Spontaneous Healing and Growth of Locked Magnetic Island Chains in Stellarator Plasmas

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Spontaneous Island Healing/Growth on LHD

- Spontaneous "healing/growth" of vacuum magnetic islands in LHD and TJ-II[a] stellarators highly reminiscent of "mode unlocking/locking" phenomena in tokamaks[b] and RFPs.[c]
  - Healing transitions triggered when amplitude of externally generated resonant magnetic field exceeds critical threshold.
  - Growth transitions triggered when field amplitude falls below second critical threshold that is significantly lower than first.
  - Island healing/growth transitions associated with substantial changes in plasma rotation.

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[a] Y. Narushima, *et al.*, NF **51**, 083030 (2011).
[b] T.C. Hender, *et al.*, NF **32**, 2091 (1992).
[c] B.E. Chapman, *et al.*, PP **11**, 2156 (2004).

Mode Locking Theory in Tokamaks[a]

- Response of plasma to external perturbation governed by ideal-MHD everywhere, except close to resonant surface.
- Close to resonant surface, plasma response governed by neoclassical four-field reduced-MHD model.
- Ultimately obtain two coupled nonlinear equations governing time evolution of island width and island phase.
- Equations possess bifurcated solutions exhibiting sudden transitions between states with wide/narrow island. Transitions associated with significant changes in plasma rotation, and exhibit strong hysteresis.

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[a] R. Fitzpatrick, NF **33**, 1049 (1993); R. Fitzpatrick, PP **5**, 3325 (1998); R. Fitzpatrick, F.L. Waelbroeck, PPCF **52**, 055006 (2010).

Application to Stellarators

- Can only use tokamak mode locking theory to study island healing/growth by treating stellarator equilibrium as axisymmetric plasma perturbed by static, externally generated, helical magnetic field.
- In reality, helical component of equilibrium field in stellarators comparable in strength with axisymmetric component.
- Tokamak analogy highly approximate in nature. Nevertheless, represents important first step.

Island Width Evolution Equation

$$\boxed{\alpha_1\,\frac{dw}{dT} = -1 + w^{-2}\,\cos\varphi + J_c\,\alpha_2\,w^{-3}}$$

- $w = W/W_v$ - ratio of actual island width to vacuum island width. φ - island phase relative to resonant external perturbation. $T = \omega_{*\,i}\,t$.
- First term on rhs due to intrinsic stability of island. Second to destabilizing influence of resonant external perturbation. Third to stabilizing/destabilizing effect of ion polarization current generated by plasma flow around island.
- $\alpha_1 = 1.65\,(\tau_R/m_\theta)\,(W_v/r_s)$.
  $\alpha_2 = (\beta_i/2\,m_\theta)\,(q_s/\epsilon_s)^2\,(L_s/L_n)^2\,(\rho_i/W_v)^3$.[a]

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[a] For full list of definitions see: `http://farside.ph.utexas.edu/papers/stellarator.pdf`

Island Phase Evolution Equation

$$\boxed{w\,\sin\varphi = J_s\,\alpha_3}$$

- Lhs is electromagnetic locking torque due to resonant external perturbation. Rhs is drag torque due to combination of plasma flow, ion perpendicular viscosity, and ion poloidal and toroidal flow damping.
- $\alpha_3 = (q_s/\epsilon_s)^2\,\alpha_2$.

Cosine and Sine Integrals

$$
\begin{aligned}
J_c &= -2\int_{-\infty}^{\infty}\oint \widehat{J_\parallel}\,\cos\zeta\,dX\,\frac{d\zeta}{2\pi},\\
J_s &= -2\int_{-\infty}^{\infty}\oint \widehat{J_\parallel}\,\sin\zeta\,dX\,\frac{d\zeta}{2\pi}.
\end{aligned}
$$

- $X = (r - r_s)/W$ - scaled radial coordinate. ζ - helical angle. $\widehat{J_\parallel}$ - normalized parallel current density.
- J_c multiplies ion polarization term in island width evolution equation.
- J_s multiplies drag torque term in island phase evolution equation.

Flow Damping Regimes

- J_c and J_s functions of normalized poloidal flow damping rate,

  $$\widehat{\nu}_\theta = (\epsilon_s/q_s)^2\,(\nu_{\theta\,i}/\omega_{*\,i}),$$

  normalized toroidal flow damping rate,

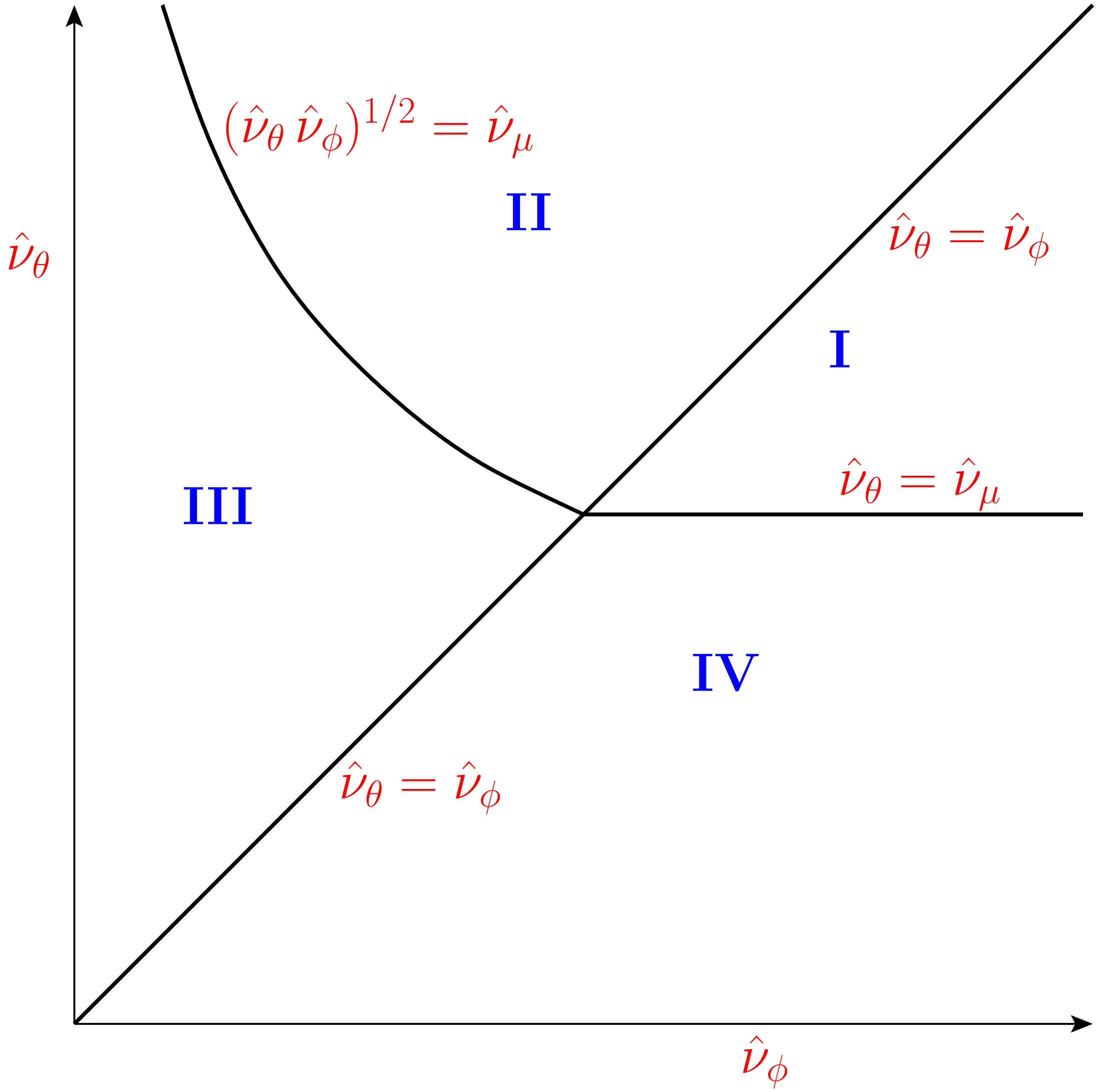
  $$\widehat{\nu}_\phi = (\epsilon_s/q_s)^2\,(\nu_{\phi\,i}/\omega_{*\,i}),$$

  and normalized viscous diffusion rate

  $$\widehat{\nu}_\mu = (r_s/W)^2\,(\nu_{\mu_{\perp\,i}}/\omega_{*\,i}).$$

- Four different regimes depending on relative sizes of $\widehat{\nu}_\theta$, $\widehat{\nu}_\phi$, and $\widehat{\nu}_\mu$.

Flow Damping Regimes



Flow Damping Regimes

Regime	J_c	J_s
I	$1.38\, v_\phi\, (v_\phi - 1)$	$-0.36\, \widehat{\nu}_\theta\, (v_\phi - v_\theta)$
II	$1.38\, v_\theta\, (v_\theta - 1)$	$-3.74\, \widehat{\nu}_\theta^{1/4}\, \widehat{\nu}_\phi^{3/4}\, (v_\phi - v_\theta)$
III	$1.38\, v_\theta\, (v_\theta - 1)$	$-4.00\, \widehat{\nu}_\phi^{1/2}\, \widehat{\nu}_\mu^{1/2}\, (v_\phi - v_\theta)$
IV	$1.38\, v_\phi\, (v_\phi - 1)$	$-4.00\, \widehat{\nu}_\theta^{1/2}\, \widehat{\nu}_\mu^{1/2}\, (v_\phi - v_\theta)$

- $v_\theta = -1.17\, \eta_i/(1+\eta_i)$.[a]
- $v_\phi = -2.37\, \eta_i/(1+\eta_i) + d\varphi/dT$ in "$1/\nu$" regime $(\nu_i \gg q_s\, V_{ExB}/R_0)$.[b]
- $v_\phi = +0.25\, \eta_i/(1+\eta_i) + d\varphi/dT$ in "ν" regime $(\nu_i \ll q_s\, V_{ExB}/R_0)$.[b]

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[a] Y.B. Kim, *et al.*, PFB **3**, 2050 (1991).

[b] K.C. Shaing, *et al.*, PP **15**, 082506 (2008).

Bifurcated Solutions

- Solutions to island evolution equations exhibit two different types of bifurcation.
- "Torque bifurcations" - due to breakdown in balance between electromagnetic and drag torques in island phase evolution equation.[a]
- "Stability bifurcations" - due to breakdown in balance between various terms on rhs of island width evolution equation.[b]
- For sake of brevity, will only discuss torque bifurcations.

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[a] C.C. Hegna, NF **51**, 113017 (2011).

[b] K. Ida, *et al.*, PRL **100**, 045003 (2008).

Torque Bifurcations

- Assumptions: Regime III; $1/\nu$ toroidal flow damping; neglect polarization term.
- Island evolution equations reduce to

$$\begin{aligned} \alpha_1\, dw/d\varphi &= -1 + w^{-2}\, \cos\varphi, \\ w\, \sin\varphi &= \alpha_4\,(v_0 - d\varphi/dT). \end{aligned}$$

Here, $v_0 = 1.2\,\eta_i/(1+\eta_i)$, $\alpha_4 = 4\,(\widehat{\nu}_\phi\,\widehat{\nu}_\mu)^{1/2}\,\alpha_3$.

Locked Island Solutions

- Wide island that is stationary in lab frame. Ion poloidal velocity at resonant surface zero.
- Island evolution equations yield

$$\begin{aligned} w &= (\cos\varphi)^{1/2}, \\ (\cos\varphi)^{1/2}\,\sin\varphi &= \alpha_4\, v_0. \end{aligned}$$

- Locked solution only possible when $\alpha_4\, v_0 < (4/27)^{1/4}$. When $\alpha_4\, v_0$ exceeds critical value $(4/27)^{1/4}$ island unlocks from vacuum perturbation, spins up (in electron diamagnetic direction), and its width decays significantly.

Rotating Island Solutions

- Narrow island that rotates in electron diamagnetic direction. Ion poloidal velocity at resonant surface also in electron direction.
- Island evolution equations yield

$$\begin{aligned} d\varphi/dT &= v, \\ w &= (3/\alpha_1\, v)^{1/3}\,|\sin\varphi|^{1/3}, \\ (\sin\varphi)^{4/3} &= \alpha_5\,(v/v_0)\,(1 - v/v_0). \end{aligned}$$

  Here, $\alpha_5 = v_0\,\alpha_4\,(v_0\,\alpha_1/3)^{1/3}$.

- Rotating island has pulsating width. Solution only possible when $\alpha_5 > (256/27)^{1/3}$. When α_5 falls below critical value $(256/27)^{1/3}$ island locks to vacuum perturbation, and its width grows significantly.

Island Healing/Growth Transitions

- Island healing transition is bifurcation from locked to rotating solution. Occurs when

$$\beta > \beta_{heal} \sim |s|^{1/2} \frac{\nu_*^{1/2}}{\rho_*^3} \left(\frac{b_{r\,vac}}{B_\phi}\right)^{3/2},$$

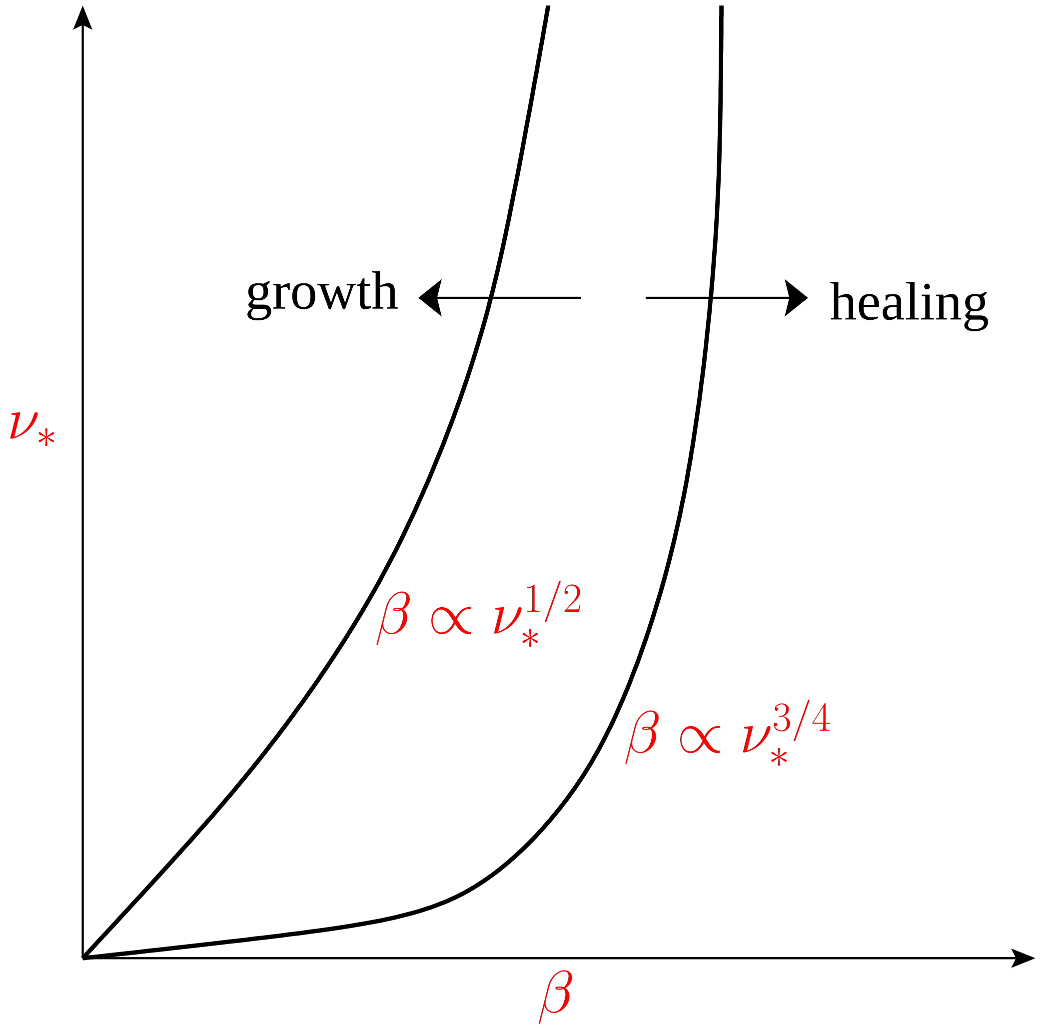
  where s is magnetic shear at resonant surface.

- Island growth transition is bifurcation from rotating to locked solution. Occurs when

$$\beta < \beta_{grow} \sim \frac{\nu_*^{3/4}}{\rho_*^4} \left(\frac{b_{r\,vac}}{B_\phi}\right)^{2} < \beta_{heal}.$$

- Locked/rotating solutions metastable when $\beta_{grow} < \beta < \beta_{heal}$.
- Healing/growth transitions associated with poloidal velocity shifts in electron/ion diamagnetic direction, respectively.

Island Healing/Growth Transitions



Summary

- Tokamak mode locking theory, applied to stellarators, yields predictions in good agreement with experimental observations of spontaneous island healing/growth:
    - Island healing/growth occurs at high/low β and low/high ν_*, respectively.
    - Healing/growth associated with polodal velocity shift in electron/ion diamagnetic direction, respectively.
    - Critical β value for healing exceeds that for growth.
- Agreement not complete. Torque bifurcations associated with healing transitions in which island unlocks from vacuum perturbation, spins up, and decays away. Spin-up not observed experimentally. Stability bifurcations do not have this problem.