Issues in Neoclassical Tearing Mode Theory

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Tearing Mode Stability in Tokamaks

- According to standard (single-fluid) tearing mode theory, all tearing modes except 2/1 should be stable in non-pathological tokamak discharges. (Neglecting, 1/1 which is special case).
- Experimentally, high temperature tokamak discharges often plagued by 3/2 (most common), and 4/5 tearing modes.
- Clearly, something is destabilizing tearing modes w.r.t. predictions of single-fluid theory.

Bootstrap Current

- In low-collisionality toroidal tokamak plasma, complicated interplay between trapped and passing electrons gives rise to non-inductive contribution to parallel current. This contribution is proportional to radial electron pressure gradient, and is called "Bootstrap Current". In fact,

$$j_{\parallel bs} \sim -\frac{1}{\sqrt{\epsilon}\,B_z}\,\frac{dp_e}{dr},$$

  where ϵ is inverse aspect-ratio, and B_z toroidal field-strength.

Perturbed Bootstrap Current

- Magnetic islands expected to flatten p_e, since they are effectively 3-D equilibria, and equilibrium pressure is flux-surface function.
- Hence, in presence of island of width W, perturbed bootstrap current is localized to island, and is of magnitude

$$\delta j_{\parallel bs} \sim \frac{1}{\sqrt{\epsilon}\, B_z} \frac{(p_e'\, W)}{W} \sim \frac{p_e'}{\sqrt{\epsilon}\, B_z},$$

where p_e' is equilibrium radial pressure gradient.

Bootstrap Modification to Rutherford Equation - I

- Ohm's law:

$$\frac{d\Psi}{dt} \sim -\eta\,(\delta j_\parallel - \delta j_{\parallel\,bs}),$$

where Ψ is reconnected flux, $\delta j_\parallel$ perturbed parallel current, $W/r_s \sim \sqrt{\Psi/(\epsilon\,B_z\,r_s)}$, and r_s is minor radius of rational surface.

- Asymptotic matching:

$$\Delta'\,\Psi \sim -\int \mu_0\,\delta j_\parallel\,dr \sim \mu_0\,\delta j_\parallel\,W.$$

- Follows that

$$\tau_R\,\frac{d(W/r_s)}{dt} \sim \Delta'\,r_s + \alpha\,\frac{\sqrt{\epsilon}\,\beta_p'}{(W/a)},$$

where $\tau_R = r_s^2\,\mu_0/\eta$, $\beta_p' = -r_s\,d\beta_p/dr$, $\beta_p = \mu_0\,p_e/(\epsilon\,B_z)^2$.

Bootstrap Modification to Rutherford Equation - II

- Bootstrap modified Rutherford equation:

$$\tau_R \frac{d(W/r_s)}{dt} \sim \Delta' r_s + \alpha \frac{\sqrt{\epsilon}\,\beta_p'}{(W/r_s)},$$

  where α is O(1) constant.

- More careful calculation shows that $\alpha > 0$. Hence, perturbed bootstrap effect is *destabilizing*.

- Effect can cause tearing-stable mode ($\Delta' < 0$) to grow to finite amplitude—so-called *neoclassical tearing mode*.

- Actually, perturbed bootstrap current term drives *all* tearing-stable modes to finite amplitude! Tearing modes now too unstable. Need stabilizing mechanism to offset bootstrap term.

Incomplete Pressure Flattening

- Argument that islands flatten pressure depends on fact that parallel heat/particle transport much greater than perpendicular transport. Hence, parallel transport forces pressure to be constant on flux-surfaces.

- However, as island width W decreases, perpendicular wave-number $k_\perp \sim W^{-1}$ increases, and parallel wave-number $k_\parallel \sim \epsilon\, W/r_s^2$ decreases. Must come critical width, W_c, below which quicker for heat/particles to flow radially across island, than to flow around island along field-lines. No pressure flattening for $W < W_c$, hence no perturbed bootstrap term in Rutherford equation.

Calculation of Critical Island Width

- Temperature in vicinity of island (no local heat sources/sinks) satisfies

$$\chi_{\parallel}\,\nabla_{\parallel}^2\,T_e + \chi_{\perp}\,\nabla_{\perp}^2\,T_e \simeq 0.$$

- Incomplete temperature flattening when two terms comparable:

$$\chi_{\parallel}\,k_{\parallel}^2\,T_e \sim \chi_{\perp}\,k_{\perp}^2\,T_e.$$

- So

$$\frac{W_c}{r_s} \sim \left(\frac{\chi_{\perp}}{\epsilon^2\,\chi_{\parallel}}\right)^{1/4}.$$

Polarization Current - I

- Diamagnetic effects cause island to propagate w.r.t. ion fluid with velocity of order $V_* \sim p'/(n_e\, e\, B_z)$.
- Ion flow around island generates perpendicular polarization current:

  $$\rho\,(\vec{V}\cdot\nabla)\vec{V} \sim \vec{j}_{\perp p} \times \vec{B},$$

  so

  $$j_{\perp p} \sim \frac{\rho\, V_*^2}{r_s\, B_z}.$$

- Current continuity demands parallel polarization current:

  $$\nabla_{\parallel}\vec{j}_{\parallel p} + \nabla_{\perp}\vec{j}_{\perp p} \sim 0,$$

  so $j_{\parallel p} \sim (k_{\perp}/k_{\parallel})\, j_{\perp p}$.

Polarization Current - II

- Contribution to Rutherford equation:

$$\Delta_p \sim \frac{\mu_0\, j_{\|p}\, W}{\Psi} = g\, \frac{(V_*/V_H)^2}{(W/r_s)^3},$$

  where $V_H \sim \epsilon B_z/\sqrt{\mu_0\,\rho}$, and g is $O(1)$ constant.

- More careful analysis shows that g is negative if island propagates between local $\vec{E} \times \vec{B}$ velocity and local ion fluid velocity, and positive otherwise.

- Ion polarization term can be stabilizing/destabilizing depending on island propagation velocity.

Rutherford Equation for Neoclassical Tearing Mode

- $$\tau_R \, \frac{d(W/r_s)}{dt} \sim \Delta' \, r_s + \alpha \, \frac{\sqrt{\epsilon} \, \beta_p' \, (W/r_s)}{(W_c/r_s)^2 + (W/r_s)^2} + g \, \frac{(V_*/V_H)^2}{(W/r_s)^3}.$$
- How much, if any, of this analysis is correct?
- Is there any way in which it can be tested numerically or experimentally?
- Are there any other major terms which need to be added to Rutherford equation?