha_T^B\left(-\eta\frac{\omega_*}{\omega}\right)\right]\Phi^{(0)}\right\}nT \\ &- (1-\zeta\alpha_n^B\epsilon^{1/2})(j_\parallel)_i + O(\epsilon),\end{aligned} \qquad (44)$$

$$\begin{aligned}(j_r)_e = {}& \zeta\epsilon^{1/2}\frac{c}{B_\theta}\left\{ik_\parallel\left(\alpha_n^P\frac{\delta n}{n}+\alpha_T^P\frac{\delta T}{T}\right) + \frac{e}{T}\left[\alpha_n^P\left(1-\frac{\omega_*}{\omega}\right)+\alpha_T^P\left(-\eta\frac{\omega_*}{\omega}\right)\right]E_\parallel^{(0)}\right\}nT \\ &- \zeta\epsilon^{1/2}(-e)\nu_c\rho_p^2\frac{\partial}{\partial r}\left\{\left(\alpha_n^D\frac{\delta n}{n}+\alpha_T^D\frac{\delta T}{T}\right) - \frac{e}{T}\left[\alpha_n^D\left(1-\frac{\omega_*}{\omega}\right)+\alpha_T^D\left(-\eta\frac{\omega_*}{\omega}\right)\right]\Phi^{(0)}\right\}n \\ &+ 2\alpha_n^D\zeta\epsilon^{1/2}\frac{\nu_c}{\Omega_p}(j_\parallel)_i + O(\epsilon),\end{aligned} \qquad (45)$$

and

$$\begin{aligned}(Q_\parallel)_e = {}& -\kappa_\parallel(1-\zeta\epsilon^{1/2})\Big\{ik_\parallel\left(\beta_n^\parallel(1-\zeta B_n\epsilon^{1/2})\frac{\delta n}{n} + \beta_T^\parallel(1-\zeta B_T\epsilon^{1/2})\frac{\delta T}{T}\right) \\ &+ \frac{e}{T}\left[\beta_n^\parallel(1-\zeta B_n\epsilon^{1/2})\left(1-\frac{\omega_*}{\omega}\right) + \beta_T^\parallel(1-\zeta B_T\epsilon^{1/2})\left(-\eta\frac{\omega_*}{\omega}\right)\right]E_\parallel^{(0)}\Big\}T \\ &- \zeta\epsilon^{1/2}\frac{c}{B_\theta}\frac{T}{(-e)}\frac{\partial}{\partial r}\left\{\left(\beta_n^B\frac{\delta n}{n}+\beta_T^B\frac{\delta T}{T}\right) - \frac{e}{T}\left[\beta_n^B\left(1-\frac{\omega_*}{\omega}\right)+\beta_T^B\left(-\eta\frac{\omega_*}{\omega}\right)\right]\Phi^{(0)}\right\}nT \\ &- \left(\frac{5}{2}-\zeta\beta_n^B\epsilon^{1/2}\right)\frac{T}{(-e)}(j_\parallel)_i + O(\epsilon),\end{aligned} \qquad (46)$$

$$(Q_r)_e = \zeta\epsilon^{1/2}\frac{c}{B_\theta}\frac{T}{(-e)}\left\{ik_\parallel\left(\beta_n^P\frac{\delta n}{n}+\beta_T^P\frac{\delta T}{T}\right)+\frac{e}{T}\left[\beta_n^P\left(1-\frac{\omega_*}{\omega}\right)+\beta_T^P\left(-\eta\frac{\omega_*}{\omega}\right)\right]E_\parallel^{(0)}\right\}nT$$
$$-\zeta\epsilon^{1/2}\nu_c\rho_p^2\frac{\partial}{\partial r}\left\{\left(\beta_n^D\frac{\delta n}{n}+\beta_T^D\frac{\delta T}{T}\right)-\frac{e}{T}\left[\beta_n^D\left(1-\frac{\omega_*}{\omega}\right)+\beta_T^D\left(-\eta\frac{\omega_*}{\omega}\right)\right]\Phi^{(0)}\right\}nT$$
$$+2\beta_n^D\zeta\epsilon^{1/2}\frac{\nu_c}{\Omega_p}\frac{T}{(-e)}(j_\parallel)_i+O(\epsilon), \tag{47}$$

where $\rho_p = v_{\rm th}/\Omega_p$. In the above equations, $\sigma_\parallel = e^2n/m\nu_c$ and $\kappa_\parallel = nT/m\nu_c$ are the parallel (electron) electrical and thermal conductivities, respectively. The parameter ζ occurs in the expansion

$$B_0^2\times\frac{3}{4}\int_0^{B^{-1}(\theta=\pi)}\lambda\,d\lambda\,\langle[1-\lambda B(\theta)]^{1/2}\rangle^{-1}$$
$$=1-\zeta\epsilon^{1/2}+O(\epsilon), \tag{48}$$

and takes the value 1.46 to two decimal places (cf. Hinton and Oberman[13]). In deriving the above equations we have made use of the fact that $\langle B\rangle\approx B_0$, $\langle B^2\rangle\approx B_0^2$, etc., to $O(\epsilon^2)$.

Tables I–IV show the values of the various transport coefficients occurring in Eqs. (44)–(47), evaluated from our model operator for $Z = 1$, 2, 4, 8, and ∞, as well as the corresponding coefficients evaluated for $Z = 1$ using the full collision operator (these are obtained from Braginskii[14] and Rosenbluth *et al.*[15]—note that only 13 out of 20 are given in the literature). We can see that all but one of our parallel transport coefficients are significantly in error (although all are an improvement on those obtained from the Lorentz approximation), while all but one of our perpendicular transport coefficients lie within a few percent of the true value.

The first term in (44) represents an Ohmic electron current driven by the perturbed electric field (but modified by the perturbed parallel pressure gradient and the presence of trapped particles) plus a current driven by a perturbed thermal force (also modified by the presence of trapped particles); the second term represents the perturbed electron bootstrap curent. The first term in (45) represents an electron current associated with the perturbed Ware pinch (modified by the perturbed parallel pressure gradient), while the second term represents the current carried by perturbed collisional cross-field diffusion of electrons. The final terms in Eqs. (44) and (45) come from momentum conservation between the ions and electrons. Note, also, that Eq. (37) represents a perturbed current driven by the principal magnetic field line curvature.

6. Discussion

In the preceding sections, we have written down various expressions for the perturbed electron currents and energy fluxes in a low beta, large aspect ratio tokamak with approximately circular flux surfaces. Let us now consider the range of plasma parameters for which these expressions are valid.

Underlying all our analysis, of course, is the assumption that the electron cyclotron frequency is far larger than any other electron frequency. The constraints on the other frequencies, required for our expressions to be strictly valid, are summed up in the inequality (13). Note, however, that very similar expressions would be obtained even if (13) were not fully satisfied, provided that the following two fundamental conditions hold.

(i) The electrons must be in the banana regime; i.e.,

$$\nu_c \ll \epsilon^{3/2}\nu_{\rm trans}. \tag{49}$$

As $\nu_c\to\epsilon^{3/2}\nu_{\rm trans}$ and the electrons enter the plateau regime, collisional cross-field transport becomes relatively much weaker, and trapped particle effects (i.e., the bootstrap current, the Ware pinch, and the reduction of the parallel conductivity due to trapped particles) gradually disappear.

(ii) The electrons must be amenable to a fluidlike description; i.e.,

$$\omega\ll\nu_c. \tag{50}$$

As $\omega\to\nu_c$ we can probably still write down an electron fluid equation, but with transport coefficients that now depend on ω. Once $\omega\gg\nu_c$, however, the electron dynamics become collisionless (except for a collisional boundary layer at the

TABLE I. The parallel current transport coefficients evaluated using the model collision operator (39).

Z	$\alpha_n^\parallel$	$\alpha_T^\parallel$	A_n	A_T	α_n^B	α_T^B
∞	3.395	8.488	0.000	0.000	1.000	1.000
8	3.191	7.775	0.041	0.020	1.093	0.984
4	3.020	7.189	0.083	0.041	1.183	0.968
2	2.751	6.280	0.172	0.088	1.355	0.939
1	2.389	5.088	0.364	0.194	1.681	0.882
True values for $Z = 1$	1.96	3.35	0.49	···	1.67	0.47

TABLE II. The parallel energy flux transport coefficients evaluated using the model collision operator (39).

Z	$\beta_n^\parallel$	$\beta_T^\parallel$	B_n	B_T	β_n^B	β_T^B
∞	13.58	47.53	0.000	0.000	2.500	5.000
8	12.56	43.06	0.027	0.010	2.751	4.957
4	11.72	39.41	0.058	0.020	2.991	4.915
2	10.41	33.79	0.121	0.044	3.446	4.836
1	8.671	26.51	0.264	0.098	4.291	4.690
True values for $Z = 1$	6.29	13.92	···	···	···	···

TABLE III. The perpendicular current transport coefficients evaluated using the model collision operator (39).

Z	α_n^P	α_T^P	α_n^D	α_T^D
∞	1.000	1.000	0.500	− 0.250
8	1.093	1.112	0.533	− 0.256
4	1.183	1.217	0.567	− 0.262
2	1.355	1.413	0.633	− 0.273
1	1.681	1.771	0.766	− 0.296
True values for $Z = 1$	1.67	⋯	0.77	− 0.29

trapped/passing boundary in velocity space) and a kinetic description is required (cf. Connor *et al.*[6] and Catto and Rosenbluth[5]).

As long as (49) and (50) are satisfied we are bound to obtain expressions for perturbed electron currents and energy fluxes similar in form to those in previous sections. If the inequality (13) is not satisfied, however, the transport coefficients will be different from those we have calculated. For instance, if $\mathbf{k}_\perp \cdot \mathbf{v}_d \to \nu_{\text{trans}}$ then zeroth-order quantities will begin to vary with θ, leading to new expressions for transport coefficients that contain integrals of these θ-varying quantities over flux surfaces.

C. Ion gyrokinetics

1. Introduction

The ion gyrokinetic equation can be written

$$\left(\omega - k_\parallel v_\parallel - \mathbf{k}_\perp \cdot (\mathbf{v}_d)_i - iC_i + i\frac{v_\parallel}{qR}\frac{\partial}{\partial\theta}\right) g_i = \omega \frac{Ze}{T_i} \int_{-\infty}^{\infty} (F_*)_i J_0^2\left(\frac{p v_\perp}{\Omega_i}\right)\left(\overline{\Phi} - \frac{v_\parallel}{c}\overline{A}_\parallel\right) e^{ipr}\, dp, \tag{51}$$

where

$$\Phi(r,\theta) = \int_{-\infty}^{\infty} \overline{\Phi}(p,\theta)\, e^{ipr}\, dp,$$
$$A_\parallel(r,\theta) = \int_{-\infty}^{\infty} \overline{A}_\parallel(p,\theta)\, e^{ipr}\, dp, \tag{52}$$

and J_0 is a standard Bessel function. Note that the driving term on the right-hand side of (51) includes ion finite Larmor radius effects. In this paper we shall consider the following two limits for the ions.

(i) The ion Larmor radius ρ_i is much greater than the layer width $|\delta_e|$; under normal circumstances this limit is the most appropriate to JET-like tokamaks. When $\rho_i \gg |\delta_e|$ the ion gyrokinetics are particularly simple because the driving term in (51) becomes very small, and thus to a very good approximation the nonadiabatic portion g_i of the gyroaveraged perturbed ion distribution function is zero.

(ii) The ion Larmor radius is much less than the layer width. This limit becomes appropriate when the magnetic shear at the rational surface is very small. There is growing evidence that this is the case for the $m_\theta = 1$ modes associated with "sawtooth" relaxations (cf. Gill *et al.*[1]). In this limit, g_i is nontrivial and Eq. (51) must be solved by expansion around the various ion frequencies. The remaining portion of Sec. II C is devoted to the details of this expansion procedure.

2. The hierarchy of ion frequencies

In general, it is quite difficult to find a completely robust hierarcy of ion frequencies suitable for the solution of Eq. (51). However, for the case of both low shear (with $m_\theta = 1$) and the parameter $Z_{\text{eff}}^{-1} T_i/T$ fairly small (i.e., $\leqslant 0.3$), the following hierarchy is not too unreasonable:

$$(\nu_{\text{trans}})_i \gg (\nu_c)_i, (\mathbf{k}_r \cdot \mathbf{v}_d)_i \gg \omega \gg k_\parallel v_\parallel \gg (\mathbf{k}_\theta \cdot \mathbf{v}_d)_i, \text{ ion FLR effects.} \tag{53}$$

Note that for Ohmic discharges we generally find that T_i is somewhat less than T, which is fine with Z_{eff} typically in the range 2–3. On the other hand, auxiliary heating often leads to $T_i > T$. In this case ω_* tends to exceed $(\nu_c)_i$, and we must replace the quasifluid treatment of the ions employed below by a kinetic treatment. Clearly, the following analysis is restricted to discharges where $T_i \leqslant T$. In fact, in this paper we shall take the limit $T_i \ll T$ when $\rho_i \ll |\delta_e|$, for the sake of simplicity. It turns out that the neglect of T_i has a negligible effect on drift-tearing modes, which are the only sort of modes we are concerned with at the moment. However, we will have to reinstate T_i when we discuss resistive g modes (driven by trapped particle effects) and ideal modes in subsequent papers.

3. The expansion of the ion gyrokinetic equation

The ion gyrokinetic equation is most conveniently written

$$(\omega - k_\parallel v_\parallel - iC_i) G_i + \left[B_0 \frac{v_\parallel}{B}\frac{\partial}{\partial r}\left(\frac{v_\parallel}{\Omega_i}\right) k_\theta + i\frac{B_0}{r}\frac{v_\parallel}{\mathrm{B}}\frac{\partial}{\partial\theta}\left(\frac{v_\parallel}{\Omega_i}\right)\frac{\partial}{\partial r} + i\frac{v_\parallel}{qR}\frac{\partial}{\partial\theta}\right] g_i = i\frac{Ze}{T_i}(F_*)_i v_\parallel E_\parallel + \omega\frac{Ze}{T_i} \times \int_{-\infty}^{\infty} (F_*)_i \left[J_0^2\left(\frac{p v_\perp}{\Omega_i}\right) - 1\right]\left(\overline{\Phi} - \frac{v_\parallel}{c}\overline{A}_\parallel\right) e^{ipr}\, dp. \tag{54}$$

TABLE IV. The perpendicular energy flux transport coefficients evaluated using the model collision operator (39).

Z	β_n^P	β_T^P	β_n^D	β_T^D
∞	2.500	5.000	0.500	0.250
8	2.623	5.148	0.544	0.283
4	2.742	5.288	0.588	0.316
2	2.971	5.549	0.677	0.383
1	3.404	6.023	0.854	0.515
True values for $Z = 1$	2.98	⋯	0.87	0.50

Note that the last term on the right-hand side of the above just contains ion finite Larmor radius effects.

The zeroth-order expansion of Eq. (54) yields

$$i\frac{v_\parallel}{qR}\frac{\partial g_i^{(0)}}{\partial\theta}=0, \tag{55}$$

implying that $g_i^{(0)}$ and $G_i^{(0)}$ are both θ independent.

The first-order expansion of (54) gives

$$-iC_i(G_i^{(0)})+i\frac{B_0}{r}\frac{v_\parallel}{B}\frac{\partial}{\partial\theta}\left(\frac{v_\parallel}{\Omega_i}\right)\frac{\partial g_i^{(0)}}{\partial r}$$
$$+i\frac{v_\parallel}{qR}\frac{\partial g_i^{(1)}}{\partial\theta}=0. \tag{56}$$

Operating on the above with the bounce/transit average operator annihilates both the radial-drift and transit terms, leaving

$$-i\widetilde{C}_i(G_i^{(0)})=0. \tag{57}$$

The only nontrivial solution of (57) is $C_i(G_i^{(0)})=0$. Now, in general, the ion–electron collision operator can be neglected with respect to the ion–ion collision operator. With $C_i\approx C_{ii}$, the most general solution of $C_i(G_i^{(0)})=0$ is a Maxwellian with terms corresponding to both a zeroth-order perturbed density and temperature, plus a term corresponding to a zeroth-order perturbed flow [i.e., a term proportional to $v_\parallel(F_M)_i$]. In fact, it turns out that a zeroth-order perturbed flow is only tenable in the presence of an ion sound wave resonance [i.e., $\omega\sim(k_\parallel v_\parallel)_i$]. Thus, for the case where $\omega\gg(k_\parallel v_\parallel)_i$, the most general expression for $G_i^{(0)}$ is

$$G_i^{(0)}=\frac{\delta n_i^{(0)}}{n_i}(F_M)_i+\frac{\delta T_i^{(0)}}{T_i}(F_M)_i\left(\frac{v^2}{(v_{th})_i^2}-\frac{3}{2}\right), \tag{58}$$

where $\delta n_i^{(0)}(r)$ and $\delta T_i^{(0)}(r)$ are the "semicollisional" contributions to the zeroth-order perturbed ion density and temperature, respectively. Equation (56) also yields

$$g_i^{(1)}=\hat{g}_i^{(1)}-\frac{B_0}{(\Omega_p)_i}\left(\frac{v_\parallel}{B}\right)\frac{\partial g_i^{(0)}}{\partial r}, \tag{59}$$

where $\hat{g}_i^{(1)}$ is θ independent.

The second-order expansion of (54) gives

$$\omega G_i^{(0)}-iC_i(G_i^{(1)})+i\frac{B_0}{r}\frac{v_\parallel}{B}\frac{\partial}{\partial\theta}\left(\frac{v_\parallel}{\Omega_i}\right)\frac{\partial g_i^{(1)}}{\partial r}$$
$$+i\frac{v_\parallel}{qR}\frac{\partial g_i^{(2)}}{\partial\theta}=0. \tag{60}$$

Operating on the above with $\langle\int(\cdots)d^3v\rangle$ and $\langle\int(\cdots)v^2\,d^3v\rangle$ (neglecting collisional energy exchange between the ions and electrons) and making use of (59), we obtain

$$\delta n_i^{(0)}=\delta T_i^{(0)}=0, \tag{61}$$

implying that

$$g_i^{(0)}=(Ze/T_i)(F_*)_i\Phi^{(0)}. \tag{62}$$

The result (61) is a consequence of the fact that, unlike the electrons, the ions are not semicollisional (i.e., $(k_\parallel v_\parallel)_i\ll[\omega_*(\nu_c)_i]^{1/2}$). Operating on (60) with the bounce/transit average operator annihilates the radial-drift and transit terms, leaving

$$\widetilde{C}_i(g_i^{(1)})=0. \tag{63}$$

Equation (63), which we shall denote the "ion neoclassical" equation, determines $\hat{g}_i^{(1)}$ and hence the parallel and radial ion currents.

The third-order expansion of (54) gives

$$\omega G_i^{(1)}-iC_i(G_i^{(2)})+i\frac{B_0}{r}\frac{v_\parallel}{B}\frac{\partial}{\partial\theta}\left(\frac{v_\parallel}{\Omega_i}\right)\frac{\partial g_i^{(2)}}{\partial r}$$
$$+i\frac{v_\parallel}{qR}\frac{\partial g_i^{(3)}}{\partial\theta}=i(Ze/T_i)(F_*)_iv_\parallel E_\parallel^{(0)}. \tag{64}$$

If we operate on $(Ze)\times(54)$ with $\langle\int(\cdots)d^3v\rangle$, we obtain the following charge-conservation equation for the ions:

$$\omega(Ze)\delta n_i-k_\parallel(j_\parallel)_i-\left(-i\frac{\partial}{\partial r}\right)(j_r)_i-k_\theta(j_c)_i$$
$$=\omega\frac{(Ze)^2}{T_i}n_i\int_{-\infty}^{\infty}F(p)\overline{\Phi}^{(0)}(p)e^{ipr}\,dp, \tag{65}$$

where

$$F(p)=\left(1-\frac{\omega_{*i}}{\omega}\right)(\Gamma_0-1)\frac{\omega_{*i}}{\omega}\eta_i\frac{p^2\rho_i^2}{2}(\Gamma_0-\Gamma_1), \tag{66}$$

with

$$\Gamma_n=\exp(-p^2\rho_i^2/2)I_n(p^2\rho_i^2/2). \tag{67}$$

Here, I_n represents a standard Bessel function. We define

$$\delta n_i=\left\langle\int G_i^{(2)}\,d^3v\right\rangle, \tag{68}$$

and

$$(j_\parallel)_i=Ze\left\langle\int v_\parallel g_i^{(1)}\,d^3v\right\rangle, \tag{69}$$

$$(j_r)_i=Ze\frac{B_0}{r}\left\langle\int\frac{v_\parallel}{B}\frac{\partial}{\partial\theta}\left(\frac{v_\parallel}{\Omega_i}\right)g_i^{(3)}\,d^3v\right\rangle, \tag{70}$$

$$(j_c)_i=-Ze\,B_0\left\langle\int\frac{v_\parallel}{B}\frac{\partial}{\partial r}\left(\frac{v_\parallel}{\Omega_i}\right)g_i^{(0)}\,d^3v\right\rangle. \tag{71}$$

Using the model collision operator (39) to invert the ion neoclassical equation, expression (69) can be shown to reduce to

$$(j_\parallel)_i=-\frac{c}{B_\theta}\left(1-\frac{\omega_{*i}}{\omega}[1+\alpha_i(1-\zeta A_i\epsilon^{1/2})\eta_i]\right)$$
$$\times\frac{Ze}{T_i}\frac{\partial\Phi^{(0)}}{\partial r}n_iT_i+O(\epsilon), \tag{72}$$

where

$$\alpha_i=-0.17\quad\text{and}\quad A_i=6.80. \tag{73}$$

Integrating Eq. (70) by parts in θ, and then substituting from (64), we obtain

$$(j_r)_i = -\frac{c}{B_\theta}\left\langle\frac{B_0}{B}\int m_i v_\| C_i(g_i^{(2)})d^3v\right\rangle - \frac{c}{B_\theta}\frac{Ze}{T_i}$$
$$\times\left(1-\frac{\omega_{*i}}{\omega}(1+\eta_i)\right)E_\|^{(0)}n_iT_i$$
$$+i\omega\frac{(Ze)^2}{T_i}n_i\frac{(\rho_p)_i^2}{2}\left(1-\frac{\omega_{*i}}{\omega}\right.$$
$$\left.\times[1+\alpha_i(1-\zeta A_i\epsilon^{1/2})\eta_i]\right)\frac{\partial\Phi^{(0)}}{\partial r}+O(\epsilon). \quad (74)$$

Finally, Eq. (71) reduces to

$$(j_c)_i = -\frac{c}{B_0}\left\langle\frac{\partial}{\partial r}\left(\frac{B_0}{B}\right)^2\right\rangle$$
$$\times\frac{Ze}{T_i}\left(1-\frac{\omega_{*i}}{\omega}(1+\eta_i)\right)\Phi^{(0)}n_iT_i. \quad (75)$$

4. Discussion

As is well known, in the small ion Larmor radius limit the trapped ions drift radially under the influence of both a Ware pinch effect (in response to a toroidal electric field) and a neoclassically enhanced polarization drift (in response to a time-varying radial electric field—cf. Hinton and Robertson[16]). Because C_i is dominated by the ion–ion collision operator (which attempts to relax the ion distribution back to a Maxwellian) the whole ion population is forced to move radially with the trapped ions, giving rise to a "bulk" ion Ware pinch effect (i.e., one in which all the ions take part) and an enhanced ion polarization drift (or, equivalently, an enhanced ion inertia). In the above analysis, the last term in (74) corresponds to a perturbed ion polarization drift which is neoclassically enhanced by a factor $(q/\epsilon)^2$ over the standard drift.

Using analogous arguments to those employed in Sec. II B 6, we can easily show that as long as the ions are in the banana regime and $\omega \ll (\nu_c)_i$, the various ion currents must be similar in form to those calculated above. Of course, the numerical constants appearing in our expressions are only accurate when the ion hierarchy (53) is strictly valid.

D. Layer equations

1. Quasineutrality and charge conservation

Quasineutrality requires that

$$(-e)n' + (Ze)n_i' = 0, \quad (76)$$

where n' and n_i' are the total perturbed electron and ion number densities, respectively. Equation (76) implies

$$(-e)\delta n + (Ze)\delta n_i = 0, \quad (77)$$

where

$$\delta n_k = \left\langle\int G_k\, d^3v\right\rangle. \quad (78)$$

If we sum Eqs. (24) and (65), making use of (77), we obtain the first of our layer equations, which is, in fact, the equation of global charge conservation:

$$i\nabla\cdot\mathbf{j} = -k_\|j_\| - \left(-i\frac{\partial}{\partial r}\right)j_r - k_\theta j_c$$
$$= \omega\frac{(Ze)^2}{T_i}n_i\int_{-\infty}^{\infty}F(p)\overline{\Phi}^{(0)}(p)e^{ipr}\,dp, \quad (79)$$

where $j_\| = (j_\|)_e + (j_\|)_i$, etc.

2. Ampère's law

The second layer equation is obtained from the linearized, flux surface averaged, parallel component of the Ampère–Maxwell equation (neglecting the displacement current):

$$\langle k_\perp^2 A_\|\rangle \approx -\frac{\partial^2 A_\|^{(0)}}{\partial r^2} = \frac{4\pi}{c}j_\|. \quad (80)$$

3. Layer boundary conditions

Let x be a radial coordinate with origin at the center of the layer. We can expand $k_\|$ across the layer as follows:

$$k_\| \approx -(k_\theta/l_s)x, \quad (81)$$

where

$$l_s = [(Rq/r)q/q']_{x=0} \quad (82)$$

is the magnetic shear length at the rational surface.

Note that only those portions of $\Phi^{(0)}(x)$ and $A_\|^{(0)}(x)$ that have, respectively, odd and even parity in x have any influence on the final dispersion relation. Since we are only interested in calculating a dispersion relation, we can restrict $\Phi^{(0)}$ and $A_\|^{(0}$ to be odd and even functions of x, respectively, without any loss of generality.

The first boundary condition can be written

$$E_\|^{(0)} = i[(\omega/c)A_\|^{(0)} - k_\|\Phi^{(0)}] \to 0 \quad \text{as} \quad |x|/|\delta_e| \to \infty, \quad (83)$$

and is obtained by requiring continuity of the "inner" solution with the "outer" ideal MHD solution at the edge of the layer.

The second layer boundary condition is obtained by requiring the asymptotic behavior of $A_\|^{(0)}$ at large $|x|/|\delta_e|$ to be physically acceptable. In practice, this means that only power-law solutions for $A_\|^{(0)}$ are allowed at the edge of the layer. In general, there are two such independent solutions, so we can write

$$A_\|^{(0)} \to c_- x^{\sigma_-}[1+O(1/x^2)] + c_+ x^{\sigma_+}[1+O(1/x^2)] \quad \text{as } |x|/|\delta_e| \to \infty, \quad (84)$$

where c_-, c_+, σ_-, and σ_+ are all constants, with $\sigma_+ > \sigma_-$. Note that if we neglect curvature effects [which are $O(\epsilon^2)$], we always have $\sigma_+ = 1$ and $\sigma_- = 0$. The well-known frequency-dependent quantity $\Delta(\omega)$, which characterizes the intrinsic stability properties of the layer, is defined in terms of the ratio of the amplitudes of the two asymptotic solutions for $A_\|^{(0)}$, as follows:

$$c_+/c_- = \Delta(\omega)/2. \quad (85)$$

Finally, the dispersion relation

$$\Delta(\omega) = \Delta' \tag{86}$$

is obtained by matching the layer solution to the "outer" ideal MHD solution. Note that in writing (86) we have neglected poloidal mode coupling (see Bussac *et al.*[17] and Connor *et al.*[18] for details of how such coupling modifies the form of the dispersion relation). The real quantity Δ' characterizes the external driving of the layer and is calculated entirely from the "outer" solution. Throughout the rest of this paper we shall assume that Δ' is given.

4. The zeroth-order electric potential

At this point it is convenient to prove that $\Phi^{(0)}$ is θ independent, as has been previously assumed. Recall from Eqs. (15) and (55) that $g_k^{(0)}$ is θ independent, where $k \equiv (e \text{ or } i)$. From (6), we may write

$$\int g_k^{(0)}\, d^3v = \delta n_k + \frac{Z_k e}{T_k} n_k \left(1 - \frac{\omega_{*k}}{\omega}\right) \Phi^{(0)}, \tag{87}$$

yielding

$$\sum_k (Z_k e) \int g_k^{(0)}\, d^3v = \sum_k (Z_k e)\, \delta n_k + \sum_k \frac{(Z_k e)^2}{T_k} n_k \left(1 - \frac{\omega_{*k}}{\omega}\right) \Phi^{(0)} = \sum_k \frac{(Z_k e)^2}{T_k} n_k \Phi^{(0)}. \tag{88}$$

In the above we have made use of quasineutrality and the definition of ω_{*k}. Since the left-hand side of (88) is plainly θ independent it follows that $\Phi^{(0)}$ must also be θ independent.

5. The zeroth-order magnetic vector potential

Let us now consider the θ dependence of $A_\parallel^{(0)}$. If we calculate $(-e)\int(14)d^3v + (Ze)\int(54)d^3v$ to lowest order we obtain

$$\frac{\partial}{\partial\theta}\left(\frac{\partial^2 A_\parallel^{(0)}}{\partial s^2} \frac{B_0}{B}\right) = -\hat{\beta}\kappa_{\rm B} \frac{\partial}{\partial\theta}\left(\frac{B_0}{B}\right)^2 \frac{d}{ds}\left[s(\delta\nu + \delta\tau) + (1+\eta)\psi\right], \tag{89}$$

where we have made use of some of the normalizations described in Sec. II E 1. Equation (89) clearly shows that geodesic magnetic field line curvature effects cause $A_\parallel^{(0)}$ to vary around flux surfaces by $O(\epsilon^2)$. The assumption that $A_\parallel^{(0)}$ is θ independent is therefore equivalent to the neglect of $O(\epsilon^2)$. In practice, this means the neglect of curvature terms in our equations. The neglect of curvature effects is quite reasonable since trapped particle effects [which are generally $O(\epsilon^{1/2})$] are always far more significant.

E. The small ion Larmor radius limit

1. The equations to the layer

Let us consider the limit in which $(\rho_i/|\delta_e|) \ll 1$. Starting from the electron fluid equations (24) and (29), and the two layer equations (79) and (80), and making use of the expressions for the electron currents and energy fluxes given in Sec. II B 3 and II B 5, plus the expressions for the ion currents given in Eqs. (72) and (74), we arrive at the following four normalized equations that fully determine the behavior of the layer:

$$\begin{aligned}
&\delta\nu + \{s^2(\delta\nu + \lambda_\alpha^\parallel \delta\tau) + [\hat{\omega} - (1 + \lambda_\alpha^\parallel \eta)](A - s\psi)\} - \kappa_{\rm B} \frac{d\psi}{ds} \\
&\quad + (\zeta\alpha_n^B)\epsilon^{1/2}(\hat{\omega}^{-1}\kappa_{\rm B}) \frac{d}{ds}\left[s(\delta\nu + \lambda_\alpha^B \delta\tau) + (1 + \lambda_\alpha^B \eta)\psi\right] \\
&\quad - (\zeta\alpha_n^P)\epsilon^{1/2}(\hat{\omega}^{-1}\kappa_{\rm B}) \frac{1}{s}\frac{d}{ds}\{s^2(\delta\nu + \lambda_\alpha^P \delta\tau) + [\hat{\omega} - (1 + \lambda_\alpha^P \eta)](A - s\psi)\} \\
&\quad + (2\zeta\alpha_n^D)\epsilon^{1/2}(\hat{\omega}^{-1}\kappa_{\rm B})(\bar{\sigma}_\parallel \kappa_{\rm B}\hat{\omega}^{-1}) \frac{1}{s}\frac{d^2}{ds^2}\left[s(\delta\nu + \lambda_\alpha^D \delta\tau) + (1 + \lambda_\alpha^D \eta)\psi\right] = 0,
\end{aligned} \tag{90}$$

$$\begin{aligned}
&\frac{3}{2}(\delta\nu + \delta\tau) + \left(\frac{\bar{\kappa}_\parallel}{\bar{\sigma}_\parallel}\right)\{s^2(\delta\nu + \lambda_\beta^\parallel \delta\tau) + [\hat{\omega} - (1 + \lambda_\beta^\parallel \eta)](A - s\psi)\} - \frac{5}{2}\kappa_{\rm B} \frac{d\psi}{ds} \\
&\quad + (\zeta\beta_n^B)\epsilon^{1/2}(\hat{\omega}^{-1}\kappa_{\rm B}) \frac{d}{ds}\left[s(\delta\nu + \lambda_\beta^B \delta\tau) + (1 + \lambda_\beta^B \eta)\psi\right] \\
&\quad - (\zeta\beta_n^P)\epsilon^{1/2}(\hat{\omega}^{-1}\kappa_{\rm B}) \frac{1}{s}\frac{d}{ds}\{s^2(\delta\nu + \lambda_\beta^P \delta\tau) + [\hat{\omega} - (1 + \lambda_\beta^P \eta)](A - s\psi)\} \\
&\quad + (2\zeta\beta_n^D)\epsilon^{1/2}(\hat{\omega}^{-1}\kappa_{\rm B})(\bar{\sigma}_\parallel \kappa_{\rm B}\hat{\omega}^{-1}) \frac{1}{s}\frac{d^2}{ds^2}\left[s(\delta\nu + \lambda_\beta^D \delta\tau) + (1 + \lambda_\beta^D \eta)\psi\right] = 0,
\end{aligned} \tag{91}$$

$$\begin{aligned}
&s\{s^2(\delta\nu + \lambda_\alpha^\parallel \delta\tau) + [\hat{\omega} - (1 + \lambda_\alpha^\parallel \eta)](A - s\psi)\} + (\zeta\alpha_n^B)\epsilon^{1/2}(\hat{\omega}^{-1}\kappa_{\rm B}) s \frac{d}{ds}\left[s(\delta\nu + \lambda_\alpha^B \delta\tau) + (1 + \lambda_\alpha^B \eta)\psi\right] \\
&\quad - (\hat{\omega}^{-1}\kappa_{\rm B}) \frac{d}{ds}\left[s^2(\delta\nu + \delta\tau) - (1+\eta)(A - s\psi)\right] = -iC_{\rm eff}^{-1} \frac{d^2\psi}{ds^2},
\end{aligned} \tag{92}$$

$$-\hat{\beta}^{-1}\hat{\omega}^{-1}\frac{d^2A}{ds^2}=\{s^2(\delta\nu+\lambda_\alpha^\parallel\,\delta\tau)+[\hat{\omega}-(1+\lambda_\alpha^\parallel\eta)](A-s\psi)\}$$

$$+(\zeta\alpha_n^B)\epsilon^{1/2}(\hat{\omega}^{-1}\kappa_B)\frac{d}{ds}[s(\delta\nu+\lambda_\alpha^B\,\delta\tau)+(1+\lambda_\alpha^B\eta)\psi]. \qquad (93)$$

The details of the normalization (which is modeled on that used by Drake *et al.*[3]) are as follows:

$$\delta_e^2=-i\left(\frac{2\omega\nu_c l_s^2}{\bar{\sigma}_\parallel v_{\rm th}^2 k_\theta^2}\right),\quad s=\frac{x}{\delta_e},\quad \hat{\omega}=\frac{\omega}{\omega_*},$$

$$A=A_\parallel^{(0)},\quad \psi=-(ck_\theta\delta_e/\omega l_s)\Phi^{(0)},$$

$$\delta\nu=i\left(\frac{mc\nu_c\omega}{(-e)k_\parallel\bar{\sigma}_\parallel\omega_*}\right)\frac{\delta n}{n},$$

$$\delta\tau=i\left(\frac{mc\nu_e\omega}{(-c)k_\parallel\bar{\sigma}_\parallel\omega_*}\right)\frac{\delta T}{T},$$

$$\beta=\frac{4\pi nT}{B^2},\quad \hat{\beta}=(l_s^2/l_n^2)\beta,$$

$$\kappa_B=\frac{B}{B_\theta}\frac{l_n}{l_s}\equiv-\frac{d(\ln q)}{d(\ln n)},$$

$$C_{\rm eff}^{-1}=\bar{\sigma}_\parallel Z^{-1}\frac{\omega_*}{\nu_c}\frac{m_{i\,\rm eff}}{m}\frac{l_n^2}{l_s^2},$$

$$m_{i\,\rm eff}=m_i(1+B^2/B_\theta^2),$$

$$\bar{\sigma}_\parallel=\alpha_n^\parallel(1-\zeta(1+A_n)\epsilon^{1/2}),$$

$$\bar{\kappa}_\parallel=\beta_n^\parallel(1-\zeta(1+B_n)\epsilon^{1/2}),$$

$$\lambda_\alpha^\parallel=(\alpha_T^\parallel/\alpha_n^\parallel)(1-\zeta(A_T-A_n)\epsilon^{1/2}),$$

$$\lambda_\beta^\parallel=(\beta_T^\parallel/\beta_n^\parallel)(1-\zeta(B_T-B_n)\epsilon^{1/2}),$$

$$\lambda_\alpha^B=\alpha_T^B/\alpha_n^B,\quad \lambda_\beta^B=\beta_T^B/\beta_n^B,$$

$$\lambda_\alpha^P=\alpha_T^P/\alpha_n^P,\quad \lambda_\beta^P=\beta_T^P/\beta_n^P,$$

$$\lambda_\alpha^D=\alpha_T^D/\alpha_n^D,\quad \lambda_\beta^D=\beta_T^D/\beta_n^D. \qquad (94)$$

Note that the radial distance has been normalized with respect to the semicollisional layer width $|\delta_e|$, and the mode frequency has been normalized with respect to the electron diamagnetic frequency ω_*. The parameter $C_{\rm eff}$ is a measure of the effective collisionality of the layer: if $C_{\rm eff}\ll 1$ the layer is semicollisional, while if $C_{\rm eff}\gg 1$ the layer is collisional. The parameter κ_B is a measure of the magnetic shear at the rational surface. Note also that the neoclassical enhancement of the ion inertia, which effectively increases the ion mass from m_i to $m_{i\,\rm eff}$, makes our layer far less collisional than a corresponding layer in cylindrical geometry.

In obtaining Eqs. (90)–(93), we have made use of the momentum conserving properties of the collision operator. We have also used the following expansion for the ion response function at small ion Larmor radius:

$$F(p)\approx-(p^2\rho_i^2/2)[1-(\omega_{*i}/\omega)(1+\eta_i)]. \qquad (95)$$

Finally, we have neglected T_i with respect to T.

Equation (90) is the electron charge-conservation equation: the first term represents the semicollisional contribution to the perturbed electron density, the second term comes from the perturbed Ohmic electron current, the third comes from momentum conservation and is associated with the bulk ion Ware pinch current, the fourth term corresponds to the perturbed electron bootstrap current, the fifth to the perturbed electron Ware pinch current, and the sixth to the perturbed collisional cross-field electron flux. Equation (91) is the electron energy-conservation equation and has analogous terms to the charge-conservation equation. Equation (92) is the quasineutrality equation: the first term corresponds to the perturbed Ohmic current, the second to the perturbed bootstrap current, the third to the perturbed bulk ion Ware pinch current, and the term on the right-hand side to the neoclassically enhanced perturbed ion polarization current. Finally, Eq. (93) is the induction equation.

The system of equations described above is eighth order in complex variables. If we neglect all trapped particle effects (including the bulk ion Ware pinch and the enhanced ion polarization drift) our equations reduce to those quoted in Drake *et al.*[3] for semicollisional tearing modes in cylindrical geometry with $T_i\ll T$. (N.B. This reduced set of equations is fourth order.) If we take the collisional limit (i.e., $C_{\rm eff}\gg 1$, $\delta\nu,\delta\tau\to 0$) and assume that the magnetic shear length is extremely large (i.e., $\kappa_B\to 0$), our equations become equivalent to those derived by Connor and Chen[10] for collisional modes in toroidal geometry, except that we have taken the limit $T_i\ll T$. (N.B. This set of equations is also fourth order.)

2. The boundary conditions to the layer

Symmetry considerations immediately give us four boundary conditions at $s=0$:

$$\frac{dA(0)}{ds}=\psi(0)=\frac{d\,\delta\nu(0)}{ds}=\frac{d\,\delta\tau(0)}{ds}=0. \qquad (96)$$

Let us now consider the power-law asymptotic solutions of Eqs. (90)–(93) at large $|s|$. The most general power-law asymptotic expansion that we can make is as follows:

$$A=\sum_{\substack{n=0\\ \rm even}}^\infty a_{(n)}s^{\sigma-n},\quad \psi=\sum_{\substack{n=0\\ \rm even}}^\infty b_{(n)}s^{\sigma-1-n},$$

$$\delta\nu=\sum_{\substack{n=0\\ \rm even}}^\infty c_{(n)}s^{\sigma-2-n},\quad \frac{\delta\tau}{\eta}=\sum_{\substack{n=0\\ \rm even}}^\infty d_{(n)}s^{\sigma-2-n}. \qquad (97)$$

Substituting the above expansion into our equations we obtain the indicial equation

$$\sigma(\sigma-1)=0, \qquad (98)$$

which has two roots

$$\sigma_-=0\quad\text{and}\quad\sigma_+=1, \qquad (99)$$

corresponding to the two independent power-law solutions expected at large $|s|$. We also have

$$\frac{b_{(0)\pm}}{a_{(0)\pm}} = \left(\frac{(\hat{\omega}-1)+\sigma_{\pm}\kappa_{\rm B}}{(\hat{\omega}-1)+\kappa_{\rm B}}\right),$$
$$\frac{c_{(0)\pm}}{a_{(0)\pm}} = \left(\frac{\kappa_{\rm B}(1-\sigma_{\pm})}{(\hat{\omega}-1)+\kappa_{\rm B}}\right)(1-\hat{\omega}),$$
$$\frac{d_{(0)\pm}}{a_{(0)\pm}} = \left(\frac{\kappa_{\rm B}(1-\sigma_{\pm})}{(\hat{\omega}-1)+\kappa_{\rm B}}\right). \quad (100)$$

Note that one of the above asymptotic solutions does not satisfy the boundary condition $E_{\parallel}^{(0)}\to 0$ as $|s|\to\infty$ (i.e., $b_{(0)-}\neq a_{(0)-}$). This is symptomatic of the fact that we have ordered the perturbed parallel (Ohmic) ion current out of our analysis by taking $(k_{\parallel}v_{\parallel})_i \ll \omega$. Now, as the radial distance from the center of the layer becomes much greater than the semicollisional layer width, the perturbed parallel electron current falls off very rapidly while the much smaller ion current remains relatively steady. Eventually, the ion current becomes dominant, leading naturally to the required cutoff in $E_{\parallel}^{(0)}$. Clearly, in order to get the correct form for the dispersion relation, we must match our semicollisional layer solution to the outer ideal MHD solution via an intermediate layer in which the perturbed parallel (Ohmic) ion current is non-negligible. This intermediate layer is characterized by $(k_{\parallel}v_{\parallel})_i\sim\omega$. More details of the matching procedure are given later in Sec. III D.

Equations (84) and (85) imply the boundary condition

$$a_{(0)+}/a_{(0)-} = e^{-i\pi/4}\hat{\omega}^{1/2}C_{\rm eff}^{1/2}\rho_{s\,\rm eff}\Delta_{\rm sc}/2, \quad (101)$$

where

$$\rho_{s\,\rm eff} = \frac{(ZT/m_{i\,\rm eff})^{1/2}}{\Omega_{i\,\rm eff}} \equiv \left(\frac{Tm_{i\,\rm eff}}{Z}\right)^{1/2}\frac{c}{eB} \quad (102)$$

is the ion Larmor radius calculated using the electron temperature and the effective ion mass. In Eq. (101), $\Delta_{\rm sc}(\omega)$ represents the value of $\Delta(\omega)$ calculated at the boundary of the semicollisional layer. The true value of Δ, appearing in the dispersion relation (86), is that calculated at the edge of the intermediate layer, and in general this will be slightly different from $\Delta_{\rm sc}$.

The eight boundary conditions implied by (96), (100), and (101) are sufficient to fully specify the solution of Eqs. (90)–(93).

F. The large ion Larmor radius limit

1. The equations to the layer

Let us next consider the case where $(\rho_i/|\delta_e|)\gg 1$. Using the fact that ion currents are negligible in this limit, we arrive at the following four equations that govern the behavior of the layer:

$$\delta\nu + \{s^2(\delta\nu+\lambda_{\alpha}^{\parallel}\delta\tau) + [\hat{\omega}-(1+\lambda_{\alpha}^{\parallel}\eta)](A-s\psi)\} + (\zeta\alpha_n^B)\epsilon^{1/2}(\hat{\omega}^{-1}\kappa_{\rm B})\frac{d}{ds}\{s(\delta\nu+\lambda_{\alpha}^{B}\delta\tau) - [\hat{\omega}-(1+\lambda_{\alpha}^{B}\eta)]\psi\}$$
$$-(\zeta\alpha_n^P)\epsilon^{1/2}(\hat{\omega}^{-1}\kappa_{\rm B})\frac{1}{s}\frac{d}{ds}\{s^2(\delta\nu+\lambda_{\alpha}^{P}\delta\tau) + [\hat{\omega}-(1+\lambda_{\alpha}^{P}\eta)](A-s\psi)\}$$
$$+(2\zeta\alpha_n^D)\epsilon^{1/2}(\hat{\omega}^{-1}\kappa_{\rm B})(\bar{\sigma}_{\parallel}\kappa_{\rm B}\hat{\omega}^{-1})\frac{1}{s}\frac{d^2}{ds^2}\{s(\delta\nu+\lambda_{\alpha}^{D}\delta\tau) - [\hat{\omega}-(1+\lambda_{\alpha}^{D}\eta)]\psi\} = 0, \quad (103)$$

$$\frac{3}{2}(\delta\nu+\delta\tau) + \left(\frac{\bar{\kappa}_{\parallel}}{\bar{\sigma}_{\parallel}}\right)\{s^2(\delta\nu+\lambda_{\beta}^{\parallel}\delta\tau) + [\hat{\omega}-(1+\lambda_{\beta}^{\parallel}\eta)](A-s\psi)\},$$
$$+(\zeta\beta_n^B)\epsilon^{1/2}(\hat{\omega}^{-1}\kappa_{\rm B})\frac{d}{ds}\{s(\delta\nu+\lambda_{\beta}^{B}\delta\tau) - [\hat{\omega}-(1+\lambda_{\beta}^{B}\eta)]\psi\}$$
$$-(\zeta\beta_n^P)\epsilon^{1/2}(\hat{\omega}^{-1}\kappa_{\rm B})\frac{1}{s}\frac{d}{ds}\{s^2(\delta\nu+\lambda_{\beta}^{P}\delta\tau) + [\omega-(1+\lambda_{\beta}^{P}\eta)](A-s\psi)\}$$
$$+(2\zeta\beta_n^D)\epsilon^{1/2}(\hat{\omega}^{-1}\kappa_{\rm B})(\bar{\sigma}_{\parallel}\kappa_{\rm B}\hat{\omega}^{-1})\frac{1}{s}\frac{d^2}{ds^2}\{s(\delta\nu+\lambda_{\beta}^{D}\delta\tau) - [\hat{\omega}-(1+\lambda_{\beta}^{D}\eta)]\psi\} = 0, \quad (104)$$

$$-s\delta\nu = -\frac{ZT}{T_i}\hat{\omega}\int_{-\infty}^{\infty}F(p)\bar{\psi}(p)e^{ipx}\,dp \approx \begin{cases}\left(\dfrac{ZT}{T_i}\hat{\omega}+1\right)\psi, & |x|\ll\rho_i,\\ -iC^{-1}\left(1+\dfrac{T_i}{ZT}\hat{\omega}^{-1}(1+\eta_i)\right)\dfrac{d^2\psi}{ds^2}, & |x|\gg\rho_i,\end{cases} \quad (105)$$

$$-\hat{\beta}^{-1}\hat{\omega}^{-1}\frac{d^2A}{ds^2} = \{s^2(\delta\nu+\lambda_{\alpha}^{\parallel}\delta\tau) + [\hat{\omega}-(1+\lambda_{\alpha}^{\parallel}\eta)](A-s\psi)\}$$
$$+(\zeta\alpha_n^B)\epsilon^{1/2}(\hat{\omega}^{-1}\kappa_{\rm B})\frac{d}{ds}\{s(\delta\nu+\lambda_{\alpha}^{B}\delta\tau) - [\hat{\omega}-(1-\lambda_{\alpha}^{B}\eta)]\psi\}, \quad (106)$$

where

$$C^{-1} = \bar{\sigma}_{\parallel} Z^{-1} \frac{\omega_*}{\nu_c} \frac{m_i}{m} \frac{l_n^2}{l_s^2} \tag{107}$$

is the unenhanced collisionality parameter.

The above system of equations is sixth order in complex variables when $|x| \ll \rho_i$ and eighth order when $|x| \gg \rho_i$. In the limit $\epsilon^{1/2} \to 0$, our equations become equivalent to those derived by Cowley, Kulsrud, and Hahm[4] for semicollisional modes in cylindrical geometry.

2. The boundary conditions to the layer

Symmetry requirements again give us the boundary conditions (96) at $s = 0$. If we look for power-law asymptotic solutions of the form (97) at large $|s|$, we obtain the following two solutions:

$$\sigma_{\pm} = \tfrac{1}{2} \pm (\tfrac{1}{4} - D)^{1/2}, \tag{108}$$

where

$$D = \frac{\hat{\beta}(\hat{\omega} - 1)[(ZT/T_i)\hat{\omega} + 1]}{(ZT/T_i + 1)}. \tag{109}$$

We also have

$$\frac{b_{(0)}}{a_{(0)}} = \frac{\hat{\omega}^{-1}(\hat{\omega} - 1)}{ZT/T_i + 1},$$
$$\frac{c_{(0)}}{a_{(0)}} = \frac{-\hat{\omega}^{-1}(\hat{\omega} - 1)[(ZT/T_i)\hat{\omega} + 1]}{ZT/T_i + 1},$$
$$\frac{d_{(0)}}{a_{(0)}} = \frac{\hat{\omega}^{-1}[(ZT/T_i)\hat{\omega} + 1]}{ZT/T_i + 1}. \tag{110}$$

Note that in calculating (109) and (110) we have assumed that $x \ll \rho_i$.

Now, we can easily see that the above asymptotic solutions do not satisfy the boundary condition $E_{\parallel}^{(0)} \to 0$ as $|s| \to \infty$. Note, however, that if we repeat the analysis for the limit $x \gg \rho_i$ we find both $E_{\parallel}^{(0)} = 0$ and $D = 0$. This implies that we must match our semicollisional layer solution to the outer ideal MHD solution via an intermediate layer whose thickness is of the order of the ion Larmor radius (cf. Cowley, Kulsrud, and Hahm[4]).

Finally, we have the boundary condition

$$a_{(0)+}/a_{(0)-} = (e^{-i\pi/4}\hat{\omega}^{1/2}C^{1/2}\rho_s)^{(1-4D)^{1/2}}(\Delta_{sc}/2), \tag{111}$$

for $|x| \ll \rho_i$, where

$$\rho_s = \frac{(ZT/m_i)^{1/2}}{\Omega_i} \equiv \left(\frac{Tm_i}{Z}\right)^{1/2} \frac{c}{eB}. \tag{112}$$

Here, $\Delta_{sc}(\omega)$ is the value of $\Delta(\omega)$ calculated at the boundary of the semicollisional layer (where $x \ll \rho_i$). The true value of Δ is calculated for $x \gg \rho_i$, and, in general, will be slightly different from Δ_{sc}.

III. CALCULATION OF THE DISPERSION RELATION FOR SMALL ION LARMOR RADIUS IN THE LIMIT $\epsilon^{1/2} \to 0$

A. The equations to the layer

Let us now consider the simplified case where $\rho_i \ll |\delta_e|$ and all terms involving $\epsilon^{1/2}$ are neglected. In this limit the equations to the layer reduce to the following:

$$s\left(\hat{\sigma}_{(0)}(s^2)(A - s\psi) + s^2\hat{\sigma}_{(1)}(s^2)\frac{d\psi}{ds}\right) - (\hat{\omega}^{-1}\kappa_B)\frac{d}{ds}\left(\hat{\sigma}_{(2)}(s^2)(A - s\psi) + s^2\hat{\sigma}_{(3)}(s^2)\frac{d\psi}{ds}\right) = -iC_{\rm eff}^{-1}\frac{d^2\psi}{ds^2}, \tag{113}$$

$$-\hat{\beta}^{-1}\hat{\omega}^{-1}\frac{d^2A}{ds^2} = \hat{\sigma}_{(0)}(s^2)(A - s\psi) + s^2\hat{\sigma}_{(1)}(s^2)\frac{d\psi}{ds}, \tag{114}$$

where

$$\hat{\sigma}_{(0)} = \left(\frac{(1 + \kappa s^2)(\hat{\omega} - 1) - \hat{\alpha}\eta}{1 + (1 + \kappa + \bar{\alpha})s^2 + \kappa s^4}\right),$$
$$\hat{\sigma}_{(1)} = \kappa_B\left(\frac{(1 + \tfrac{2}{3}\hat{\alpha}) + \kappa s^2}{1 + (1 + \kappa + \bar{\alpha})s^2 + \kappa s^4}\right),$$
$$\hat{\sigma}_{(2)} = -\left(\frac{\hat{\omega}(1 + \tfrac{2}{3}\hat{\alpha} + \kappa s^2)s^2 + [1 + (\kappa + \bar{\alpha} - \tfrac{2}{3}\hat{\alpha})s^2] + [1 + (1 - \hat{\alpha})s^2]\eta}{1 + (1 + \kappa + \bar{\alpha})s^2 + \kappa s^4}\right),$$
$$\hat{\sigma}_{(3)} = \kappa_B\left(\frac{\tfrac{5}{3} + (\tfrac{5}{3} + \kappa + \bar{\alpha} - \tfrac{4}{3}\hat{\alpha})s^2}{1 + (1 + \kappa + \bar{\alpha})s^2 + \kappa s^4}\right), \tag{115}$$

and

$$\hat{\alpha} = \lambda_{\alpha}^{\parallel}, \quad \bar{\alpha} = \tfrac{2}{3}\hat{\alpha}^2, \quad \kappa = \tfrac{2}{3}(\bar{\kappa}_{\parallel}/\bar{\sigma}_{\parallel})(\lambda_{\beta}^{\parallel} - \lambda_{\alpha}^{\parallel}). \tag{116}$$

The above equations are a fairly straightforward generalization of those of Drake *et al.*[3] to include both the bulk ion Ware pinch effect and the neoclassically enhanced ion polarization drift. The systems of equations (113) and (114) is fourth order in complex variables.

B. The asymptotic forms at small $|s|$

The physically acceptable asymptotic forms at small $|s|$, which satisfy the symmetry requirements $A(-s) = A(s)$ and $\psi(-s) = -\psi(s)$, can be written as follows:

$$A = a_0 + a_2 s^2 + a_4 s^4 + O(s^6),$$
$$\psi = b_0 s + b_2 s^3 + O(s^5), \tag{117}$$

where

$$a_2 = -(\hat{\beta}\hat{\omega}/2)\alpha_{(0)}a_0,$$
$$a_4 = [\hat{\beta}\hat{\omega}/(3\cdot 4)][\beta_{(0)}a_0 + \alpha_{(0)}a_2 - (\alpha_{(0)} - \alpha_{(1)})b_0],$$
$$b_2 = [i/(2\cdot 3)](\hat{\omega}^{-1}\kappa_{\rm B})C_{\rm eff}\{[(\hat{\omega}/\kappa_{\rm B})\alpha_{(0)} - 2\beta_{(2)}]a_0 - 2\alpha_{(2)}(a_2 - b_0) - 2\alpha_{(3)}b_0\}. \quad (118)$$

Here, a_0 and b_0 are two arbitrary constants. We also have

$$\hat{\sigma}_{(n)} \approx \alpha_{(n)} + \beta_{(n)}s^2 + O(s^4), \quad (119)$$

for $|s| \ll 1$.

C. The asymptotic forms at large $|s|$

Using Eqs. (97), (99), (100), and (101), the two power-law asymptotic forms at large $|s|$ can be written

$$A \simeq a_{\infty\,\rm sc}[\hat{\Delta}_{\rm sc}s + 1 + O(1/s)],$$
$$\psi \simeq a_{\infty\,\rm sc}\left[\hat{\Delta}_{\rm sc} + \left(\frac{(\hat{\omega}-1)}{(\hat{\omega}-1)+\kappa_{\rm B}}\right)\frac{1}{s} + O\left(\frac{1}{s^2}\right)\right], \quad (120)$$

where $a_{\infty\,\rm sc}$ is an arbitrary constant, and

$$\hat{\Delta}_{\rm sc} = e^{-i\pi/4}\hat{\omega}^{1/2}C_{\rm eff}^{1/2}\rho_{s\,\rm eff}\Delta_{\rm sc}/2. \quad (121)$$

The two remaining asymptotic forms are exponential and can be written as follows:

$$A \sim e^{s/\lambda_\pm}, \quad \psi \sim K_\pm\, e^{s/\lambda_\pm}, \quad (122)$$

where

$$\lambda_\pm^2 = \frac{iC_{\rm eff}^{-1} - \kappa_{\rm B}(\hat{\omega}^{-1}\kappa_{\rm B})(\frac{2}{3} + \kappa + \bar{\alpha} - \frac{4}{3}\hat{\alpha})/\kappa}{(\kappa_{\rm B}^2\hat{\beta})\hat{\omega} + (\hat{\omega}-1) + \kappa_{\rm B}} \quad (123)$$

and

$$K_\pm = -(\hat{\beta}^{-1}\hat{\omega}^{-1}/\kappa_{\rm B})(1/\lambda_\pm). \quad (124)$$

One of the above pair of asymptotic forms (by definition e^{s/λ_+}) will always grow along the x axis (which corresponds to the "real" space axis) and must therefore be suppressed in any physical solution.

D. The effect of the "intermediate" layer

As was mentioned in Sec. II E 2, we cannot match our layer solution to the outer ideal MHD solution without explicitly including the Ohmic ion current in our analysis. When we attempt to take this effect into account we are led to generalized equations that have the following schematic form:

$$s\left[\hat{\sigma}_{(0)}(A - s\psi) + s^2\hat{\sigma}_{(1)}\left(\frac{d\psi}{ds} + \frac{(\bar{\sigma}_\parallel)_i\hat{\omega}}{\kappa_{\rm B}}(A - s\psi)\right)\right] - (\hat{\omega}^{-1}\kappa_{\rm B})\frac{d}{ds}\left[\hat{\sigma}_{(2)}(A - s\psi) + s^2\hat{\sigma}_{(3)} \times\left(\frac{d\psi}{ds} + \frac{(\bar{\sigma}_\parallel)_i\hat{\omega}}{\kappa_{\rm B}}(A - s\psi)\right)\right] = -iC_{\rm eff}^{-1}\frac{d^2\psi}{ds^2}, \quad (125)$$

$$-\hat{\beta}^{-1}\hat{\omega}^{-1}\frac{d^2A}{ds^2} = \hat{\sigma}_{(0)}(A - s\psi) + s^2\hat{\sigma}_{(1)} \times\left(\frac{d\psi}{ds} + \frac{(\bar{\sigma}_\parallel)_i\hat{\omega}}{\kappa_{\rm B}}(A - s\psi)\right), \quad (126)$$

where

$$(\bar{\sigma}_\parallel)_i \sim O\left(\frac{\sigma_{\parallel i}}{\sigma_{\parallel e}}\right) \sim \left(\frac{m}{m_i}\right)^{1/2}\left(\frac{T_i}{T}\right)^{3/2} \quad (127)$$

is a small positive constant.

The power-law asymptotic forms at large $|s|$ are now given by

$$A \sim a_\infty[\hat{\Delta}s + 1 + O(1/s)],$$
$$\psi \sim a_\infty\left[\hat{\Delta} + \left(\frac{(\hat{\omega}-1) + \hat{\omega}(\bar{\sigma}_\parallel)_i s^2}{(\hat{\omega}-1) + \kappa_{\rm B} + \hat{\omega}(\bar{\sigma}_\parallel)_i s^2}\right)\frac{1}{s} + O\left(\frac{1}{s^2}\right)\right]. \quad (128)$$

Clearly, the boundary condition $E_\parallel[\propto(A - s\psi)] \to 0$ as $|s| \to \infty$ is satisfied once $s \gg (\bar{\sigma}_\parallel)_i^{-1/2}$.

In order to get some idea of exactly how the power-law asymptotic forms transform across the "intermediate" region linking our semicollisional layer solution to the outer ideal MHD solution, we shall consider the limit in which this intermediate region becomes infinitesimally small. To achieve this, the Ohmic ion current terms are neglected in the inner region and then suddenly "switched on" at $s = s_i$, where $(\bar{\sigma}_\parallel)_i s_i^2 \gg 1$. If we integrate Eqs. (113) and (114) across the discontinuity thus created at $s = s_i$, we obtain

$$[\psi]_{s_{i-}}^{s_{i+}} = -(1/s_i)[A - s\psi]_{s_{i-}}^{s_{i+}},$$
$$\left[\frac{dA}{ds}\right]_{s_{i-}}^{s_{i+}} = -\kappa_{\rm B}\hat{\beta}\hat{\omega}[\psi]_{s_{i-}}^{s_{i+}}. \quad (129)$$

Equations (129) imply that $a_{\infty\,\rm sc} \to a_{\infty\, i}$ and $\hat{\Delta}_{\rm sc} \to \hat{\Delta}_i$ as we pass from the inner region to the outer ideal MHD region beyond $s = s_i$, where

$$a_{\infty\, i} = \left(\frac{(\kappa_{\rm B}^2\hat{\beta})\hat{\omega} + (\hat{\omega}-1) + \kappa_{\rm B}}{(\hat{\omega}-1) + \kappa_{\rm B}}\right)a_{\infty\,\rm sc}, \quad (130)$$

$$\hat{\Delta}_i = \left(\frac{-(\kappa_{\rm B}^2\hat{\beta})\hat{\omega}}{(\kappa_{\rm B}^2\hat{\beta})\hat{\omega} + (\hat{\omega}-1) + \kappa_{\rm B}}\right)\frac{1}{s_i} + \left(\frac{(\hat{\omega}-1) + \kappa_{\rm B}}{(\kappa_{\rm B}^2\hat{\beta})\hat{\omega} + (\hat{\omega}-1) + \kappa_{\rm B}}\right)\hat{\Delta}_{\rm sc}. \quad (131)$$

We can easily show that the right-hand side of (130) and the second term on the right-hand side of (131) are completely independent of the form of the intermediate layer. On the other hand, the first term on the right-hand side of (131) is strongly dependent on the behavior of the solution within the intermediate layer; this is likely to be very complicated because of the presence of an ion sound wave resonance there. Fortunately, however, we can probably neglect this term, provided that $\hat{\Delta}_{\rm sc}$ is not too small, since we always have $s_i \gg 1$. Equation (131) implies that the relation between $\Delta_{\rm sc}(\omega)$ and the true $\Delta(\omega)$ for the layer is

$$\Delta = \{[(\hat{\omega} - 1) + \kappa_B]/[(\kappa_B^2\hat{\beta})\hat{\omega} + (\hat{\omega} - 1) + \kappa_B]\}\Delta_{sc}, \quad (132)$$

where we have neglected $O(s_i^{-1})$. Recall that $\Delta(\omega)$ is related to the ideal MHD parameter Δ' via the dispersion relation (86).

We can easily show that the two $e^{s/\lambda}$ asymptotic forms are transformed into weak $e^{s^2/\lambda}$ forms beyond $s \gg (\bar{\sigma}_\parallel)_i^{-1/2}$.

E. Numerical methods

We can obtain a dispersion relation from Eqs. (113) and (114) using the following numerical procedure.

(i) Choose a value for $\hat{\omega}$.

(ii) Choose a convenient axis (the y axis, say) that passes through the origin and lies in the region of the complex s plane where e^{s/λ_+} is growing. Note that λ_+ is defined such that e^{s/λ_+} always grows along the positive x axis. Now, we can easily show that in the half-plane in which e^{s/λ_+} is growing there are two points where the system of equations (113) and (114) becomes singular—these singular points are roots of the quartic

$$s^2\hat{\sigma}_{(3)}(s^2)(\hat{\omega}^{-1}\kappa_B) - iC_{\rm eff}^{-1} = 0. \quad (133)$$

Of course, we must ensure that our y axis does not pass too close to one of these singular points, otherwise we are bound to encounter severe numerical difficulties. We must also ensure that no singular point lies in the region of the half-plane separating the x axis from the y axis, otherwise we will be unable to transform our solution between these two axes.

(iii) Numerically integrate Eqs. (113) and (114) along the y axis from the origin to large positive y. Since the system is undetermined to an arbitrary multiplicative constant we only have one free parameter at the origin, namely, b_0/a_0. Naturally, we must choose b_0/a_0 such that the unphysical e^{s/λ_+} asymptotic form is eliminated from our solution at large y.

(iv) Fit the power-law asymptotic forms (128) to our numerical solution at large y in order to obtain values for the parameters $a_{\infty\,\rm sc}$ and $\hat{\Delta}_{\rm sc}$. Equations (132) and (86) then give us the value of Δ'. Finally, we must vary $\mathrm{Re}(\hat{\omega})$ until Δ' is real, since a complex Δ' is completely unphysical.

(v) After steps (i)–(iv) have been carried out we are left with the dispersion relation in the form $\Delta'[\mathrm{Im}(\hat{\omega})]$ and $\mathrm{Re}(\hat{\omega})[\mathrm{Im}(\hat{\omega})]$.

F. Results

(i) Figure 1 shows the effect of variations in κ_B $[\equiv -d(\ln q)/d(\ln n_e)]$ on the stability properties of the layer. We have plotted curves of the critical Δ' for marginal stability (denoted Δ'_c—if $\Delta' > \Delta'_c$ the mode grows and vice versa) against the effective collisionality parameter $C_{\rm eff}$ [defined in Eq. (94)], for various values of κ_B. Note that Δ'_c is normalized with respect to $\rho_{s\,\rm eff}^{-1}$, where $\rho_{s\,\rm eff}/$(minor radius) $\sim \frac{1}{30}$ for typical JET parameters.

The limit $\kappa_B \to 0.0$ corresponds to one of the cases considered by Drake *et al.*[3] Unfortunately, numerical difficulties have prevented us from calculating the dispersion relation with $C_{\rm eff}$ either small enough, or large enough, for any of the analytic results quoted in Ref. 3 to be valid. Because of this, a direct quantitative comparison of our results with those of Drake *et al.* is impossible. However, it is encouraging to note that both sets of results are in good qualitative agreement, i.e., both imply that the layer is neutral ($\Delta'_c \to 0$) in the collisional limit, but that in the semicollisional regime there is strong stabilization ($\Delta'_c > 0$).

As κ_B is increased from zero, a moderate destabilizing effect ($\Delta'_c < 0$), originating from neoclassical layer physics, manifests itself in the transition region between the collisional and semicollisional regimes. This effect reaches a maximum when $\kappa_B \sim 2.0$. For $\kappa_B \gtrsim 5.0$, we find stabilization in the transition region; this presumably comes from the large magnetic shear associated with higher values of κ_B. Of

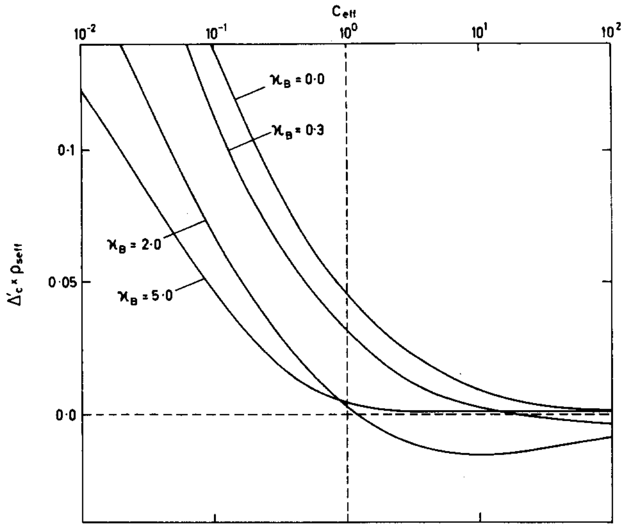

FIG. 1. The critical Δ' for marginal stability Δ'_c plotted as a function of the effective collisionality $C_{\rm eff}$ for various values of κ_B, with $\hat{\beta} = 0.1$ and $\eta = 1.0$.

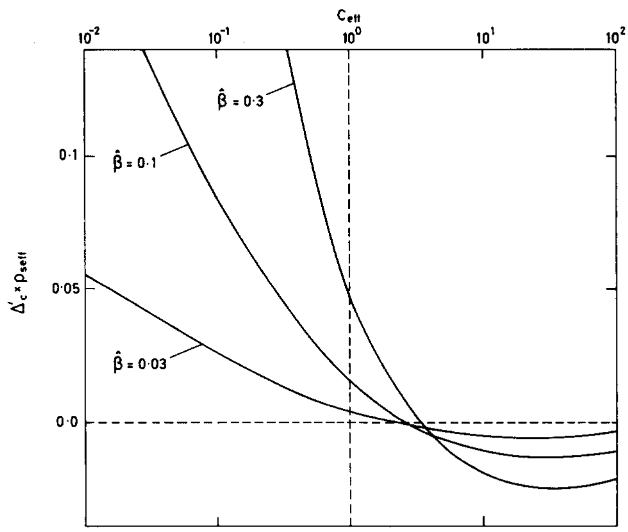

FIG. 2. The critical Δ' for marginal stability Δ'_c plotted as a function of the effective collisionality $C_{\rm eff}$ for various values of $\hat{\beta}$, with $\kappa_B = 1.0$ and $\eta = 1.0$.

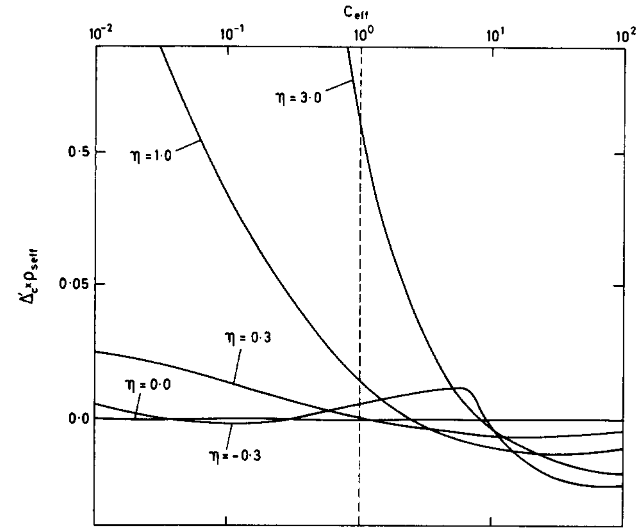

FIG. 3. The critical Δ′ for marginal stability Δ'_c plotted as a function of the effective collisionality $C_{\rm eff}$ for various values of η, with $\kappa_B = 1.0$ and $\hat{\beta} = 0.1$.

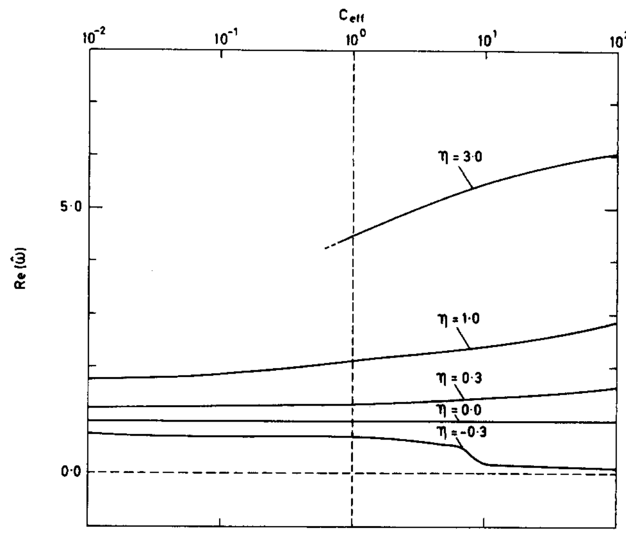

FIG. 4. The normalized real frequency $\mathrm{Re}(\hat{\omega})$ plotted as a function of the effective collisionality $C_{\rm eff}$ for various values of η, with $\kappa_B = 1.0$ and $\hat{\beta} = 0.1$.

course, the major difference between our toroidal results and the cylindrical results of Ref. 3 is the significant reduction in effective collisionality we obtain as a result of the neoclassical enhancement of ion inertia.

(ii) Figure 2 shows the effect of variations in $\hat{\beta}$ $[\equiv 4\pi(nT)_e/\kappa_B^2 B_\theta^2]$ on the stability properties of the layer. Note that both the semicollisional stabilization and the neoclassical destabilization decrease with decreasing $\hat{\beta}$, both effects tending to zero as $\hat{\beta} \to 0$.

(iii) Figure 3 shows the effect of variations in η $[\equiv d(\ln T_e)/d(\ln n_e)]$ on the stability properties of the layer. Note that for $\eta > 0$, both the semicollisional stabilization and the neoclassical destabilization decrease markedly with decreasing η, both effects tending to zero as $\eta \to 0$. For $\eta < 0$, we obtain weak semicollisional stabilization and relatively strong neoclassical destabilization, with some rather complicated behavior in the transition region.

In Fig. 4 we plot the real frequencies of the marginally stable modes displayed in Fig. 3.

IV. CONCLUSION

The layer equations obtained in Sec. II are probably the most physically realistic set yet derived for low mode number tearing modes in a tokamak. One very significant feature of these equations, in the small ion Larmor radius limit, is a reduction in the effective collisionality of the system, by a factor $(1 + B^2/B_\theta^2)$, due to neoclassical ion dynamics. In Sec. III, we solved our equations numerically in the limits of small ion Larmor radius and small $\epsilon^{1/2}$. We found strong stabilization in the semicollisional regime, much as expected, plus somewhat weaker destabilization in the transition regime. This last effect comes directly from neoclassical ion dynamics. We intend to present the solution of the full set of layer equations (i.e., with finite $\epsilon^{1/2}$), for both large and small ion Larmor radius limits, in subsequent papers.

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