Determination of Non-Ideal Response of a High Temperature Plasma to a Static External Magnetic Perturbation via Asymptotic Matching

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Error Fields

- Tokamak plasmas highly sensitive to externally generated, static, helical magnetic perturbations—a.k.a. “error fields”.

- Error fields drive magnetic reconnection in otherwise tearing stable plasmas, giving rise to formation of non-rotating magnetic island chains on rational magnetic surfaces—a.k.a. “locked modes”.

- Locked modes severely degrade plasma energy confinement, and often trigger disruptions.
Plasma Response to Error Fields

• Response very different to that predicted by naively superimposing vacuum perturbation onto equilibrium magnetic field.

• *Shielding currents* excited at rational surfaces by plasma rotation, and/or combination of pressure gradients and favorable average field-line curvature, act to suppress driven magnetic reconnection.

• *Distributed currents* generated by ideal response of realistic (i.e., highly elongated, moderate aspect-ratio, high-\(\beta\)) plasma equilibrium to perturbation profoundly modify perturbation structure. Consequently, cylindrical models, or models that rely on large aspect-ratio, low-\(\beta\) orderings, do very poor job of predicting plasma response.
Approaches to Error Field Response Calculation

1. Solve resistive-MHD equations throughout whole plasma. Highly inefficient because resistivity and inertia only important close to rational surfaces. Plasma response elsewhere governed by much simpler equations of ideal-MHD.


My Approach to Error Field Response Calculation

- Use recently developed TOMUHAWC code to calculate ideal-MHD response of realistic plasma equilibrium everywhere apart from immediate vicinity of rational surfaces.

- Asymptotically match ideal-MHD data obtained from TOMUHAWC to Glasser-Greene-Johnson linear layers at various rational surfaces. (GGJ linear layer response theory is simplest non-ideal response model that is consistent with realistic plasma equilibrium.)
Plasma Equilibrium

\[
\begin{align*}
\epsilon &= 0.3 \\
\kappa &= 1.8 \\
\delta &= 0.25 \\
q_0 &= 1.05 \\
q_a &= 3.95 \\
\beta_N &= 0.0–2.95
\end{align*}
\]
Safety Factor Profile

$q = 2 \quad q = 3$

plasma boundary
Coordinate System

- Adopt right-handed flux coordinate system: $r$, $\theta$, $\phi$.
  - $r$ - flux-surface label
  - $\theta$ - “straight” poloidal angle
  - $\phi$ - geometric toroidal angle
- Jacobian:
  \[
  (\nabla r \times \nabla \theta \cdot \nabla \phi)^{-1} = r R^2,
  \]
  where $R$ is major radius.
- Flux-surface average operator:
  \[
  \langle \cdots \rangle \equiv \frac{1}{2\pi} \int \cdots \, d\theta.
  \]
Perturbed Magnetic Field

• Let

\[ \delta B \cdot \nabla r = i \sum_j \frac{\psi_j(r)}{r R^2} \exp[i (m_j \theta - n \phi)]. \]

• In immediate vicinity of kth rational surface \([q(r_k) = m_k/n]\)

\[ \psi_k(r) = \Psi_k F_k |x_k|^{\nu_L k} + \Delta \Psi_k^\pm F_k \text{sgn}(x_k) |x_k|^{\nu_S k} + A_k x_k, \]

\[ x_k = \frac{(r - r_k)}{r_k}, \]

\[ F_k = \left( \frac{m_k^2 \langle |\nabla r|^2 \rangle + n^2 r^2}{2 \sqrt{D_{I_k}}} \right)^{1/2}_{r_k}, \]

\[ \nu_{L k} = 1/2 - \sqrt{D_{I_k}}, \quad \nu_{S k} = 1/2 + \sqrt{D_{I_k}}, \] and \( D_{I_k} \) is GGJ ideal stability index at kth rational surface.
Asymptotic Matching Parameters

- $\Psi_k$ - measures tearing parity reconnected magnetic flux at kth rational surface.

- $\Delta\Psi_k \equiv \Delta\Psi_k^+ + \Delta\Psi_k^-$ - measures tearing parity shielding currents excited at kth rational surface.

- $\Delta_k \equiv \Delta\Psi_k / \Psi_k$ - determined by non-ideal tearing parity layer solution at kth rational surface.
Homogeneous Tearing Parity Dispersion Relation

- Dispersion relation:

\[ \sum_{k'} \left( E_{kk'} - \delta_{kk'} \Delta_{k'} \right) \Psi_{k'} = 0. \]

- **TOMUHAWC** code calculates elements of hermitian \( E \)-matrix, \( E_{kk'} \). Diagonal elements of matrix are toroidal generalizations of \( \Delta' \) parameter of Furth-Killeen-Rosenbluth theory.

- Toroidal electromagnetic torque at \( k \)th surface:

\[ \delta T_k = 2 n \pi^2 \text{Im}(\Delta_k) |\Psi_k|^2. \]

Fact that \( E \)-matrix is hermitian ensures that zero net torque exerted on plasma.
Error Field Generation

- Perfectly conducting wall subject to small amplitude, static, helical displacement:

\[ \xi = \delta r \frac{|\nabla r|}{|\nabla r|^2}, \]

where

\[ \delta r(\theta, \phi) = \sum_j \Xi_j \exp[i(m_j \theta - n \phi)]. \]
Inhomogeneous Tearing Parity Dispersion Relation

- Dispersion relation:

\[
\sum_{k'} \left( E_{kk'} - \delta_{kk'} \Delta_{k'} \right) \Psi_{k'} = \chi_k,
\]

where

\[
\chi_k = \sum_j \Xi_j \xi_{k,j}.
\]

- TOMUHAWC code calculates \( \xi_{k,j} \) parameters from homogeneous tearing eigenfunctions.
Shielding Factor

- Assume that $k$th rational surface locked to error field, whereas plasma at other rational surfaces rotating sufficiently rapidly to suppress driven reconnection.

- Reconnected flux driven at $k$th surface:

$$\Psi_k = \frac{\chi_k}{E_{kk}} S_k,$$

where

$$S_k = \frac{E_{kk}}{E_{kk} - \Delta_k}$$

is so-called shielding factor—i.e., factor by which driven reconnection at $k$th rational surface suppressed by shielding currents.
Optimum Wall Displacement

- Optimum wall displacement for driving reconnected flux at kth rational surface is

\[ \Xi_j = \delta \hat{\xi}_{k,j}, \]

where

\[ \hat{\xi}_{k,j} = \frac{\xi_{k,j}}{||\xi_k||}, \]

and

\[ ||\xi_k|| = \sum_j |\xi_{k,j}|^2. \]

Here, \( \delta \) is mean wall displacement (in \( r \)).
Overlap Factor

- Overlap factor

\[ \alpha_{12} = \left| \sum_j \hat{\xi}_{1j} \hat{\xi}_{2j}^* \right| \]

measures extent to which optimum wall displacement for driving reconnected flux at rational surface 1 is similar to optimal wall displacement for driving reconnected flux at rational surface 2. If \( \alpha_{12} = 1 \) then wall displacements are identical. If \( \alpha_{12} = 0 \) then wall displacements are independent.
Shielding Factor at $q = 2$ surface

Top to bottom: $S_1 = 10^6$, $S_1 = 10^7$, $S_1 = 10^8$, $S_1 = 10^9$, $S_1 = 10^{10}$. 
Shielding Factor at \( q = 3 \) surface

Top to bottom: \( S_2 = 10^6, S_2 = 10^7, S_2 = 10^8, S_2 = 10^9, S_2 = 10^{10} \).
Optimum Wall Displacement: $q = 2$, Low-$\beta$
Optimum Wall Displacement: \( q = 3, \text{ Low-} \beta \)
Optimum Wall Displacement: $q = 2$, High-$\beta$
Optimum Wall Displacement: $q = 3$, High-$\beta$
$q = 2/q = 3 \text{ Overlap Parameter}$

\begin{figure}
\centering
\includegraphics[width=\textwidth]{overlap_parameter}
\end{figure}
Future Plans

- Include plasma rotation in linear GGJ layer response calculation.
- Generalize GGJ linear layer response model to equivalent nonlinear island response model, so as to determine threshold error-field strengths for penetration.
- Include two-fluid and low collisionality effects in linear layer response model.
- Use TOMUHAWC to calculate error-field induced drag torque due to neoclassical toroidal viscosity.