Introduction to Magnetic Island Theory

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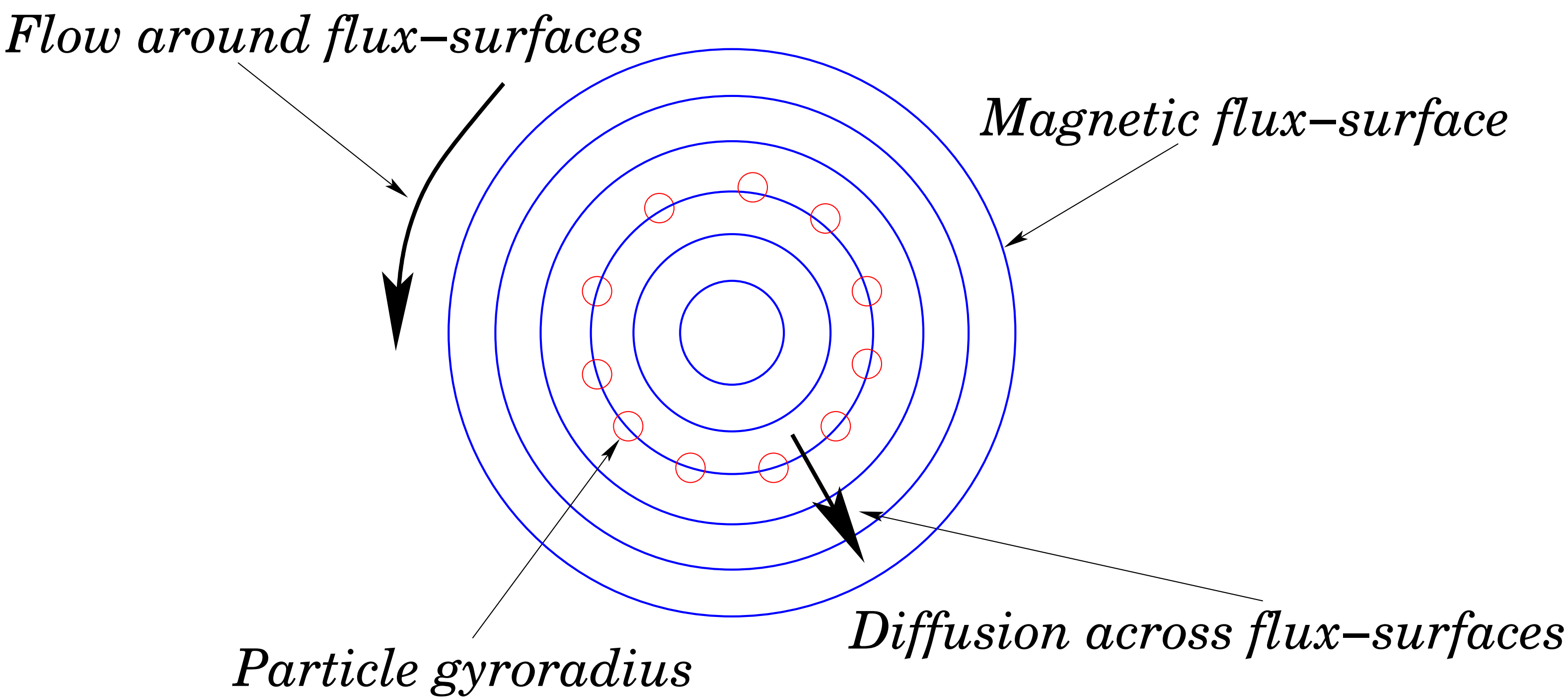
Introduction: Toroidal Magnetic Confinement

- Toroidal magnetic confinement devices designed to trap hot plasma within set of toroidally nested magnetic flux-surfaces.
- Basic principle—charged particles free to stream along field-lines, but "stick" to magnetic flux-surfaces due to (relatively) small gyroradii.
- Energy flows rapidly along field-lines, but can only diffuse across flux-surfaces relatively slowly. Diffusion rate controlled by small-scale plasma turbulence.

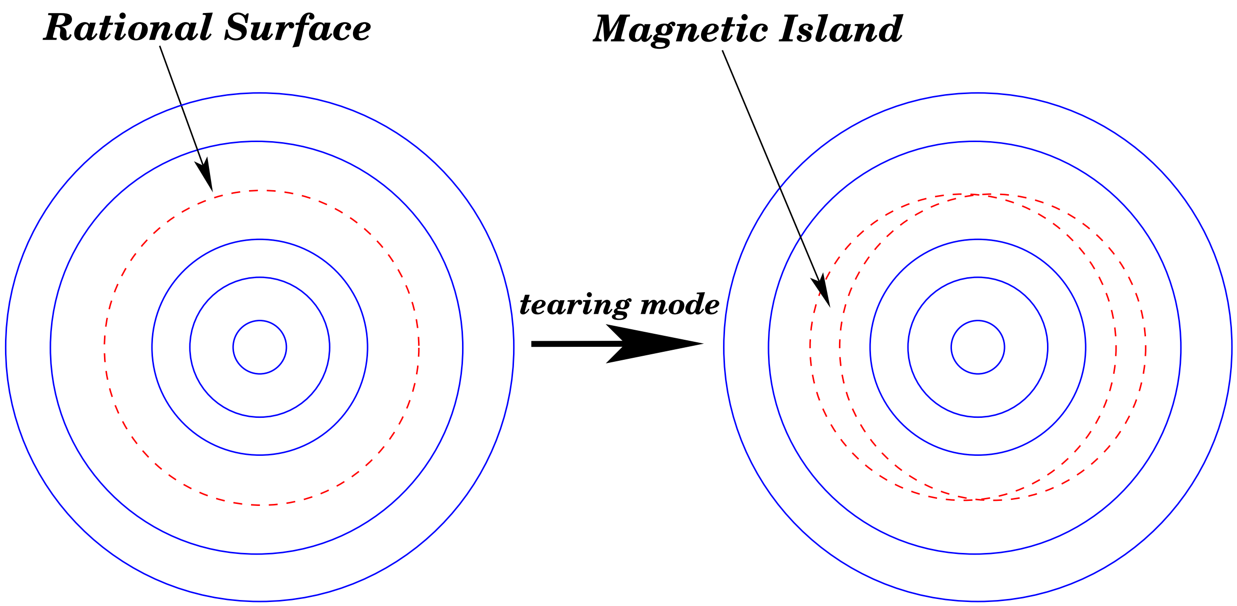
Poloidal Cross–Section of Toroidal Confinement Device



Introduction: Magnetohydrodynamical Instabilities

- Two main types of MHD instability in toroidal confinement device:
  - Catastrophic "ideal" (*i.e.*, non-reconnecting) instabilities, which destroy plasma in matter of micro-seconds—we know how to avoid these.
  - Slowly growing "tearing" instabilities, which reconnect magnetic flux-surfaces to form *magnetic islands*, thereby degrading their confinement properties—much harder to avoid.

Introduction: Magnetic Islands



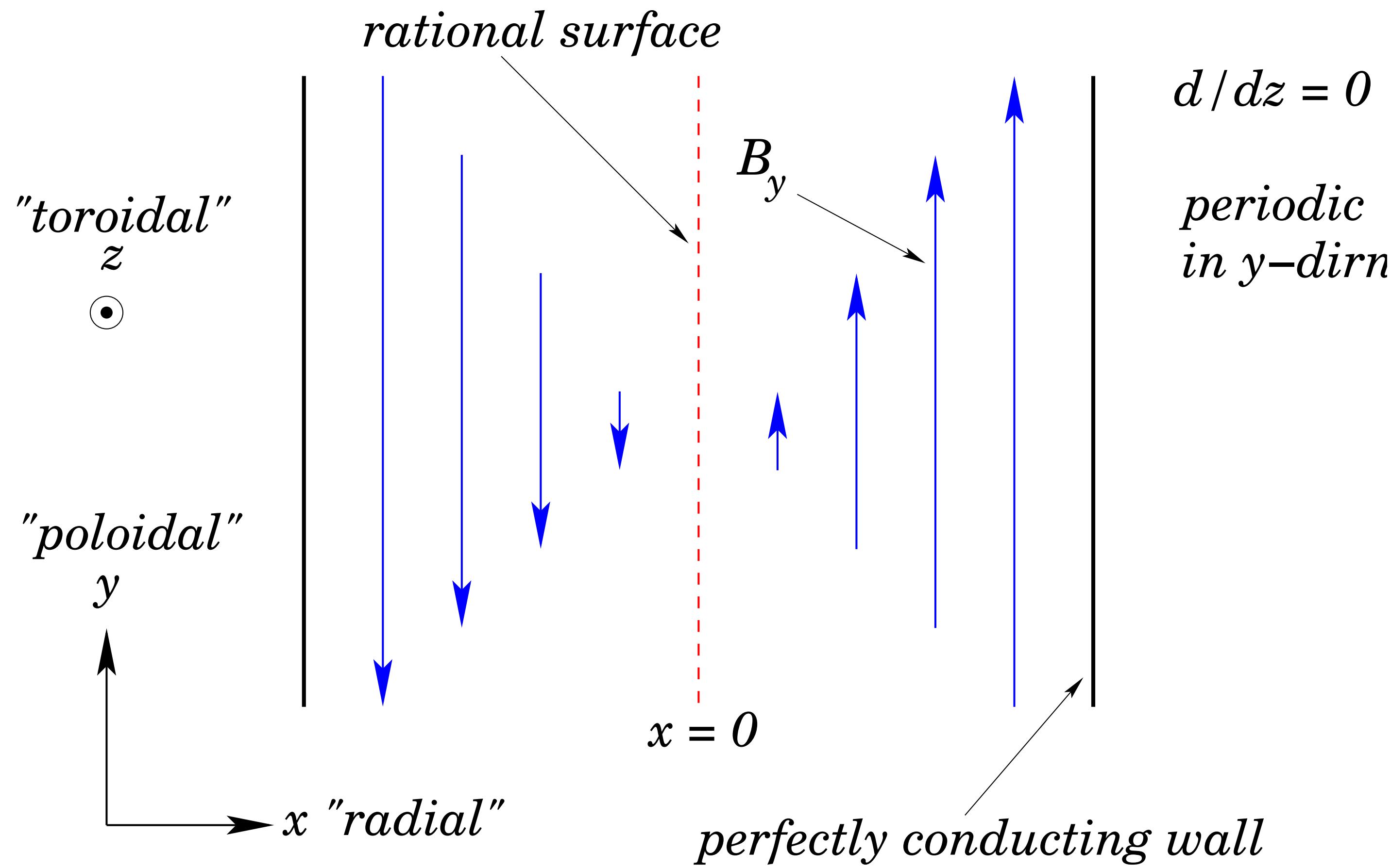
- Centered on *rational flux-surfaces* which satisfy $\vec{k} \cdot \vec{B} = 0$, where $\vec{k}$ is wave-number, and $\vec{B}$ is equilibrium magnetic field.
- Effectively "short-circuit" confinement by allowing heat to transit island region by rapidly flowing along field-lines, rather than slowly diffusing across flux-surfaces.

Introduction: Physics of Magnetic Islands

- Magnetic island formation associated with *non-linear* phase of tearing mode growth (*i.e.*, when island width becomes greater than linear layer width).

- Lectures discuss following aspects of magnetic island theory:
  - MHD effects - Rutherford island width evolution equation, island velocity, island saturation.
  - Drift-MHD effects - diamagnetic current, ion polarization current.
  - Neoclassical effects - poloidal flow-damping, bootstrap current.

Slab Model



rational surface
d/dz = 0
B_y
periodic
in y–dirn
"toroidal"
z
"poloidal"
y
x = 0
x "radial"
perfectly conducting wall

MHD Effects: Slab Model

- Cartesian coordinates: (x, y, z). Let $\partial/\partial z \equiv 0$.
- All lengths/magnetic field-strengths normalized to convenient length-scale/scale field-strength. All times normalized to associated Alfvén time.
- Harris pinch equilibrium: $B_y = \tanh x$.
- Perfect wall boundary conditions at $x = \pm a$.
- Wave-number of tearing instability: $\vec{k} = (0, k, 0)$, so $\vec{k} \cdot \vec{B} = 0$ at $x = 0$. Hence, rational surface at $x = 0$.

MHD effects: Model MHD equations

- Let $\vec{B} = \nabla\psi \times \vec{z}$ and $\vec{V} = \nabla\phi \times \vec{z}$, where $\vec{V}$ is $\vec{E} \times \vec{B}$ velocity.
- $\vec{B} \cdot \nabla\psi = \vec{V} \cdot \nabla\phi = 0$, so ψ maps magnetic flux-surfaces, and ϕ maps stream-lines of $\vec{E} \times \vec{B}$ fluid.
- Incompressible MHD equations:

$$\frac{\partial\psi}{\partial t} = [\phi, \psi] + \eta\, J,$$
$$\frac{\partial U}{\partial t} = [\phi, U] + [J, \psi] + \mu\, \nabla^2 U,$$

where $J = \nabla^2\psi$, $U = \nabla^2\phi$, and $[A, B] = A_x\, B_y - A_y\, B_x$. Here, η is resistivity, and μ is viscosity. In normalized units: $\eta, \mu \ll 1$.

- First equation is z-component of Ohm's law. Second equation is z-component of curl of plasma equation of motion.

MHD Effects: Outer Region

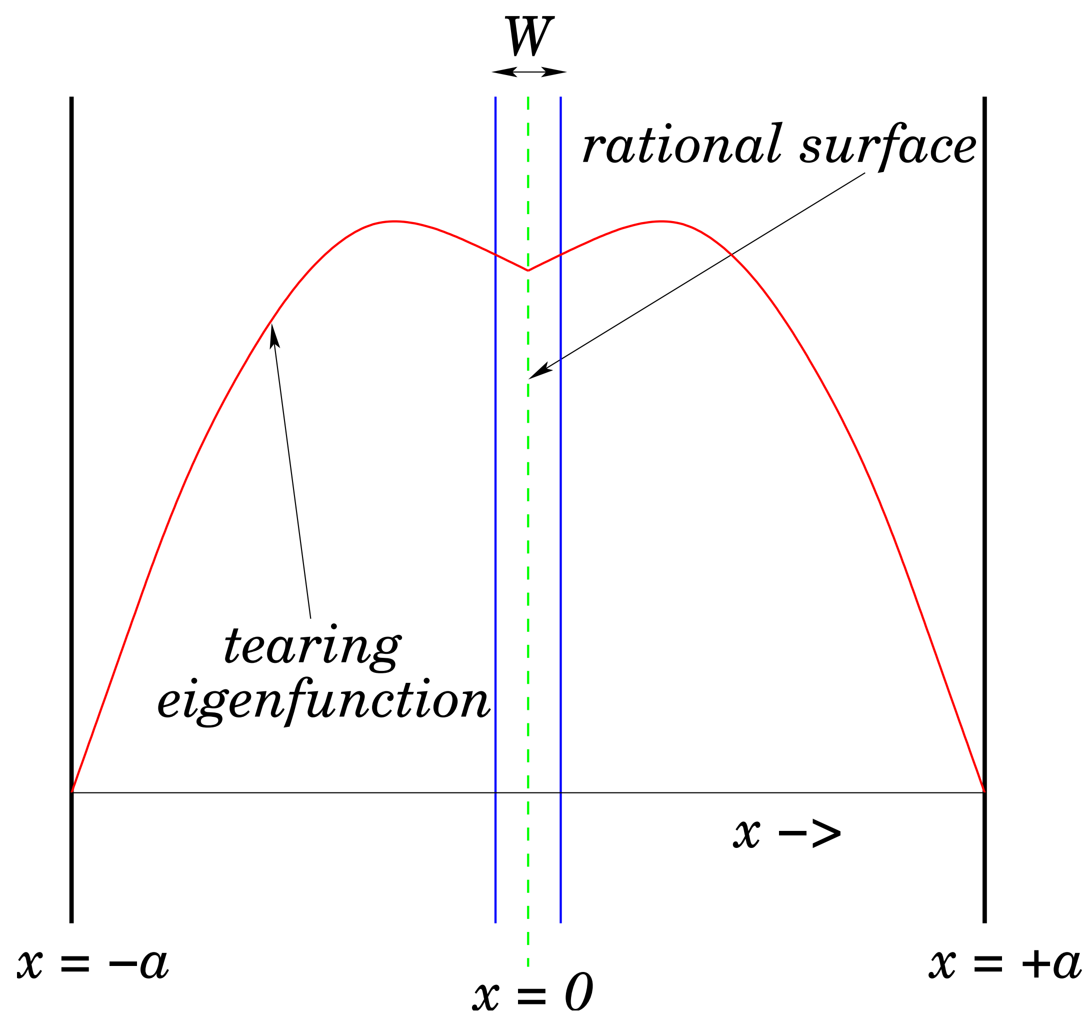
- In "outer region", which comprises most of plasma, non-linear, non-ideal (η and μ), and inertial effects ($\partial/\partial t$ and $\vec{V}\cdot\nabla$), negligible.
- Vorticity equation reduces to

$$[J, \psi] \simeq 0.$$

- When linearized, obtain $\psi(x, y) = \psi^{(0)}(x) + \psi^{(1)}(x)\,\cos(k\,y)$, where $\psi_0(x) = -\ln[\cosh(x)]$, and

$$\left(\frac{d^2}{dx^2} - k^2\right)\psi^{(1)} - \left(\frac{d^2B_y^{(0)}/dx^2}{B_y^{(0)}}\right)\psi^{(1)} = 0.$$

- Equation is *singular* at rational surface, $x = 0$, where $B_y^{(0)} = 0$.


W
rational surface
tearing
eigenfunction
x ->
$x = -a$
$x = 0$
$x = +a$

MHD Effects: Tearing Stability Index

- Find tearing eigenfunction, $\psi^{(1)}(x)$, which is continuous, has tearing parity $[\psi^{(1)}(-x) = \psi^{(1)}(x)]$, and satisfies boundary condition $\psi^{(1)}(a) = 0$ at conducting wall.
- In general, eigenfunction has *gradient discontinuity* across rational surface (at $x = 0$). Allowed because tearing mode equation singular at rational surface.
- Tearing stability index:

$$\Delta' = \left[\frac{d \ln \psi^{(1)}}{dx}\right]_{0-}^{0+}.$$

- According to conventional MHD theory,[a] tearing mode is unstable if $\Delta' > 0$.

[a] H.P. Furth, J. Killeen, and M.N. Rosenbluth, Phys. Fluids **6**, 459 (1963).

MHD Effects: Inner Region

- Inner region centered on rational surface, $x = 0$. Of extent, $W \ll 1$, where W is magnetic island width (in x).
- In inner region, non-ideal effects, non-linear effects, and plasma inertia can all be important.
- Inner solution must be asymptotically matched to outer solution already obtained.

MHD Effects: Constant-ψ Approximation

- $\psi^{(1)}(x)$ generally does not vary significantly in x over inner region:

$$|\psi^{(1)}(W) - \psi^{(1)}(0)| \ll |\psi^{1}(0)|.$$

- *Constant-ψ approximation*: treat $\psi^{(1)}(x)$ as constant in x over inner region.

- Approximation valid provided

$$|\Delta'|\, W \ll 1,$$

which is easily satisfied for conventional tearing modes.

MHD Effects: Constant-ψ Magnetic Island

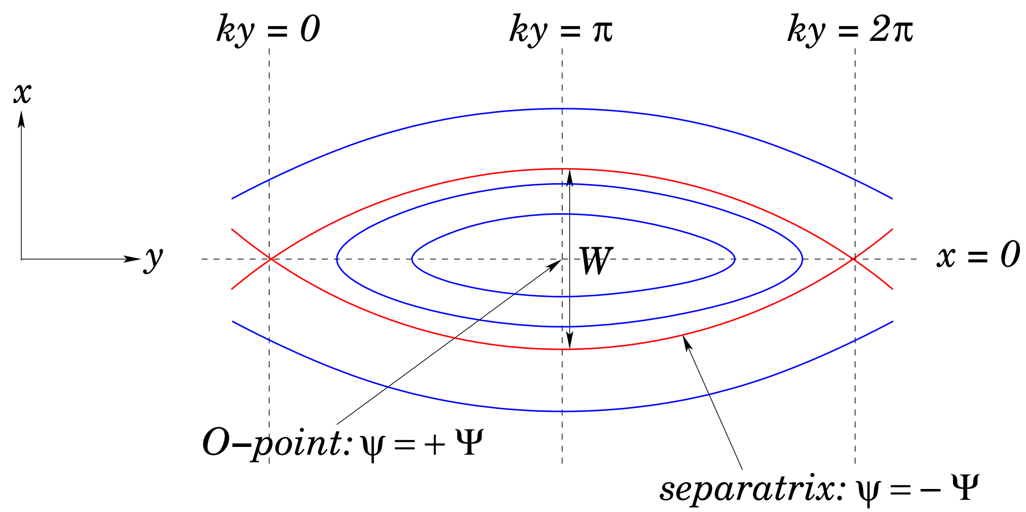
- In vicinity of rational surface:

$$\psi(x, y, t) \simeq -\frac{1}{2}\,x^2 + \Psi(t)\,\cos\theta,$$

where $\Psi = \psi^{(1)}(0)$ is "reconnected flux, and $\theta = k\,y$.

- Full island width

$$W = 4\,\sqrt{\Psi}.$$



MHD Effects: Flux-Surface Average Operator

- Flux-surface average operator is annihilator of Poisson bracket $[A, \psi] \equiv \vec{B} \cdot \nabla A \equiv k\, x\, (\partial A/\partial\theta)_\psi$ for any A: *i.e.*,

$$\langle [A, \psi] \rangle \equiv 0.$$

- Outside separatrix:

$$\langle f(\psi, \theta) \rangle = \oint \frac{f(\psi, \theta)}{|x|}\, \frac{d\theta}{2\pi}.$$

- Inside separatrix:

$$\langle f(s, \psi, \theta) \rangle = \int_{-\theta_0}^{\theta_0} \frac{f(s, \psi, \theta) + f(-s, \psi, \theta)}{2\,|x|}\, \frac{d\theta}{2\pi},$$

where $s = \mathrm{sgn}(x)$, and $x(s, \psi, \theta_0) = 0$.

MHD Effects: MHD Flow -I

- Move to island frame. Look for steady-state solution: $\partial/\partial t = 0$.[a]
- Ohm's law:

$$0 \simeq [\phi, \psi] + \eta\, J.$$

- Since $\eta \ll 1$, first term potentially much larger than second.
- To lowest order:

$$[\phi, \psi] \simeq 0.$$

- Follows that

$$\phi = \phi(\psi);$$

i.e., MHD flow constrained to be around flux-surfaces.

[a] F.L. Waelbroeck, and R. Fitzpatrick, Phys. Rev. Lett. **78**, 1703 (1997).

MHD Effects: MHD Flow - II

- Let

$$M(\psi) = \frac{d\phi}{d\psi}.$$

- Easily shown that

$$V_y = x\,M.$$

- By symmetry, $M(\psi)$ is *odd* function of x. Hence,

$$M = 0$$

inside separatrix: *i.e.*, no flow inside separatrix in island frame. Plasma *trapped* within magnetic separatrix.

MHD Effects: MHD Flow - III

- Vorticity equation:

$$0 \simeq -M\,[U,\psi] + [J,\psi] + \mu\,\nabla^4\phi.$$

- Flux-surface average, recalling that $\langle [A,\psi]\rangle = 0$:

$$\langle \nabla^4\phi\rangle \equiv -\frac{d^2}{d\psi^2}\left(\langle x^4\rangle\,\frac{dM}{d\psi}\right) \simeq 0.$$

- Solution outside separatrix:

$$M(\psi) = \mathrm{sgn}(x)\,M_0\,\int_{-\Psi}^{\psi} d\psi/\langle x^4\rangle \bigg/ \int_{-\Psi}^{-\infty} d\psi/\langle x^4\rangle.$$

MHD Effects: MHD Flow - IV

- Note

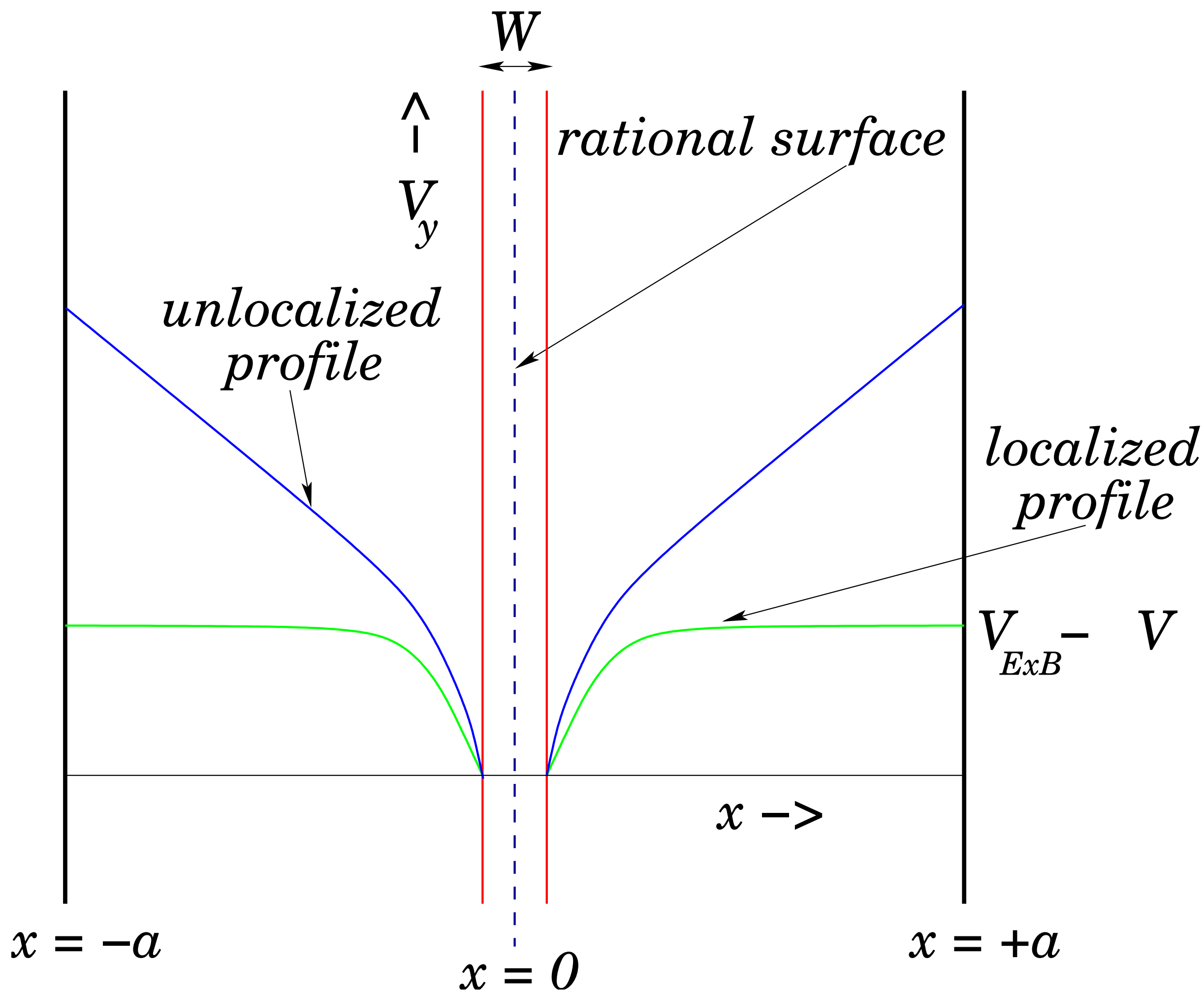
  $$V_y = x\,M \to |x|\,M_0$$

  as $|x|/W \to \infty$.

- V-shaped velocity profile which extends over whole plasma.

- Expect *isolated* magnetic island to have *localized* velocity profile. Suggests that $M_0 = 0$ for isolated island.

- Hence, zero MHD flow in island frame: *i.e.*, island propagates at local $\vec{E} \times \vec{B}$ velocity.


W
V
y
rational surface
unlocalized
profile
localized
profile
V
ExB
− V
x −>
x = −a
x = 0
x = +a

MHD Effects: Rutherford Equation - I

- Asymptotic matching between inner and outer regions yields:

$$\Delta' \Psi = -4 \int_{+\Psi}^{-\infty} \langle J \cos\theta \rangle \, d\psi.$$

- In island frame, in absence of MHD flow, vorticity equation reduces to

$$[J, \psi] \simeq 0.$$

- Hence,

$$J = J(\psi).$$

MHD Effects: Rutherford Equation - II

- Ohm's law:
$$\frac{d\Psi}{dt}\cos\theta \simeq [\phi,\psi] + \eta\,J(\psi).$$
- Have shown there is no MHD-flow [*i.e.*, $\phi \sim O(1)$], but can still be *resistive flow* [*i.e.*, $\phi \sim O(\eta)$].
- Eliminate resistive flow by flux-surface averaging:
$$\frac{d\Psi}{dt}\,\langle\cos\theta\rangle \simeq \eta\,J(\psi)\,\langle 1\rangle.$$
- Hence,
$$\Delta'\,\Psi \simeq -\frac{4}{\eta}\frac{d\Psi}{dt}\int_{+\Psi}^{-\infty}\frac{\langle\cos\theta\rangle^2}{\langle 1\rangle}\,d\psi.$$

MHD Effects: Rutherford Equation - III

- Use $W = 4\sqrt{\Psi}$, and evaluate integral. Obtain *Rutherford island width evolution equation*:[a]

$$\frac{0.823}{\eta}\frac{dW}{dt} \simeq \Delta'.$$

- According to Rutherford equation, island grows *algebraically* on *resistive time-scale*.
- Rutherford equation does not predict island saturation.

[a] P.H. Rutherford, Phys. Fluids **16**, 1903 (1973).

MHD Effects: Rutherford Equation - IV

- Higher order asymptotic matching between inner and outer regions yields:[a]

$$\frac{0.823}{\eta}\frac{dW}{dt} \simeq \Delta' - 0.41\left(-\frac{d^4B_y^{(0)}/dx^4}{d^2B_y^{(0)}/dx^2}\right)_{x=0} W.$$

- Hence, saturated $(d/dt = 0)$ island width is

$$W_0 = \frac{\Delta'}{0.41}\left(-\frac{d^2B_y^{(0)}/dx^2}{d^4B_y^{(0)}/dx^4}\right)_{x=0}.$$

[a] F. Militello, and F. Porcelli, Phys. Plasmas **11**, L13 (2004); D.F. Escande, and M. Ottaviani, Physics Lett. A **323**, 278 (2004).

MHD Effects: Summary

- Tearing mode unstable if $\Delta' > 0$.
- Island propagates at local $\vec{E} \times \vec{B}$ velocity at rational surface.
- Island grows algebraically on resistive time-scale.
- Saturated island width:

$$W_0 = \frac{\Delta'}{0.41} \left(- \frac{d^2 B_y^{(0)}/dx^2}{d^4 B_y^{(0)}/dx^4} \right)_{x=0}.$$

Drift Effects: Introduction

- Retain slab model, for sake of simplicity.
- Physics of outer region identical to MHD model, only physics of inner region modified.
- Main new effect: *diamagnetic current* due to pressure gradient across island region:

$$\vec{J}_* \times \vec{B} \simeq \nabla P.$$

  In MHD approximation, $\vec{J}_*$ assumed to be negligible.
- Diamagnetic current causes ion and electron fluids to flow in y-direction at substantially *different velocities*. Does island propagate with ion or electron fluid?

Drift Effects: Basic Definitions

- Variables:
  - ψ - magnetic flux-function.
  - ϕ - *guiding-center* (*i.e.*, MHD) stream-function.
  - P - (normalized) plasma pressure (also $\propto \delta B_z$).
  - V_z - (normalized) parallel ion velocity.
  - $J = \nabla^2\psi$, $U = \nabla^2\phi$.
- Parameters:
  - ρ_s - ion Larmor radius calculated with electron temperature.
  - c_s - sound speed.
  - $\tau = T_i/T_e$. η - resistivity. κ - (perpendicular) thermal conductivity. $\mu_{i/e}$ - (perpendicular) ion/electron viscosity.

Drift Effects: Model Drift-MHD Equations - I

- Drift-MHD equations:[a]

$$\frac{\partial\psi}{\partial t} = [\phi - \rho_s P, \psi] + \eta\, J,$$

$$\frac{\partial P}{\partial t} = [\phi, P] + [c_s V_z + \rho_s J, \psi] + \kappa\, \nabla^2 P,$$

$$\frac{\partial U}{\partial t} = [\phi, U] - \frac{\rho_s \tau}{2}\left\{\nabla^2[\phi, P] + [U, P] + [\nabla^2 P, \phi]\right\} + [J, \psi] + \mu_i \nabla^4(\phi + \rho_s \tau\, P) + \mu_e \nabla^4(\phi - \rho_s P),$$

$$\frac{\partial V_z}{\partial t} = [\phi, V_z] + c_s [P, \psi] + \mu_i \nabla^2 V_z.$$

- Reduce to MHD equations in limit $\rho_s \to 0$.

[a] R. Fitzpatrick, and F.L. Waelbroeck, Phys. Plasmas **12**, 022307 (2005).

Drift Effects: Model Drift-MHD Equations - II

- Boundary conditions in island rest-frame. As $|x|/W \to \infty$:
  - $\psi \to -\frac{1}{2}\,x^2 + \Psi\,\cos\theta$.
  - $P \to -\frac{V_*}{\rho_s\,(1+\tau)}\,x$.
  - $\phi \to -(V_{EB} - V)\,x$.
  - $V_z \to 0$.
- Here, $V_* = J_*/n\,e$ is local diamagnetic velocity, V_{EB} is local $\vec{E} \times \vec{B}$ velocity (in lab. frame), and V is island phase-velocity (in lab. frame).

Drift Effects: Island Propagation - I

- Look for steady-state ($\partial/\partial t = 0$) solution in island frame.
- Ohm's law:

$$0 = [\phi - \rho_s\, P, \psi] + \eta\, J.$$

- Since $\eta \ll 1$, first term potentially much larger than second.
- To lowest order:

$$[\phi - \rho_s\, P, \psi] \simeq 0.$$

- Follows that

$$\phi - \rho_s\, P = F(\psi) :$$

i.e., electron stream-function $\phi_e = \phi - \rho_s\, P$ is flux-surface function. Electron fluid flow constrained to be around flux-surfaces.

Drift Effects: Island Propagation - II

- Parallel flow equation:

$$0 = [\phi, V_z] + c_s [P, \psi] + \mu_i \nabla^2 V_z.$$

- Highlighted term dominant provided

$$\frac{W\, c_s}{V_*} \sim \frac{k_\parallel\, c_s}{\omega_*} \gg 1.$$

- Follows that to lowest order

$$P = P(\psi) :$$

i.e., island sufficiently wide that sound-waves able to flatten pressure profile along field-lines.

Drift Effects: Island Propagation - III

- To lowest order:

$$\phi = \phi(\psi), \;\; P = P(\psi).$$

- Follows that both electron stream-function, $\phi_e = \phi - \rho_s\, P$, and ion stream-function, $\phi_i = \phi + \rho_s\, \tau\, P$, are flux-surface functions. Both electron and ion fluid flow constrained to follow field-lines.

- Let

$$M(\psi) = d\phi/d\psi, \;\; L(\psi) = dP/d\psi.$$

- Follows that

$$V_{E\times B\, y} = x\, M, \;\; V_{e\, y} = x\, (M - \rho_s L), \;\; V_{i\, y} = x\, (M + \rho_s \tau\, L).$$

Drift Effects: Island Propagation - IV

- By symmetry, both $M(\psi)$ and $L(\psi)$ are *odd* functions of x. Hence,

$$M(\psi) = L(\psi) = 0$$

  inside separatrix: *i.e.*, no electron/ion flow within separatrix in island frame.

- Electron/ion fluids constrained to propagate with island inside separatrix.

- Pressure profile *flattened* within separatrix.

Drift Effects: Island Propagation - V

- Pressure equation:

$$0 \simeq [c_s V_z + \rho_s J, \psi] + \kappa \nabla^2 P.$$

- Vorticity equation:

$$\begin{aligned} 0 &\simeq \left[-M\,U - (\rho_s \tau/2)(L\,U + M\,\nabla^2 P) + J, \psi\right] \\ &\quad + \mu_i \nabla^4(\phi + \rho_s \tau\,P) + \mu_e \nabla^4(\phi - \rho_s P). \end{aligned}$$

- Flux-surface average, recalling that $\langle [A, \psi] \rangle = 0$.

Drift Effects: Island Propagation - VI

- Obtain

$$\langle \nabla^2 P \rangle \simeq 0,$$

and

$$(\mu_i + \mu_e)\langle \nabla^4 \phi \rangle + \rho_s(\mu_i\,\tau - \mu_e)\langle \nabla^4 P \rangle \simeq 0.$$

- Solution outside separatrix:

$$M(\psi) = -\frac{\rho_s(\mu_i\,\tau - \mu_e)}{(\mu_i + \mu_e)} L(\psi) + F(\psi),$$

where

$$L(\psi) = -\mathrm{sgn}(x)\, L_0/\langle x^2 \rangle,$$

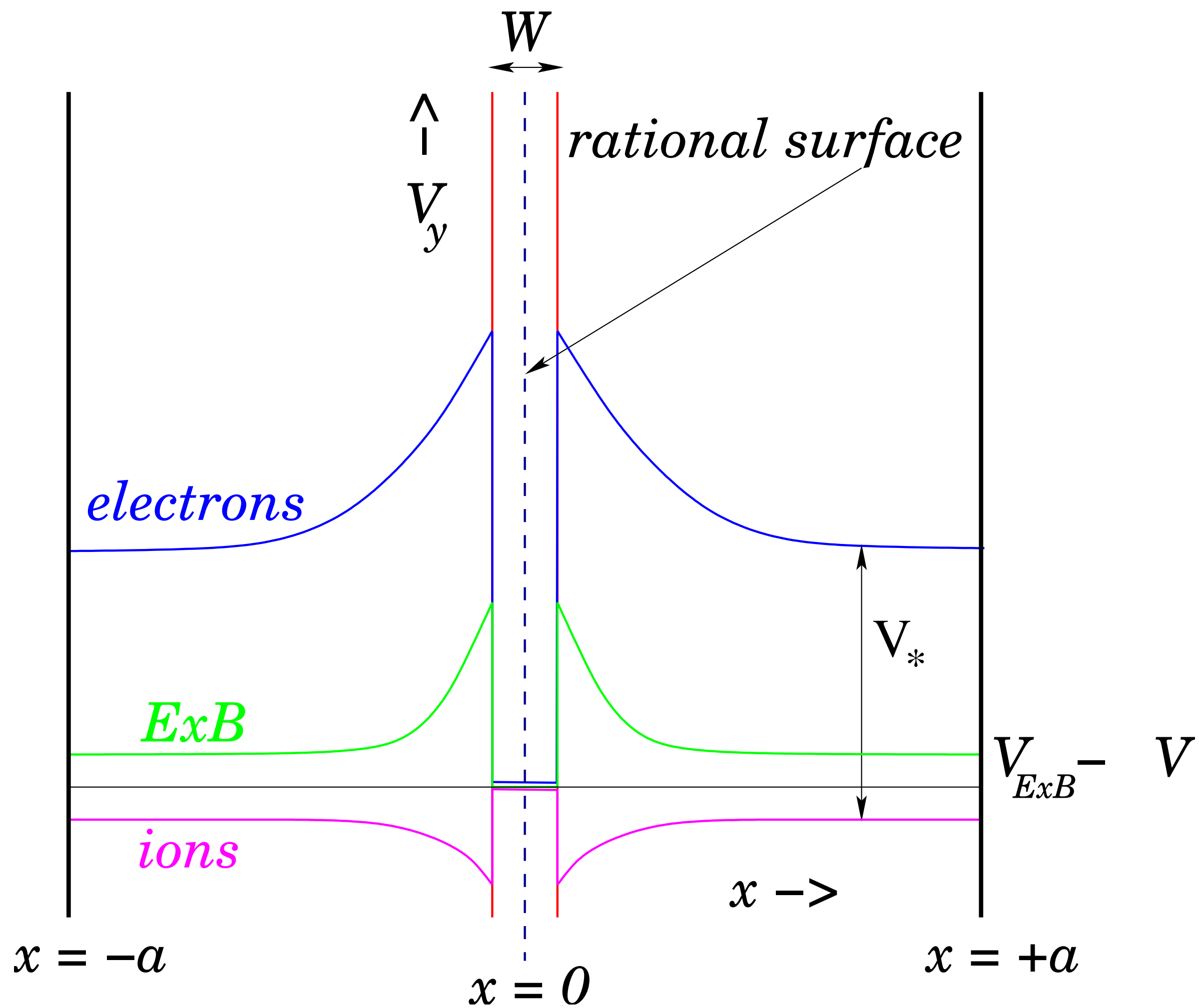
and $F(\psi)$ is previously obtained MHD profile:

$$F(\psi) = \mathrm{sgn}(x)\, F_0 \int_{-\Psi}^{\psi} d\psi/\langle x^4 \rangle \Bigg/ \int_{-\Psi}^{-\infty} d\psi/\langle x^4 \rangle.$$

Drift Effects: Island Propagation - VII

- As $|x|/W \to \infty$ then $x\,L \to L_0$ and $x\,F \to |x|\,F_0$.
- $L(\psi)$ corresponds to *localized* velocity profile. $F(\psi)$ corresponds to *non-localized* profile. Require localized profile, so $F_0 = 0$.
- Velocity profiles outside separatrix (using b.c. on P):

$$
\begin{aligned}
V_{y\,i} &\simeq +\frac{\mu_e}{\mu_i+\mu_e}\,\frac{|x|}{\langle x^2\rangle}\,V_*,\\
V_{y\,E\times B} &\simeq -\frac{(\mu_i\,\tau-\mu_e)}{(1+\tau)\,(\mu_i+\mu_e)}\,\frac{|x|}{\langle x^2\rangle}\,V_*,\\
V_{y\,e} &= -\frac{\mu_i}{\mu_i+\mu_e}\,\frac{|x|}{\langle x^2\rangle}\,V_*.
\end{aligned}
$$


W
rational surface
^
|
V_y
electrons
ExB
V_*
$V_{ExB} - V$
ions
x ->
x = −a
x = 0
x = +a

Drift Effects: Island Propagation - VIII

- As $|x|/W \to \infty$ expect $V_{y\,E\times B} \to V_{EB} - V$, where V_{EB} is unperturbed (*i.e.*, no island) $\vec{E} \times \vec{B}$ velocity at rational surface (in lab. frame), and V is island phase-velocity (in lab. frame).
- Hence
$$V = V_{EB} + \frac{(\mu_i\,\tau - \mu_e)}{(1+\tau)\,(\mu_i + \mu_e)}\,V_*.$$
- But unperturbed ion/electron fluid velocities (in lab. frame):
$$V_i = V_{EB} + \tau\,V_*/(1+\tau), \quad V_e = V_{EB} - V_*/(1+\tau).$$
- Hence
$$V = \frac{\mu_i}{\mu_i + \mu_e}\,V_i + \frac{\mu_e}{\mu_i + \mu_e}\,V_e.$$
So, island phase-velocity is *viscosity weighted average* of unperturbed ion/electron fluid velocities.

Drift Effects: Polarization Term - I

- Vorticity equation yields:

$$J_c \simeq \frac{1}{2}\left(x^2 - \frac{\langle x^2\rangle}{\langle 1\rangle}\right)\frac{d[M\,(M+\rho_s\tau\,L)]}{d\psi} + I(\psi)$$

outside separatrix, where J_c is part of J with $\cos\theta$ symmetry.

- As before, flux-surface average of Ohm's law yields:

$$\langle J_c\rangle = I(\psi)\langle 1\rangle = \eta^{-1}\frac{d\Psi}{dt}\langle\cos\theta\rangle.$$

- Hence

$$J_c \simeq \frac{1}{2}\left(x^2 - \frac{\langle x^2\rangle}{\langle 1\rangle}\right)\frac{d[M\,(M+\rho_s\tau\,L)]}{d\psi} + \eta^{-1}\frac{d\Psi}{dt}\frac{\langle\cos\theta\rangle}{\langle 1\rangle}.$$

Drift Effects: Polarization Term - II

- Asymptotic matching between inner and outer regions yields:

$$\Delta'\,\Psi = -4\int_{+\Psi}^{-\infty}\langle J_c\,\cos\theta\rangle\,d\psi.$$

- Evaluating flux-surface integrals, making use of previous solutions for M and L, obtain modified Rutherford equation:

$$\frac{0.823}{\eta}\frac{dW}{dt} \simeq \Delta' + 1.38\,\frac{(V - V_{EB})\,(V - V_i)}{(W/4)^3}.$$

- New term is due to *polarization current* associated with ion fluid flow around curved island flux-surfaces (in island frame). Obviously, new term is zero if island propagates with ion fluid: *i.e.*, $V = V_i$.

Drift Effects: Polarization Term - III

- Polarization term can be significant, especially for small islands.
- Term stabilizing if island propagates between unperturbed $\vec{E} \times \vec{B}$ velocity and unperturbed ion velocity. Destabilizing otherwise.
- Recalling that island phase-velocity is viscosity weighted average of unperturbed ion and electron fluid velocities, follows that polarization term is *stabilizing* provided ion (perpendicular) viscosity *exceeds* electron (perpendicular) viscosity (which is what we expect), and destabilizing otherwise.

Drift Effects: Summary

- Results limited to large islands: *i.e.*, large enough for sound waves to flatten pressure profile.
- Island propagates at (perpendicular) viscosity weighted average of unperturbed (no island) ion and electron fluid velocities.
- Polarization term in Rutherford equation is stabilizing provided ion (perpendicular) viscosity exceeds electron (perpendicular) viscosity (which is what we expect), and destabilizing otherwise.

Neoclassical Effects: Poloidal Flow Damping

- In toroidal plasma, poloidal flow *strongly damped* by parallel ion viscosity.
- So if

$$\vec{V}_i = \vec{V}_{\perp\, i} + \vec{V}_{\parallel\, i}\, \vec{b}$$

then

$$\vec{V}_{\parallel\, i} \rightarrow -\vec{V}_{\perp\, i\, \theta}\, \frac{B_\phi}{B_\theta}.$$

- Strong parallel flow results in *enhancement* of polarization term in Rutherford equation by factor $(B_\phi/B_\theta)^2$.[a]

[a] A.I. Smolyakov, and E. Lazzaro, Phys. Plasmas **11**, 4353 (2004).

Neoclassical Effects: Bootstrap Current - I

- In toroidal plasma friction between trapped and passing electrons leads to appearance of non-inductive *bootstrap current* in Ohm's law: [a]

$$\frac{d\Psi}{dt}\cos\theta \simeq [\phi,\psi] + \eta\,[J(\psi) - J_{\rm boot}],$$

where

$$J_{\rm boot} = -1.46\,\sqrt{\epsilon}\,B_\theta^{-1}\,\frac{\partial P}{\partial r}.$$

Here, ϵ is inverse aspect-ratio, $1.46\,\sqrt{\epsilon}$ is measure of fraction of trapped-particles, and P is plasma pressure.

[a] M.N. Rosenbluth, R.D. Hazeltine, and F.L. Hinton, Phys. Fluids **15**, 116 (1972).

Neoclassical Effects: Bootstrap Current - II

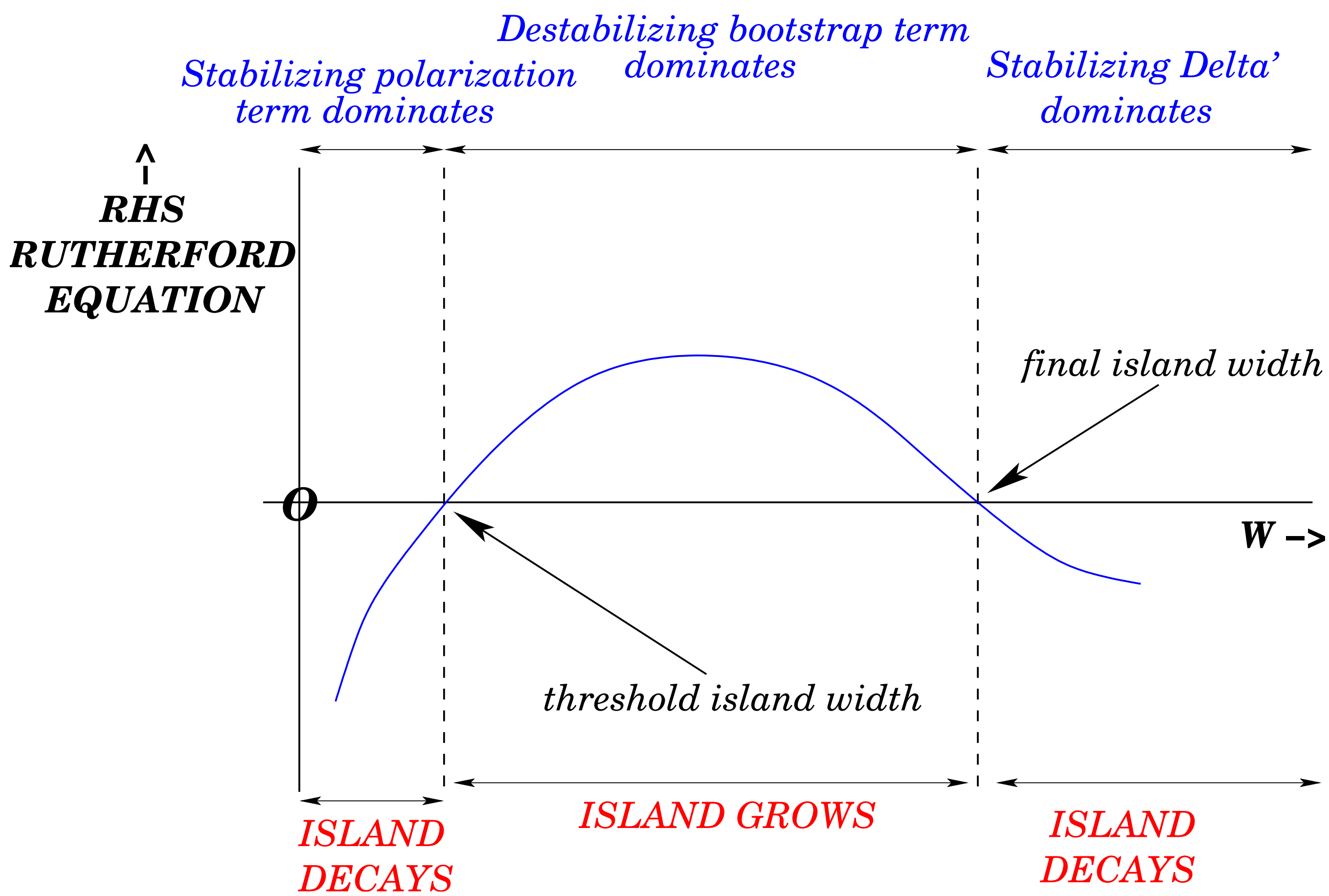
- Have seen that pressure profile *flattened* inside island separatrix.
- Bootstrap current consequently disappears inside separatrix.
- Absence of bootstrap current inside separatrix and continued presence outside leads to *destabilizing* term in Rutherford island equation: [a]

$$\frac{0.823}{\eta}\frac{dW}{dt} \simeq \Delta' + 1.38\left(\frac{B_\phi}{B_\theta}\right)^2 \frac{(V - V_{EB})\,(V - V_i)}{(W/4)^3}$$
$$-2.31\sqrt{\epsilon}\,\frac{(r\,P'/B_\theta^2)}{(W/4)}$$

[a] R. Fitzpatrick, Phys. Plasmas **2**, 825 (1995).

Neoclassical Effects: Neoclassical Tearing Modes - I

- A *neoclassical tearing mode* is an *intrinsically stable* ($\Delta' < 0$) tearing mode destabilized by bootstrap term.



Neoclassical Effects: Neoclassical Tearing Modes - II

- Neoclassical tearing mode must exceed *threshold island width* in order to grow and saturate: *i.e.*, mode is *meta-stable*.
- Threshold island width set by relative strengths of bootstrap and polarization terms in Rutherford island equation.

Neoclassical Effects: Summary

- Neoclassical poloidal flow damping leads to *large enhancement* in stabilizing polarization term in Rutherford equation.
- Neoclassical bootstrap current leads to appearance of large new *destabilizing* term in Rutherford island equation.
- – Bad news: Large magnetic island can be driven in intrinsically tearing-stable plasma by bootstrap term. Bootstrap drive proportional to P'—the very quantity we wish to maximize!
  – Good news: Island must exceed threshold size before it can grow.