

# Introduction to Magnetic Island Theory

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## Introduction: Toroidal Magnetic Confinement

- Toroidal magnetic confinement devices designed to trap hot plasma within set of toroidally nested magnetic flux-surfaces.
- Basic principle—charged particles free to stream along field-lines, but “stick” to magnetic flux-surfaces due to (relatively) small gyroradii.
- Energy flows rapidly along field-lines, but can only diffuse across flux-surfaces relatively slowly. Diffusion rate controlled by small-scale plasma turbulence.

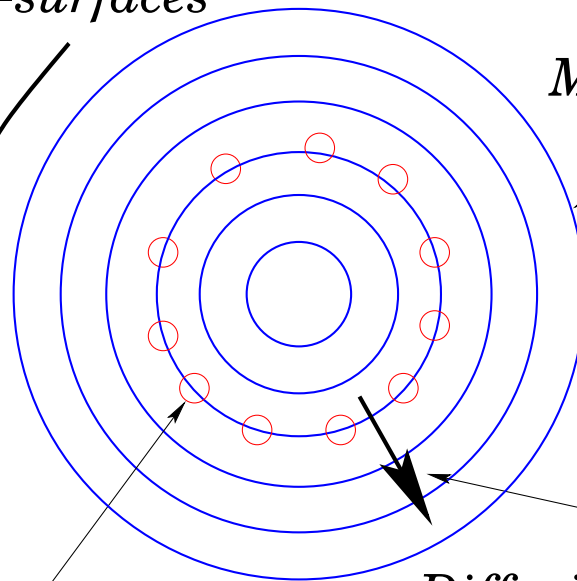
## ***Poloidal Cross-Section of Toroidal Confinement Device***

*Flow around flux-surfaces*

*Magnetic flux-surface*

*Particle gyroradius*

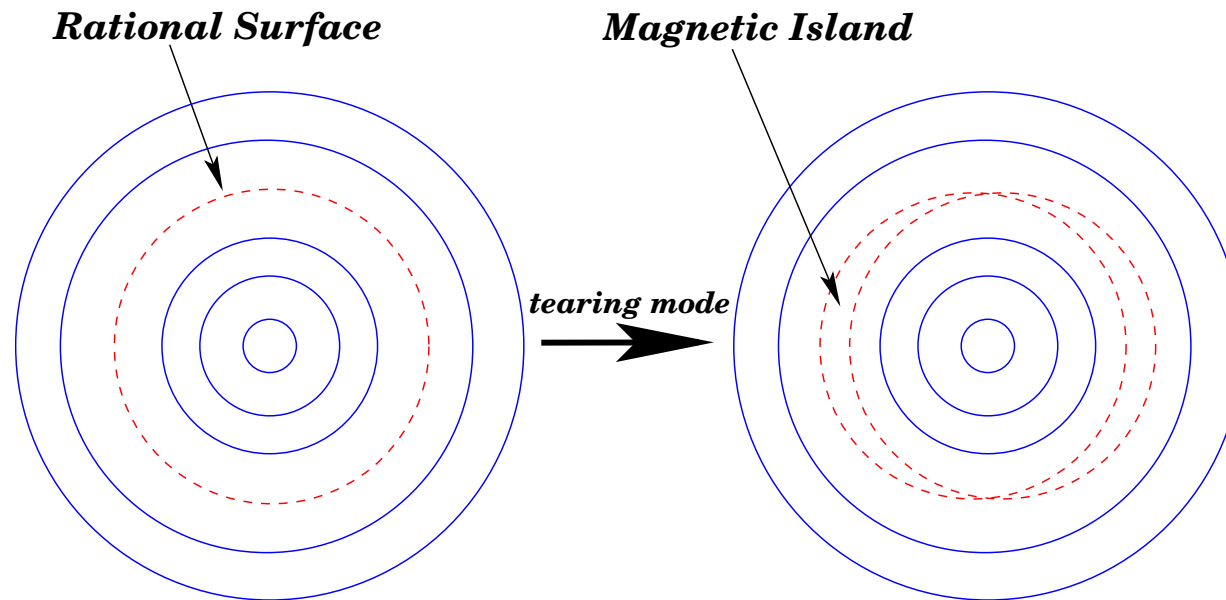
*Diffusion across flux-surfaces*



## Introduction: Magnetohydrodynamical Instabilities

- Two main types of MHD instability in toroidal confinement device:
  - Catastrophic “ideal” (*i.e.*, non-reconnecting) instabilities, which destroy plasma in matter of micro-seconds—we know how to avoid these.
  - Slowly growing “tearing” instabilities, which reconnect magnetic flux-surfaces to form *magnetic islands*, thereby degrading their confinement properties—much harder to avoid.

## Introduction: Magnetic Islands

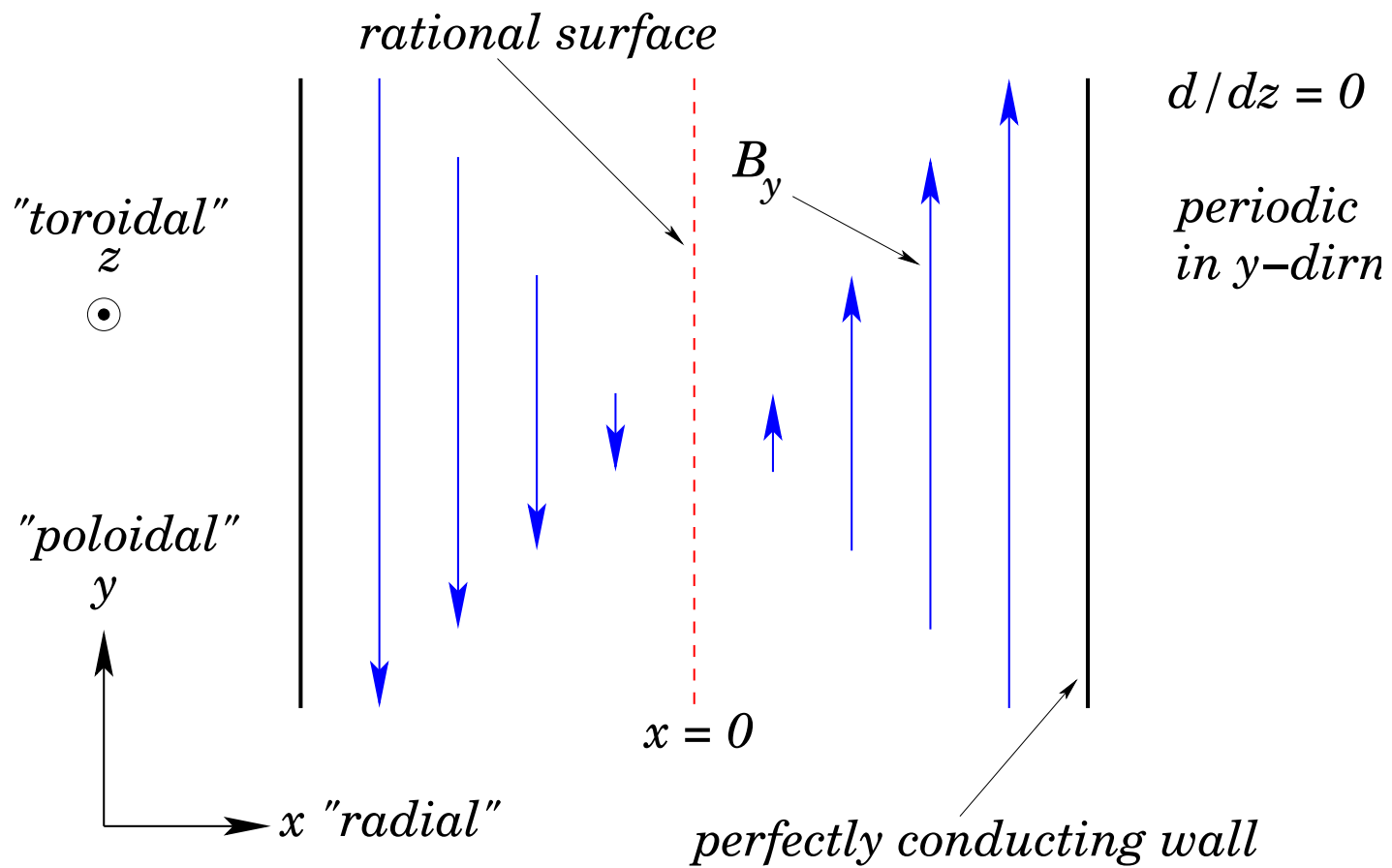


- Centered on *rational flux-surfaces* which satisfy  $\vec{k} \cdot \vec{B} = 0$ , where  $\vec{k}$  is wave-number, and  $\vec{B}$  is equilibrium magnetic field.
- Effectively “short-circuit” confinement by allowing heat to transit island region by rapidly flowing along field-lines, rather than slowly diffusing across flux-surfaces.

## Introduction: Physics of Magnetic Islands

- Magnetic island formation associated with *non-linear* phase of tearing mode growth (*i.e.*, when island width becomes greater than linear layer width).
- Lectures discuss following aspects of magnetic island theory:
  - **MHD effects** - Rutherford island width evolution equation, island velocity, island saturation.
  - **Drift-MHD effects** - diamagnetic current, ion polarization current.
  - **Neoclassical effects** - poloidal flow-damping, bootstrap current.

## *Slab Model*



## MHD Effects: Slab Model

- Cartesian coordinates:  $(x, y, z)$ . Let  $\partial/\partial z \equiv 0$ .
- All lengths/magnetic field-strengths normalized to convenient length-scale/scale field-strength. All times normalized to associated Alfvén time.
- Harris pinch equilibrium:  $B_y = \tanh x$ .
- Perfect wall boundary conditions at  $x = \pm a$ .
- Wave-number of tearing instability:  $\vec{k} = (0, k, 0)$ , so  $\vec{k} \cdot \vec{B} = 0$  at  $x = 0$ . Hence, rational surface at  $x = 0$ .

## MHD effects: Model MHD equations

- Let  $\vec{B} = \nabla\psi \times \vec{z}$  and  $\vec{V} = \nabla\phi \times \vec{z}$ , where  $\vec{V}$  is  $\vec{E} \times \vec{B}$  velocity.
- $\vec{B} \cdot \nabla\psi = \vec{V} \cdot \nabla\phi = 0$ , so  $\psi$  maps magnetic flux-surfaces, and  $\phi$  maps stream-lines of  $\vec{E} \times \vec{B}$  fluid.
- Incompressible MHD equations:

$$\frac{\partial\psi}{\partial t} = [\phi, \psi] + \eta J,$$

$$\frac{\partial\mathcal{U}}{\partial t} = [\phi, \mathcal{U}] + [J, \psi] + \mu \nabla^2 \mathcal{U},$$

where  $J = \nabla^2\psi$ ,  $\mathcal{U} = \nabla^2\phi$ , and  $[A, B] = A_x B_y - A_y B_x$ . Here,  $\eta$  is resistivity, and  $\mu$  is viscosity. In normalized units:  $\eta, \mu \ll 1$ .

- First equation is  $z$ -component of Ohm's law. Second equation is  $z$ -component of curl of plasma equation of motion.

## MHD Effects: Outer Region

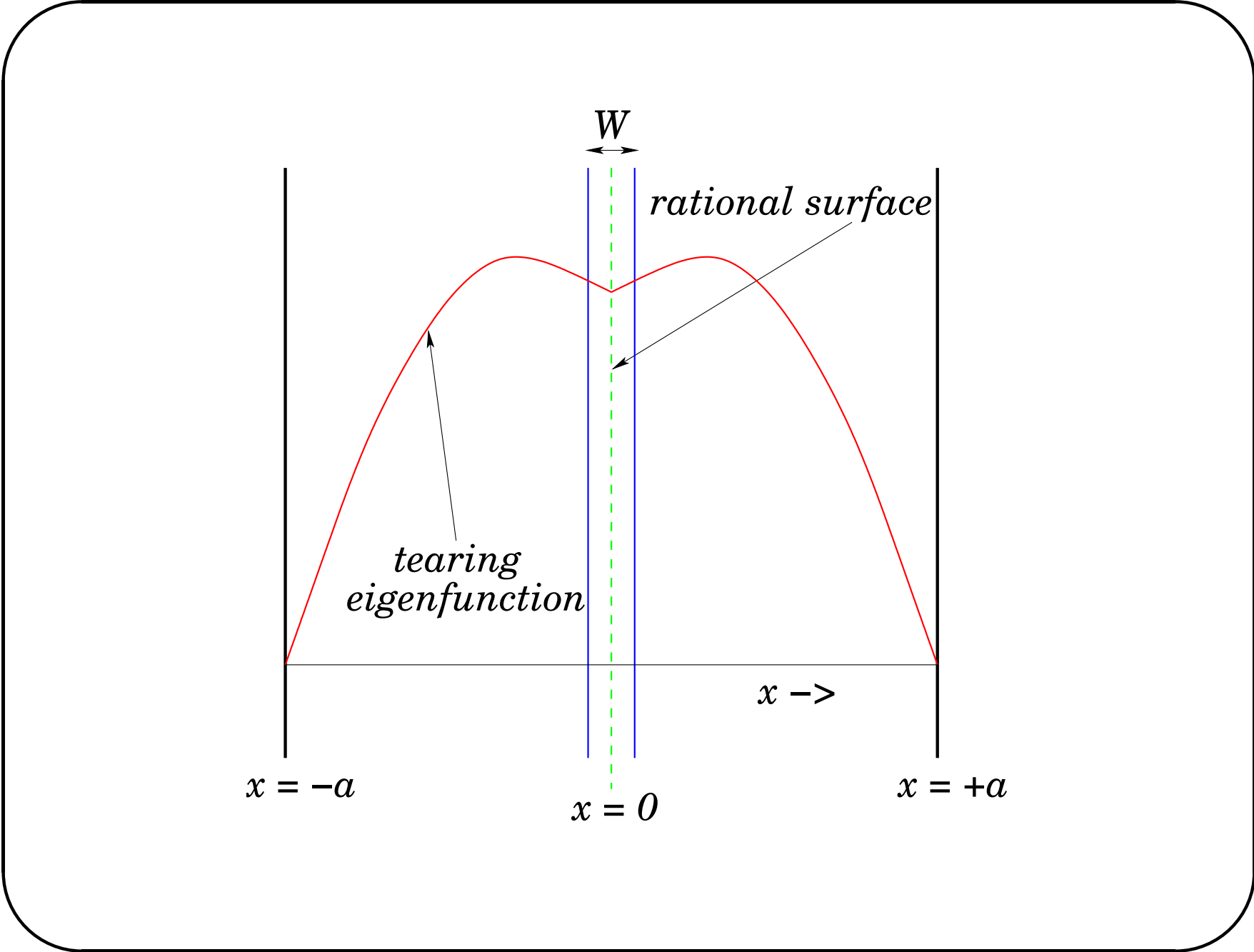
- In “outer region”, which comprises most of plasma, non-linear, non-ideal ( $\eta$  and  $\mu$ ), and inertial effects ( $\partial/\partial t$  and  $\vec{V} \cdot \nabla$ ), negligible.
- Vorticity equation reduces to

$$[J, \psi] \simeq 0.$$

- When linearized, obtain  $\psi(x, y) = \psi^{(0)}(x) + \psi^{(1)}(x) \cos(ky)$ , where  $\psi_0(x) = -\ln[\cosh(x)]$ , and

$$\left( \frac{d^2}{dx^2} - k^2 \right) \psi^{(1)} - \left( \frac{d^2 B_y^{(0)} / dx^2}{B_y^{(0)}} \right) \psi^{(1)} = 0.$$

- Equation is *singular* at rational surface,  $x = 0$ , where  $B_y^{(0)} = 0$ .



## MHD Effects: Tearing Stability Index

- Find tearing eigenfunction,  $\psi^{(1)}(x)$ , which is continuous, has tearing parity [ $\psi^{(1)}(-x) = \psi^{(1)}(x)$ ], and satisfies boundary condition  $\psi^{(1)}(a) = 0$  at conducting wall.
- In general, eigenfunction has *gradient discontinuity* across rational surface (at  $x = 0$ ). Allowed because tearing mode equation singular at rational surface.
- Tearing stability index:

$$\Delta' = \left[ \frac{d \ln \psi^{(1)}}{dx} \right]_{0-}^{0+}.$$

- According to conventional MHD theory,<sup>a</sup> tearing mode is unstable if  $\Delta' > 0$ .

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<sup>a</sup>H.P. Furth, J. Killeen, and M.N. Rosenbluth, Phys. Fluids **6**, 459 (1963).

## MHD Effects: Inner Region

- Inner region centered on rational surface,  $x = 0$ . Of extent,  $W \ll 1$ , where  $W$  is magnetic island width (in  $x$ ).
- In inner region, non-ideal effects, non-linear effects, and plasma inertia can all be important.
- Inner solution must be asymptotically matched to outer solution already obtained.

## MHD Effects: Constant- $\psi$ Approximation

- $\psi^{(1)}(x)$  generally does not vary significantly in  $x$  over inner region:

$$|\psi^{(1)}(W) - \psi^{(1)}(0)| \ll |\psi^{(1)}(0)|.$$

- *Constant- $\psi$  approximation*: treat  $\psi^{(1)}(x)$  as constant in  $x$  over inner region.
- Approximation valid provided

$$|\Delta'|W \ll 1,$$

which is easily satisfied for conventional tearing modes.

## MHD Effects: Constant- $\psi$ Magnetic Island

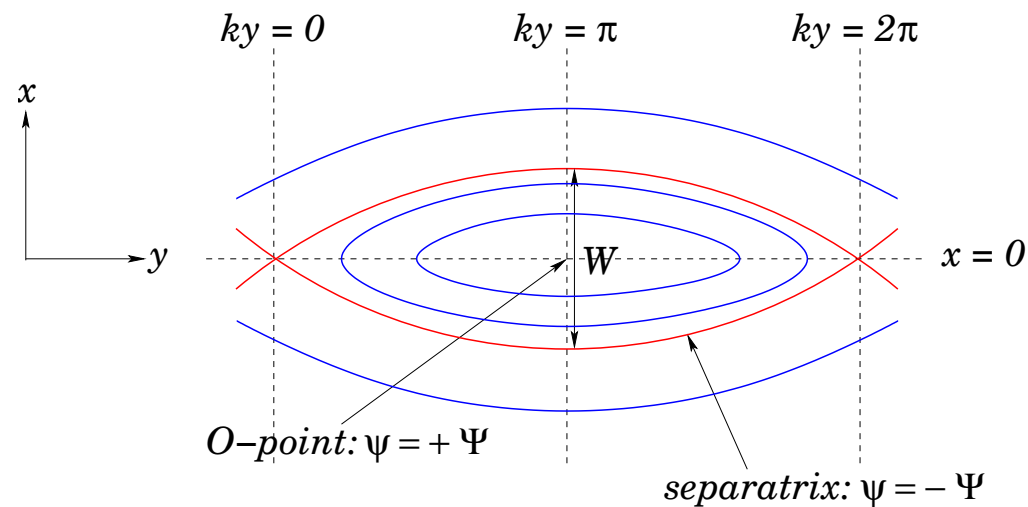
- In vicinity of rational surface:

$$\psi(x, y, t) \simeq -\frac{1}{2} x^2 + \Psi(t) \cos \theta,$$

where  $\Psi = \psi^{(1)}(0)$  is “reconnected flux, and  $\theta = ky$ .

- Full island width

$$W = 4 \sqrt{\Psi}.$$



## MHD Effects: Flux-Surface Average Operator

- Flux-surface average operator is annihilator of Poisson bracket  $[A, \psi] \equiv \vec{B} \cdot \nabla A \equiv k \times (\partial A / \partial \theta)_\psi$  for any  $A$ : *i.e.*,

$$\langle [A, \psi] \rangle \equiv 0.$$

- Outside separatrix:

$$\langle f(\psi, \theta) \rangle = \oint \frac{f(\psi, \theta)}{|\mathbf{x}|} \frac{d\theta}{2\pi}.$$

- Inside separatrix:

$$\langle f(s, \psi, \theta) \rangle = \int_{-\theta_0}^{\theta_0} \frac{f(s, \psi, \theta) + f(-s, \psi, \theta)}{2|\mathbf{x}|} \frac{d\theta}{2\pi},$$

where  $s = \text{sgn}(\mathbf{x})$ , and  $\mathbf{x}(s, \psi, \theta_0) = 0$ .

## MHD Effects: MHD Flow -I

- Move to island frame. Look for steady-state solution:  $\partial/\partial t = 0$ .<sup>a</sup>
- Ohm's law:

$$0 \simeq [\phi, \psi] + \eta J.$$

- Since  $\eta \ll 1$ , first term potentially much larger than second.
- To lowest order:

$$[\phi, \psi] \simeq 0.$$

- Follows that

$$\phi = \phi(\psi);$$

*i.e.*, MHD flow constrained to be around flux-surfaces.

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<sup>a</sup>F.L. Waelbroeck, and R. Fitzpatrick, Phys. Rev. Lett. **78**, 1703 (1997).

## MHD Effects: MHD Flow - II

- Let

$$M(\psi) = \frac{d\phi}{d\psi}.$$

- Easily shown that

$$V_y = x M.$$

- By symmetry,  $M(\psi)$  is *odd* function of  $x$ . Hence,

$$M = 0$$

inside separatrix: *i.e.*, no flow inside separatrix in island frame.  
Plasma *trapped* within magnetic separatrix.

## MHD Effects: MHD Flow - III

- Vorticity equation:

$$0 \simeq -M [\mathbf{U}, \psi] + [J, \psi] + \mu \nabla^4 \phi.$$

- Flux-surface average, recalling that  $\langle [A, \psi] \rangle = 0$ :

$$\langle \nabla^4 \phi \rangle \equiv -\frac{d^2}{d\psi^2} \left( \langle x^4 \rangle \frac{dM}{d\psi} \right) \simeq 0.$$

- Solution outside separatrix:

$$M(\psi) = \text{sgn}(x) M_0 \int_{-\Psi}^{\psi} d\psi / \langle x^4 \rangle \bigg/ \int_{-\Psi}^{-\infty} d\psi / \langle x^4 \rangle.$$

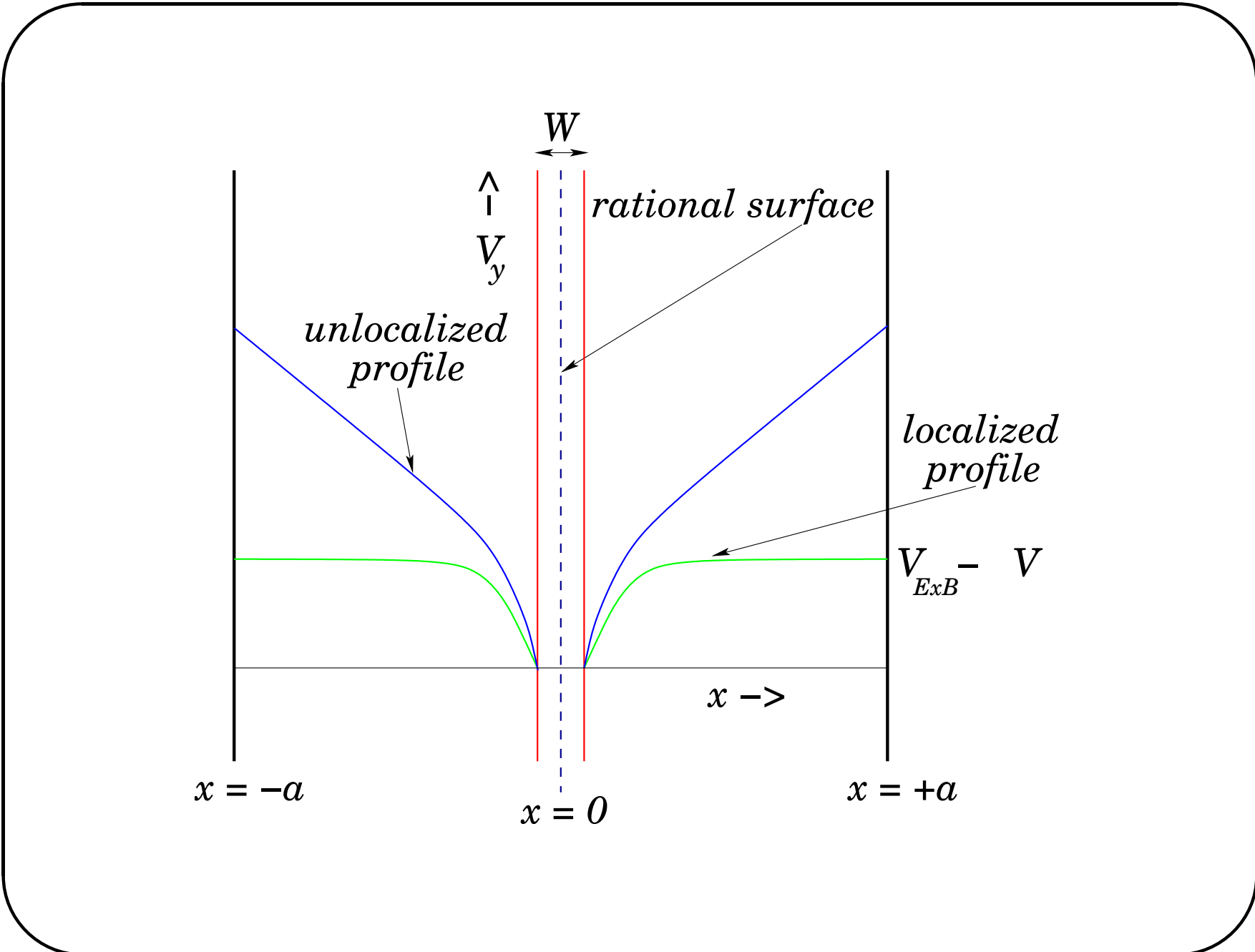
## MHD Effects: MHD Flow - IV

- Note

$$V_y = x M \rightarrow |x| M_0$$

as  $|x|/W \rightarrow \infty$ .

- V-shaped velocity profile which extends over whole plasma.
- Expect *isolated* magnetic island to have *localized* velocity profile. Suggests that  $M_0 = 0$  for isolated island.
- Hence, zero MHD flow in island frame: *i.e.*, island propagates at local  $\vec{E} \times \vec{B}$  velocity.



## MHD Effects: Rutherford Equation - I

- Asymptotic matching between inner and outer regions yields:

$$\Delta' \Psi = -4 \int_{+\Psi}^{-\infty} \langle J \cos \theta \rangle d\psi.$$

- In island frame, in absence of MHD flow, vorticity equation reduces to

$$[J, \psi] \simeq 0.$$

- Hence,

$$J = J(\psi).$$

## MHD Effects: Rutherford Equation - II

- Ohm's law:

$$\frac{d\Psi}{dt} \cos \theta \simeq [\phi, \psi] + \eta J(\psi).$$

- Have shown there is no MHD-flow [*i.e.*,  $\phi \sim O(1)$ ], but can still be *resistive flow* [*i.e.*,  $\phi \sim O(\eta)$ ].
- Eliminate resistive flow by flux-surface averaging:

$$\frac{d\Psi}{dt} \langle \cos \theta \rangle \simeq \eta J(\psi) \langle 1 \rangle.$$

- Hence,

$$\Delta' \Psi \simeq -\frac{4}{\eta} \frac{d\Psi}{dt} \int_{+\Psi}^{-\infty} \frac{\langle \cos \theta \rangle^2}{\langle 1 \rangle} d\psi.$$

## MHD Effects: Rutherford Equation - III

- Use  $W = 4 \sqrt{\Psi}$ , and evaluate integral. Obtain *Rutherford island width evolution equation*:<sup>a</sup>

$$\frac{0.823}{\eta} \frac{dW}{dt} \simeq \Delta'$$

- According to Rutherford equation, island grows *algebraically* on *resistive time-scale*.
- Rutherford equation does not predict island saturation.

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<sup>a</sup>P.H. Rutherford, Phys. Fluids **16**, 1903 (1973).

## MHD Effects: Rutherford Equation - IV

- Higher order asymptotic matching between inner and outer regions yields:<sup>a</sup>

$$\frac{0.823}{\eta} \frac{dW}{dt} \simeq \Delta' - 0.41 \left( -\frac{d^4 B_y^{(0)} / dx^4}{d^2 B_y^{(0)} / dx^2} \right)_{x=0} W.$$

- Hence, saturated ( $d/dt = 0$ ) island width is

$$W_0 = \frac{\Delta'}{0.41} \left( -\frac{d^2 B_y^{(0)} / dx^2}{d^4 B_y^{(0)} / dx^4} \right)_{x=0}.$$

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<sup>a</sup>F. Militello, and F. Porcelli, Phys. Plasmas **11**, L13 (2004); D.F. Escande, and M. Ottaviani, Physics Lett. A **323**, 278 (2004).

## MHD Effects: Summary

- Tearing mode unstable if  $\Delta' > 0$ .
- Island propagates at local  $\vec{E} \times \vec{B}$  velocity at rational surface.
- Island grows algebraically on resistive time-scale.
- Saturated island width:

$$W_0 = \frac{\Delta'}{0.41} \left( -\frac{d^2 B_y^{(0)} / dx^2}{d^4 B_y^{(0)} / dx^4} \right)_{x=0} .$$

## Drift Effects: Introduction

- Retain slab model, for sake of simplicity.
- Physics of outer region identical to MHD model, only physics of inner region modified.
- Main new effect: *diamagnetic current* due to pressure gradient across island region:

$$\vec{J}_* \times \vec{B} \simeq \nabla P.$$

In MHD approximation,  $\vec{J}_*$  assumed to be negligible.

- Diamagnetic current causes ion and electron fluids to flow in  $y$ -direction at substantially *different velocities*. Does island propagate with ion or electron fluid?

## Drift Effects: Basic Definitions

- Variables:
  - $\psi$  - magnetic flux-function.
  - $\phi$  - *guiding-center* (*i.e.*, MHD) stream-function.
  - $P$  - (normalized) plasma pressure (also  $\propto \delta B_z$ ).
  - $V_z$  - (normalized) parallel ion velocity.
  - $J = \nabla^2 \psi$ ,  $U = \nabla^2 \phi$ .
- Parameters:
  - $\rho_s$  - ion Larmor radius calculated with electron temperature.
  - $c_s$  - sound speed.
  - $\tau = T_i/T_e$ .  $\eta$  - resistivity.  $\kappa$  - (perpendicular) thermal conductivity.  $\mu_{i/e}$  - (perpendicular) ion/electron viscosity.

## Drift Effects: Model Drift-MHD Equations - I

- Drift-MHD equations:<sup>a</sup>

$$\frac{\partial \psi}{\partial t} = [\phi - \rho_s P, \psi] + \eta J,$$

$$\frac{\partial P}{\partial t} = [\phi, P] + [c_s V_z + \rho_s J, \psi] + \kappa \nabla^2 P,$$

$$\begin{aligned} \frac{\partial \mathbf{U}}{\partial t} = & [\phi, \mathbf{U}] - \frac{\rho_s \tau}{2} \{ \nabla^2 [\phi, P] + [\mathbf{U}, P] + [\nabla^2 P, \phi] \} \\ & + [J, \psi] + \mu_i \nabla^4 (\phi + \rho_s \tau P) + \mu_e \nabla^4 (\phi - \rho_s P), \end{aligned}$$

$$\frac{\partial V_z}{\partial t} = [\phi, V_z] + c_s [P, \psi] + \mu_i \nabla^2 V_z.$$

- Reduce to MHD equations in limit  $\rho_s \rightarrow 0$ .

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<sup>a</sup>R. Fitzpatrick, and F.L. Waelbroeck, Phys. Plasmas **12**, 022307 (2005).

## Drift Effects: Model Drift-MHD Equations - II

- Boundary conditions in island rest-frame. As  $|x|/W \rightarrow \infty$ :
  - $\psi \rightarrow -\frac{1}{2} x^2 + \Psi \cos \theta$ .
  - $P \rightarrow -\frac{V_*}{\rho_s (1+\tau)} x$ .
  - $\phi \rightarrow -(V_{EB} - V) x$ .
  - $V_z \rightarrow 0$ .
- Here,  $V_* = J_*/n e$  is local diamagnetic velocity,  $V_{EB}$  is local  $\vec{E} \times \vec{B}$  velocity (in lab. frame), and  $V$  is island phase-velocity (in lab. frame).

## Drift Effects: Island Propagation - I

- Look for steady-state ( $\partial/\partial t = 0$ ) solution in island frame.
- Ohm's law:

$$0 = [\phi - \rho_s P, \psi] + \eta J.$$

- Since  $\eta \ll 1$ , first term potentially much larger than second.
- To lowest order:

$$[\phi - \rho_s P, \psi] \simeq 0.$$

- Follows that

$$\phi - \rho_s P = F(\psi) :$$

*i.e.*, electron stream-function  $\phi_e = \phi - \rho_s P$  is flux-surface function. Electron fluid flow constrained to be around flux-surfaces.

## Drift Effects: Island Propagation - II

- Parallel flow equation:

$$0 = [\phi, V_z] + c_s [P, \psi] + \mu_i \nabla^2 V_z.$$

- Highlighted term dominant provided

$$\frac{W c_s}{V_*} \sim \frac{k_{\parallel} c_s}{\omega_*} \gg 1.$$

- Follows that to lowest order

$$P = P(\psi) :$$

*i.e.*, island sufficiently wide that sound-waves able to flatten pressure profile along field-lines.

## Drift Effects: Island Propagation - III

- To lowest order:

$$\phi = \phi(\psi), \quad P = P(\psi).$$

- Follows that both electron stream-function,  $\phi_e = \phi - \rho_s P$ , and ion stream-function,  $\phi_i = \phi + \rho_s \tau P$ , are flux-surface functions. Both electron and ion fluid flow constrained to follow field-lines.

- Let

$$M(\psi) = d\phi/d\psi, \quad L(\psi) = dP/d\psi.$$

- Follows that

$$V_{E \times B y} = x M, \quad V_{e y} = x (M - \rho_s L), \quad V_{i y} = x (M + \rho_s \tau L).$$

## Drift Effects: Island Propagation - IV

- By symmetry, both  $M(\psi)$  and  $L(\psi)$  are *odd* functions of  $x$ .  
Hence,

$$M(\psi) = L(\psi) = 0$$

inside separatrix: *i.e.*, no electron/ion flow within separatrix in island frame.

- Electron/ion fluids constrained to propagate with island inside separatrix.
- Pressure profile *flattened* within separatrix.

## Drift Effects: Island Propagation - V

- Pressure equation:

$$0 \simeq [c_s V_z + \rho_s J, \psi] + \kappa \nabla^2 P.$$

- Vorticity equation:

$$0 \simeq [-M U - (\rho_s \tau / 2)(L U + M \nabla^2 P) + J, \psi] \\ + \mu_i \nabla^4 (\phi + \rho_s \tau P) + \mu_e \nabla^4 (\phi - \rho_s P).$$

- Flux-surface average, recalling that  $\langle [A, \psi] \rangle = 0$ .

## Drift Effects: Island Propagation - VI

- Obtain

$$\langle \nabla^2 P \rangle \simeq 0,$$

and

$$(\mu_i + \mu_e) \langle \nabla^4 \phi \rangle + \rho_s (\mu_i \tau - \mu_e) \langle \nabla^4 P \rangle \simeq 0.$$

- Solution outside separatrix:

$$M(\psi) = -\frac{\rho_s (\mu_i \tau - \mu_e)}{(\mu_i + \mu_e)} L(\psi) + F(\psi),$$

where

$$L(\psi) = -\text{sgn}(x) L_0 / \langle x^2 \rangle,$$

and  $F(\psi)$  is previously obtained MHD profile:

$$F(\psi) = \text{sgn}(x) F_0 \int_{-\Psi}^{\psi} d\psi / \langle x^4 \rangle \bigg/ \int_{-\Psi}^{-\infty} d\psi / \langle x^4 \rangle.$$

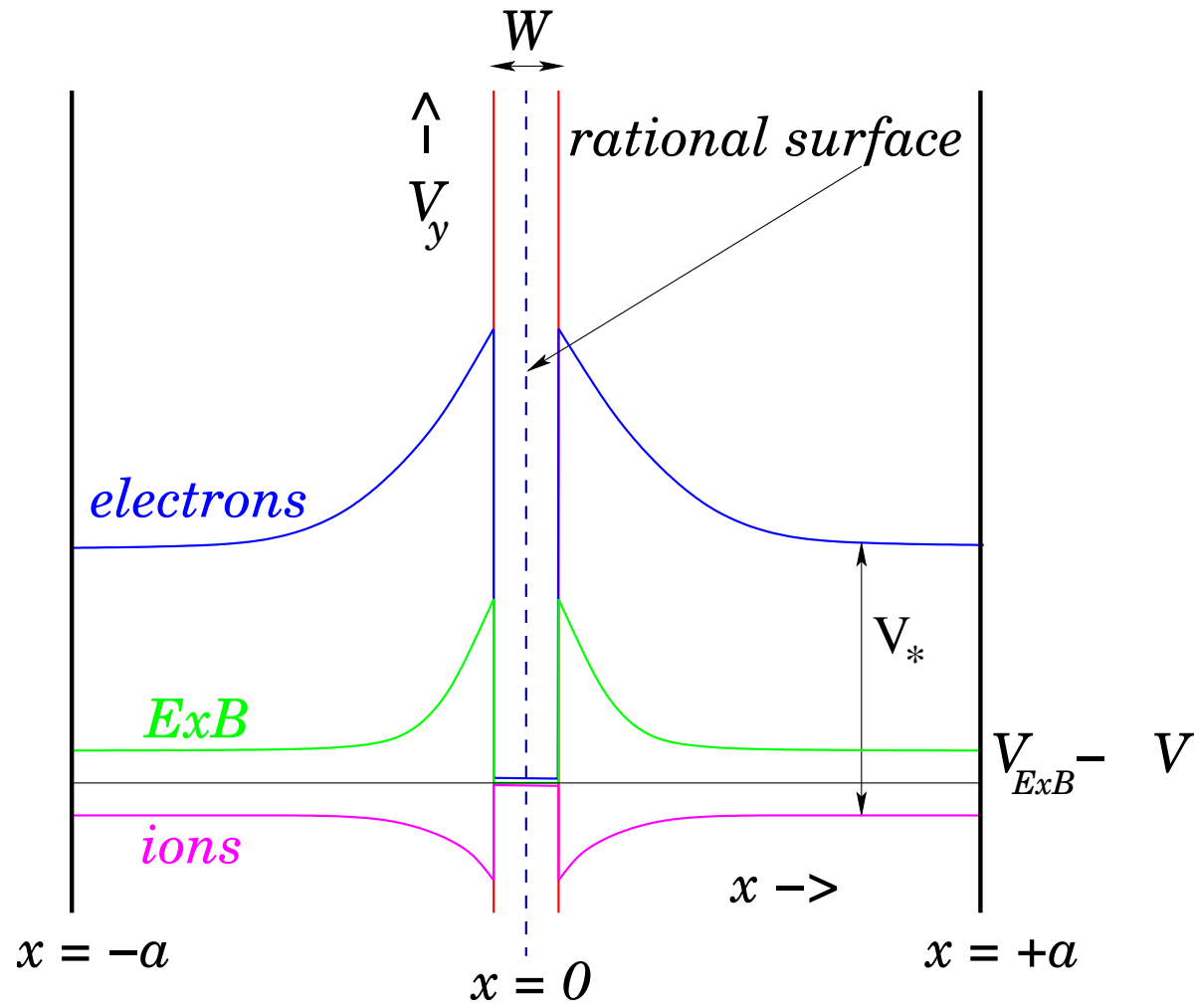
## Drift Effects: Island Propagation - VII

- As  $|x|/W \rightarrow \infty$  then  $xL \rightarrow L_0$  and  $xF \rightarrow |x|F_0$ .
- $L(\psi)$  corresponds to *localized* velocity profile.  $F(\psi)$  corresponds to *non-localized* profile. Require localized profile, so  $F_0 = 0$ .
- Velocity profiles outside separatrix (using b.c. on P):

$$V_{y i} \simeq + \frac{\mu_e}{\mu_i + \mu_e} \frac{|x|}{\langle x^2 \rangle} V_*,$$

$$V_{y E \times B} \simeq - \frac{(\mu_i \tau - \mu_e)}{(1 + \tau)(\mu_i + \mu_e)} \frac{|x|}{\langle x^2 \rangle} V_*,$$

$$V_{y e} = - \frac{\mu_i}{\mu_i + \mu_e} \frac{|x|}{\langle x^2 \rangle} V_*.$$



## Drift Effects: Island Propagation - VIII

- As  $|x|/W \rightarrow \infty$  expect  $V_{y \text{ E} \times \text{B}} \rightarrow V_{\text{EB}} - V$ , where  $V_{\text{EB}}$  is unperturbed (*i.e.*, no island)  $\vec{E} \times \vec{B}$  velocity at rational surface (in lab. frame), and  $V$  is island phase-velocity (in lab. frame).

- Hence

$$V = V_{\text{EB}} + \frac{(\mu_i \tau - \mu_e)}{(1 + \tau)(\mu_i + \mu_e)} V_*.$$

- But unperturbed ion/electron fluid velocities (in lab. frame):

$$V_i = V_{\text{EB}} + \tau V_*/(1 + \tau), \quad V_e = V_{\text{EB}} - V_*/(1 + \tau).$$

- Hence

$$V = \frac{\mu_i}{\mu_i + \mu_e} V_i + \frac{\mu_e}{\mu_i + \mu_e} V_e.$$

So, island phase-velocity is *viscosity weighted average* of unperturbed ion/electron fluid velocities.

## Drift Effects: Polarization Term - I

- Vorticity equation yields:

$$J_c \simeq \frac{1}{2} \left( x^2 - \frac{\langle x^2 \rangle}{\langle 1 \rangle} \right) \frac{d[M (M + \rho_s \tau L)]}{d\psi} + I(\psi)$$

outside separatrix, where  $J_c$  is part of  $J$  with  $\cos \theta$  symmetry.

- As before, flux-surface average of Ohm's law yields:

$$\langle J_c \rangle = I(\psi) \langle 1 \rangle = \eta^{-1} \frac{d\Psi}{dt} \langle \cos \theta \rangle.$$

- Hence

$$J_c \simeq \frac{1}{2} \left( x^2 - \frac{\langle x^2 \rangle}{\langle 1 \rangle} \right) \frac{d[M (M + \rho_s \tau L)]}{d\psi} + \eta^{-1} \frac{d\Psi}{dt} \frac{\langle \cos \theta \rangle}{\langle 1 \rangle}.$$

## Drift Effects: Polarization Term - II

- Asymptotic matching between inner and outer regions yields:

$$\Delta' \Psi = -4 \int_{+\Psi}^{-\infty} \langle J_c \cos \theta \rangle d\psi.$$

- Evaluating flux-surface integrals, making use of previous solutions for  $M$  and  $L$ , obtain modified Rutherford equation:

$$\frac{0.823}{\eta} \frac{dW}{dt} \simeq \Delta' + 1.38 \frac{(V - V_{EB})(V - V_i)}{(W/4)^3}.$$

- New term is due to *polarization current* associated with ion fluid flow around curved island flux-surfaces (in island frame). Obviously, new term is zero if island propagates with ion fluid: *i.e.*,  $V = V_i$ .

## Drift Effects: Polarization Term - III

- Polarization term can be significant, especially for small islands.
- Term stabilizing if island propagates between unperturbed  $\vec{E} \times \vec{B}$  velocity and unperturbed ion velocity. Destabilizing otherwise.
- Recalling that island phase-velocity is viscosity weighted average of unperturbed ion and electron fluid velocities, follows that polarization term is *stabilizing* provided ion (perpendicular) viscosity *exceeds* electron (perpendicular) viscosity (which is what we expect), and destabilizing otherwise.

## Drift Effects: Summary

- Results limited to large islands: *i.e.*, large enough for sound waves to flatten pressure profile.
- Island propagates at (perpendicular) viscosity weighted average of unperturbed (no island) ion and electron fluid velocities.
- Polarization term in Rutherford equation is stabilizing provided ion (perpendicular) viscosity exceeds electron (perpendicular) viscosity (which is what we expect), and destabilizing otherwise.

## Neoclassical Effects: Poloidal Flow Damping

- In toroidal plasma, poloidal flow *strongly damped* by parallel ion viscosity.

- So if

$$\vec{V}_i = \vec{V}_{\perp i} + \vec{V}_{\parallel i} \vec{b}$$

then

$$\vec{V}_{\parallel i} \rightarrow -\vec{V}_{\perp i} \theta \frac{B_{\phi}}{B_{\theta}}.$$

- Strong parallel flow results in *enhancement* of polarization term in Rutherford equation by factor  $(B_{\phi}/B_{\theta})^2$ .<sup>a</sup>

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<sup>a</sup>A.I. Smolyakov, and E. Lazzaro, Phys. Plasmas **11**, 4353 (2004).

## Neoclassical Effects: Bootstrap Current - I

- In toroidal plasma friction between trapped and passing electrons leads to appearance of non-inductive *bootstrap current* in Ohm's law:<sup>a</sup>

$$\frac{d\Psi}{dt} \cos \theta \simeq [\phi, \psi] + \eta [J(\psi) - J_{\text{boot}}],$$

where

$$J_{\text{boot}} = -1.46 \sqrt{\epsilon} B_{\theta}^{-1} \frac{\partial P}{\partial r}.$$

Here,  $\epsilon$  is inverse aspect-ratio,  $1.46 \sqrt{\epsilon}$  is measure of fraction of trapped-particles, and  $P$  is plasma pressure.

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<sup>a</sup>M.N. Rosenbluth, R.D. Hazeltine, and F.L. Hinton, Phys. Fluids **15**, 116 (1972).

## Neoclassical Effects: Bootstrap Current - II

- Have seen that pressure profile *flattened* inside island separatrix.
- Bootstrap current consequently disappears inside separatrix.
- Absence of bootstrap current inside separatrix and continued presence outside leads to *destabilizing* term in Rutherford island equation:<sup>a</sup>

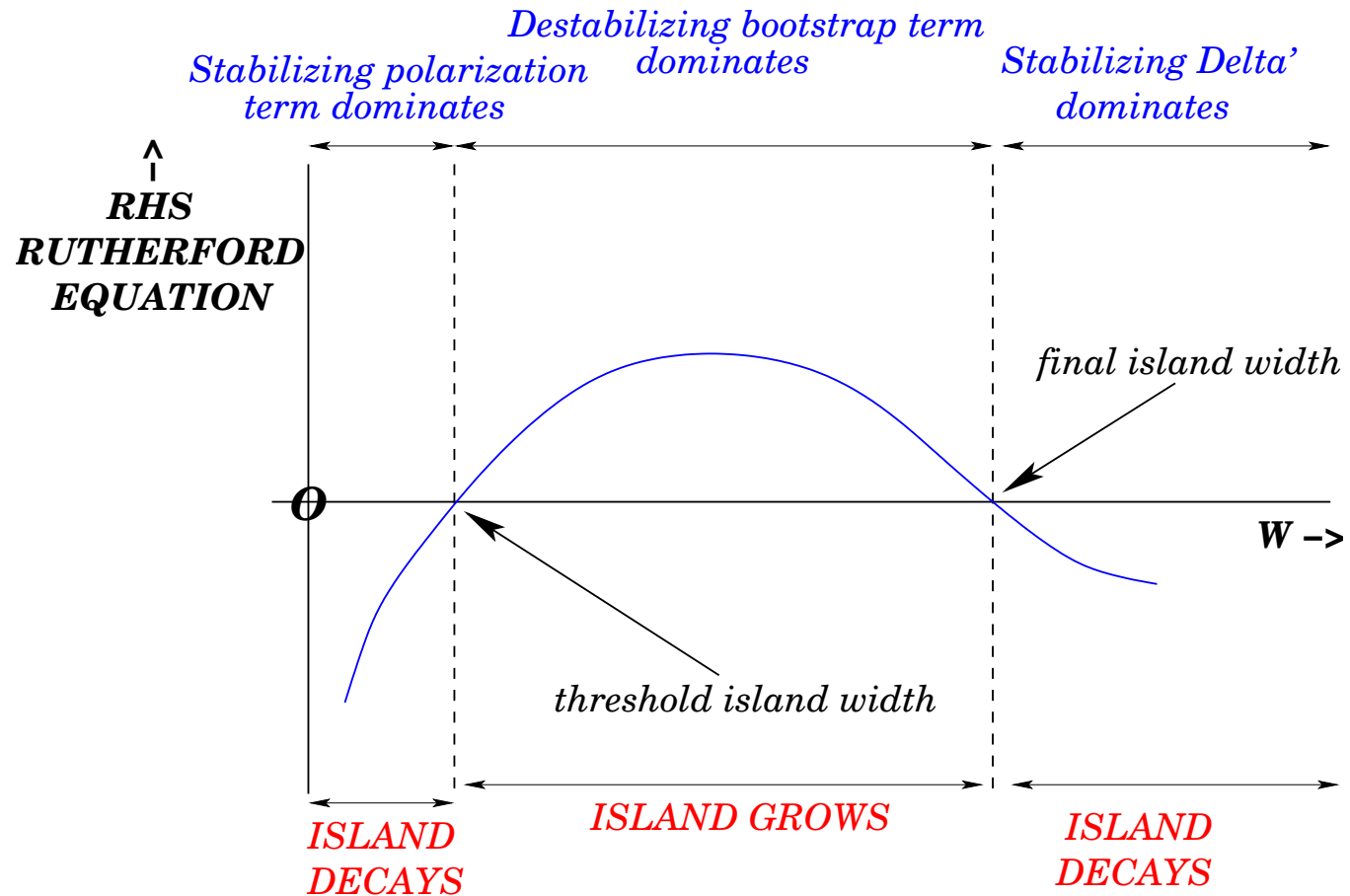
$$\frac{0.823}{\eta} \frac{dW}{dt} \simeq \Delta' + 1.38 \left( \frac{B_\phi}{B_\theta} \right)^2 \frac{(V - V_{EB})(V - V_i)}{(W/4)^3} - 2.31 \sqrt{\epsilon} \frac{(r P' / B_\theta^2)}{(W/4)}$$

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<sup>a</sup>R. Fitzpatrick, Phys. Plasmas **2**, 825 (1995).

# Neoclassical Effects: Neoclassical Tearing Modes - I

- A *neoclassical tearing mode* is an *intrinsically stable* ( $\Delta' < 0$ ) tearing mode destabilized by bootstrap term.



## Neoclassical Effects: Neoclassical Tearing Modes - II

- Neoclassical tearing mode must exceed *threshold island width* in order to grow and saturate: *i.e.*, mode is *meta-stable*.
- Threshold island width set by relative strengths of bootstrap and polarization terms in Rutherford island equation.

## Neoclassical Effects: Summary

- Neoclassical poloidal flow damping leads to *large enhancement* in stabilizing polarization term in Rutherford equation.
- Neoclassical bootstrap current leads to appearance of large new *destabilizing* term in Rutherford island equation.
- – **Bad news**: Large magnetic island can be driven in intrinsically tearing-stable plasma by bootstrap term. Bootstrap drive proportional to  $P'$ —the very quantity we wish to maximize!
- – **Good news**: Island must exceed threshold size before it can grow.