Spontaneous Healing and Growth of Locked Magnetic Island Chains in Stellarator Plasmas

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Spontaneous Island Healing/Growth on LHD

- Spontaneous “healing/growth” of vacuum magnetic islands in LHD and TJ-II\textsuperscript{a} stellarators highly reminiscent of “mode unlocking/locking” phenomena in tokamaks\textsuperscript{b} and RFPs\textsuperscript{c}.
  - Healing transitions triggered when amplitude of externally generated resonant magnetic field exceeds critical threshold.
  - Growth transitions triggered when field amplitude falls below second critical threshold that is significantly lower than first.
  - Island healing/growth transitions associated with substantial changes in plasma rotation.

\textsuperscript{a} Y. Narushima, \textit{et al.}, NF \textbf{51}, 083030 (2011).
\textsuperscript{b} T.C. Hender, \textit{et al.}, NF \textbf{32}, 2091 (1992).
Mode Locking Theory in Tokamaks \(^a\)

- Response of plasma to external perturbation governed by ideal-MHD everywhere, except close to resonant surface.
- Close to resonant surface, plasma response governed by neoclassical four-field reduced-MHD model.
- Ultimately obtain two coupled nonlinear equations governing time evolution of island width and island phase.
- Equations possess bifurcated solutions exhibiting sudden transitions between states with wide/narrow island. Transitions associated with significant changes in plasma rotation, and exhibit strong hysteresis.

Application to Stellarators

- Can only use tokamak mode locking theory to study island healing/growth by treating stellarator equilibrium as axisymmetric plasma perturbed by static, externally generated, helical magnetic field.

- In reality, helical component of equilibrium field in stellarators comparable in strength with axisymmetric component.

- Tokamak analogy highly approximate in nature. Nevertheless, represents important first step.
Island Width Evolution Equation

\[ \alpha_1 \frac{dw}{dT} = -1 + w^{-2} \cos \varphi + J_c \alpha_2 w^{-3} \]

- \( w = W/W_v \) - ratio of actual island width to vacuum island width. \( \varphi \) - island phase relative to resonant external perturbation. \( T = \omega_{*i} t \).

- First term on rhs due to intrinsic stability of island. Second to destabilizing influence of resonant external perturbation. Third to stabilizing/destabilizing effect of ion polarization current generated by plasma flow around island.

- \( \alpha_1 = 1.65 (\tau_R/m_\theta) (W_v/r_s) \).

- \( \alpha_2 = (\beta_i/2 m_\theta) (q_s/\epsilon_s)^2 (L_s/L_n)^2 (\rho_i/W_v)^3 \).

\(^a\)For full list of definitions see:
Island Phase Evolution Equation

\[ w \sin \varphi = J_s \alpha_3 \]

- Lhs is electromagnetic locking torque due to resonant external perturbation. Rhs is drag torque due to combination of plasma flow, ion perpendicular viscosity, and ion poloidal and toroidal flow damping.

- \( \alpha_3 = \left( \frac{q_s}{\epsilon_s} \right)^2 \alpha_2 \).
Cosine and Sine Integrals

\[ J_c = -2 \int_{-\infty}^{\infty} \oint \hat{\mathcal{J}}_|| \cos \zeta \, dX \frac{d\zeta}{2\pi}, \]
\[ J_s = -2 \int_{-\infty}^{\infty} \oint \hat{\mathcal{J}}_|| \sin \zeta \, dX \frac{d\zeta}{2\pi}. \]

- \( X = (r - r_s)/W \) - scaled radial coordinate. \( \zeta \) - helical angle. \( \hat{\mathcal{J}}_|| \) - normalized parallel current density.

- \( J_c \) multiplies ion polarization term in island width evolution equation.

- \( J_s \) multiplies drag torque term in island phase evolution equation.
Flow Damping Regimes

- $J_c$ and $J_s$ functions of normalized poloidal flow damping rate,

$$\tilde{\nu}_\theta = \left(\frac{\epsilon_s}{q_s}\right)^2 \left(\frac{\nu_{\theta i}}{\omega_{*i}}\right),$$

normalized toroidal flow damping rate,

$$\tilde{\nu}_\phi = \left(\frac{\epsilon_s}{q_s}\right)^2 \left(\frac{\nu_{\phi i}}{\omega_{*i}}\right),$$

and normalized viscous diffusion rate

$$\tilde{\nu}_\mu = \left(\frac{r_s}{W}\right)^2 \left(\frac{\nu_{\mu i}}{\omega_{*i}}\right).$$

- Four different regimes depending on relative sizes of $\tilde{\nu}_\theta$, $\tilde{\nu}_\phi$, and $\tilde{\nu}_\mu$. 
Flow Damping Regimes

\[
(\hat{\nu}_\theta \hat{\nu}_\phi)^{1/2} = \hat{\nu}_\mu
\]

\[
\hat{\nu}_\theta = \hat{\nu}_\phi
\]

\[
\hat{\nu}_\theta = \hat{\nu}_\mu
\]
## Flow Damping Regimes

<table>
<thead>
<tr>
<th>Regime</th>
<th>$J_c$</th>
<th>$J_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$1.38 v_\phi (v_\phi - 1)$</td>
<td>$-0.36 \tilde{v}<em>\theta (v</em>\phi - v_\theta)$</td>
</tr>
<tr>
<td>II</td>
<td>$1.38 v_\theta (v_\theta - 1)$</td>
<td>$-3.74 \tilde{v}<em>\theta^{1/4} \tilde{v}</em>\phi^{3/4} (v_\phi - v_\theta)$</td>
</tr>
<tr>
<td>III</td>
<td>$1.38 v_\theta (v_\theta - 1)$</td>
<td>$-4.00 \tilde{v}<em>\phi^{1/2} \tilde{v}</em>\mu^{1/2} (v_\phi - v_\theta)$</td>
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</tr>
</tbody>
</table>

- $v_\theta = -1.17 \eta_i/(1 + \eta_i)$.$^a$
- $v_\phi = -2.37 \eta_i/(1 + \eta_i) + d\varphi/dT$ in “1/ν” regime ($v_i \gg q_s V_{E} B / R_0$).$^b$
- $v_\phi = +0.25 \eta_i/(1 + \eta_i) + d\varphi/dT$ in “ν” regime ($v_i \ll q_s V_{E} B / R_0$).$^b$

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Bifurcated Solutions

• Solutions to island evolution equations exhibit two different types of bifurcation.

• “Torque bifurcations” - due to breakdown in balance between electromagnetic and drag torques in island phase evolution equation.\(^a\)

• “Stability bifurcations” - due to breakdown in balance between various terms on rhs of island width evolution equation.\(^b\)

• For sake of brevity, will only discuss torque bifurcations.

\(^a\)C.C. Hegna, NF 51, 113017 (2011).
Torque Bifurcations

• Assumptions: Regime III; $1/\nu$ toroidal flow damping; neglect polarization term.

• Island evolution equations reduce to

\[
\begin{align*}
\alpha_1 \frac{dw}{d\varphi} &= -1 + w^{-2} \cos \varphi, \\
w \sin \varphi &= \alpha_4 (v_0 - d\varphi/dT).
\end{align*}
\]

Here, $v_0 = 1.2 \eta_i/(1 + \eta_i)$, $\alpha_4 = 4 (\hat{v}_\phi \hat{v}_\mu)^{1/2} \alpha_3$. 
Locked Island Solutions

- Wide island that is stationary in lab frame. Ion poloidal velocity at resonant surface zero.

- Island evolution equations yield

\[ w = (\cos \varphi)^{1/2}, \]
\[ (\cos \varphi)^{1/2} \sin \varphi = \alpha_4 v_0. \]

- Locked solution only possible when \( \alpha_4 v_0 < (4/27)^{1/4} \). When \( \alpha_4 v_0 \) exceeds critical value \((4/27)^{1/4}\) island unlocks from vacuum perturbation, spins up (in electron diamagnetic direction), and its width decays significantly.
Rotating Island Solutions

• Narrow island that rotates in electron diamagnetic direction. Ion poloidal velocity at resonant surface also in electron direction.

• Island evolution equations yield

\[
\frac{d\varphi}{dT} = v, \\
w = (3/\alpha_1 v)^{1/3} |\sin \varphi|^{1/3}, \\
(sin \varphi)^{4/3} = \alpha_5 (v/v_0) (1 - v/v_0).
\]

Here, \( \alpha_5 = v_0 \alpha_4 (v_0 \alpha_1/3)^{1/3} \).

• Rotating island has pulsating width. Solution only possible when \( \alpha_5 > (256/27)^{1/3} \). When \( \alpha_5 \) falls below critical value \((256/27)^{1/3}\) island locks to vacuum perturbation, and its width grows significantly.
Island Healing/Growth Transitions

• Island healing transition is bifurcation from locked to rotating solution. Occurs when

$$\beta > \beta_{heal} \sim |s|^{1/2} \frac{\nu_*^{1/2}}{\rho_*^3} \left( \frac{br_{vac}}{B_\phi} \right)^{3/2}$$

where $s$ is magnetic shear at resonant surface.

• Island growth transition is bifurcation from rotating to locked solution. Occurs when

$$\beta < \beta_{grow} \sim \frac{\nu_*^{3/4}}{\rho_*^4} \left( \frac{br_{vac}}{B_\phi} \right)^2 < \beta_{heal}.$$  

• Locked/rotating solutions metastable when $\beta_{grow} < \beta < \beta_{heal}$.

• Healing/growth transitions associated with poloidal velocity shifts in electron/ion diamagnetic direction, respectively.
Island Healing/Growth Transitions

\[ \beta \propto \nu^{1/2} \]

\[ \beta \propto \nu_*^{3/4} \]

\[ \nu_* \]

growth \quad \rightarrow \quad \text{healing}
Summary

• Tokamak mode locking theory, applied to stellarators, yields predictions in good agreement with experimental observations of spontaneous island healing/growth:
  – Island healing/growth occurs at high/low $\beta$ and low/high $v_*$, respectively.
  – Healing/growth associated with polodal velocity shift in electron/ion diamagnetic direction, respectively.
  – Critical $\beta$ value for healing exceeds that for growth.

• Agreement not complete. Torque bifurcations associated with healing transitions in which island unlocks from vacuum perturbation, spins up, and decays away. Spin-up not observed experimentally. Stability bifurcations do not have this problem.