Magnetic Island Theory - II

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Introduction: Non-MHD Effects

- This lecture discusses influence of non-MHD effects on Rutherford island equation. Effects considered are:
  - Diamagnetic flow. Causes ion and electron fluids to propagate at different speeds. Island propagates at intermediate speed. Leads to stabilizing ion polarization term in Rutherford eq.
  - Ion parallel stress tensor. Damps poloidal ion flow. Causes large parallel flow. Leads to large enhancement in polarization term in Rutherford equation.
  - Electron parallel stress tensor. Causes to Bootstrap current. Leads to destabilizing Bootstrap term in Rutherford equation.
  - Finite parallel heat and particle transport. Allows density and pressure to not be flux-surface functions at small island widths. Leads to disappearance of Bootstrap term in Rutherford eq.

Diamagnetic Effects

- According to two-fluid theory:

$$\vec{V}_{\perp\, i} \simeq \vec{V}_{\perp\, \mathrm{MHD}},$$
$$\vec{V}_{\perp\, e} \simeq \vec{V}_{\perp\, \mathrm{MHD}} + \vec{V}_{\perp\, *}.$$

- So-called diamagnetic velocity takes form:

$$\vec{V}_{\perp\, *} \simeq -\frac{dP/dr}{n\, e\, B_\phi}\, \vec{e}_\theta,$$

where P is plasma pressure, n electron number density, B_ϕ toroidal field-strength.

Pressure Flattening - I [a]

- In immediate vicinity of magnetic island (rational surface radius: r_s), unperturbed pressure takes form:

$$P(r) \simeq P_s' \, (r - r_s).$$

- How is this profile modified in presence of island?

- Pressure profile determined by competition between parallel and perpendicular thermal/particle transport:

$$\chi_\parallel \, \nabla_\parallel^2 P + \chi_\perp \, \nabla_\perp^2 \, P \simeq 0.$$

Note that $\chi_\parallel \gg \chi_\perp$.

[a] R. Fitzpatrick, Phys. Plasmas **2**, 825 (1995).

Pressure Flattening - II

- Parallel transport term attempts to make pressure a flux-surface function. Cannot have odd flux-surface function inside island separatrix. So, if $P = P(\psi)$ then pressure is flattened inside island.
- Perpendicular transport term attempts to relax pressure profile to unperturbed profile. Opposes pressure flattening.
- Pressure is flattened across island if parallel transport term dominates perpendicular transport term.

Pressure Flattening - III

- Have
$$\nabla_{\|} \simeq \frac{B_\theta}{B_\phi}\,\frac{k_\theta\, W}{l_s},$$
where l_s is typical radial variation length-scale of B_θ.

- Also
$$\nabla_\perp \simeq \frac{1}{W}.$$

- Hence, parallel transport dominates perpendicular transport when
$$W \gg W_c,$$
where
$$k_\theta\, W_c \simeq \left(\frac{\chi_\perp}{\chi_\|}\right)^{1/4} \left(k_\theta\, l_s\, \frac{B_\phi}{B_\theta}\right)^{1/2}.$$

Pressure Flattening - IV

- Critical island width fairly small (few cm) in tokamak plasmas.
- Hence, if island width exceeds critical width then pressure flattened inside separatrix.
- Implies that diamagnetic velocity zero inside separatrix.
- Hence, ion fluid, electron fluid, and island velocities identical inside separatrix.