Locked Magnetic Islands in Toroidally Flow-Damped Tokamak Plasmas

Richard Fitzpatrick

Institute for Fusion Studies

Department of Physics

University of Texas at Austin

Classification of Error-Field Harmonics

- What is response of tokamak plasma to static, non-axisymmetric, magnetic perturbation—a.k.a. *"error-field"*?
- Typical error-field consists of superposition of resonant and non-resonant helical harmonics:
  - *Resonant* harmonic has $\mathbf{k}\cdot\mathbf{B}=0$ at some *rational* magnetic flux-surface within plasma.
  - *Non-resonant* harmonic has $\mathbf{k}\cdot\mathbf{B}\neq 0$ throughout plasma.

Response to Resonant Harmonic

- Plasma response to resonant harmonic *strongly peaked* at rational surface.
- Nature of response depends on amount of equilibrium *plasma flow* at surface.[a]
- High Flow :- Narrow (radially) helical *current sheet* forms at surface. Shields interior of plasma from resonant harmonic.
- Low Flow :- Wide helical *magnetic island chain* forms at surface. Chain *locked* to resonant harmonic. Overlapping chains generate *magnetic ergodicity*.

[a] R. Fitzpatrick, Phys. Plasmas **5**, 3325 (1998).

Resonant Electromagnetic Torque

- Resonant harmonic also produces highly localized *electromagnetic torque* at rational surface, which acts to arrest local plasma flow.[a]
- If amplitude of resonant harmonic exceeds critical threshold then EM torque overcomes plasma flow, leading to sudden transition from helical current sheet to locked island chain:- process known as *mode penetration*.
- If amplitude of resonant harmonic falls below (lower) critical threshold then plasma flow overcomes EM torque, leading to sudden transition from locked island chain to helical current sheet:- process known as *mode expulsion*.

[a] R. Fitzpatrick, Nucl. Fusion **33**, 1049 (1993).

Response to Non-Resonant Harmonics

- Plasma response to non-resonant harmonics of error-field is *non-localized*.
- Non-resonant harmonics perturb axial symmetry of plasma equilibrium. Resulting non-ambipolar particle fluxes produce distributed *toroidal flow-damping torque*.[a]
- Damping torque acts to relax toroidal ion flow to fixed *neoclassical profile* (function of collisionality, density and temperature profiles, fraction of trapped particles, *etc.*, but independent of E_r).[b]
- Poloidal ion flow already relaxed to fixed neoclassical profile, due to naturally occurring *poloidal flow damping*.

[a] K.C. Shaing, S.P. Hirshman, and J.D. Callen, Phys. Fluids, **29**, 521 (1986).
[b] A.M. Garofalo, K.H. Burrell, *et al.*, Phys. Rev. Lett. **101**, 195005 (2008).

Toroidal Flow Damping

- In conventional tokamak plasma, poloidal ion velocity profile *fixed*, but toroidal profile *free* to vary (*i.e.*, E_r free to vary).
- But, suppose toroidal flow damping large enough to relax toroidal flow to neoclassical profile.
- Poloidal and toroidal velocity profiles both *fixed functions* of collisionality, density and temperature profiles, *etc.* Implies that radial electric field profile also fixed. In essence,

$$\text{tokamak} + \text{error-field} = \text{stellarator}.$$

Mode Penetration with Toroidal Flow Damping

- According to Cole, *et al.*,[a] threshold for mode penetration *significantly modified* when toroidal velocity profile not free to vary.
- What effect does toroidal flow damping have on locked island chain? Is mode expulsion threshold also significantly modified?

[a] A.J. Cole, C.C. Hegna, and J.D. Callen, Phys. Plasmas **15**, 056102 (2008).

Locked Island Chain and Neoclassical Flow

- If radial width of locked island chain much larger than ρ_i then neoclassical ion flow cannot cross magnetic separatrix.

- So, island chain presents obstacle to neoclassical flow unless

$$V_p^{nc} \equiv V_{\theta\,i}^{nc} - (\epsilon/q)\,V_{\varphi\,i}^{nc} = 0.$$

  $V_{\theta\,i}^{nc}$ - neoclassical ion poloidal velocity; $V_{\varphi\,i}^{nc}$ - neoclassical ion toroidal velocity; ϵ - inverse aspect-ratio; q - safety-factor. All quantities evaluated at rational surface.

- Quantity V_p^{nc} called *neoclassical phase velocity*, and is phase velocity of island chain which is convected by neoclassical ion flow.

Mode Expulsion w/o Toroidal Flow Damping

- In absence of toroidal flow damping, ion poloidal velocity at rational surface *fixed*, whilst toroidal velocity *free* to vary.
- Local ion toroidal velocity, $V_{\varphi\,i}$ adjusts itself such that

$$V_{\theta\,i}^{nc} - (\epsilon/q)\,V_{\varphi\,i} = 0.$$

- Ion toroidal velocity profile subsequently *relaxes* across whole plasma under influence of *perpendicular viscosity*.
- Small residual velocity gradients at rational surface generate weak *viscous torque* and *ion polarization current*.[a]
- Viscous torque and polarization current can lead to mode expulsion, but with *very low* threshold.

[a]F.L. Waelbroeck, and R. Fitzpatrick, Phys. Rev. Lett. **78**, 1703 (1997).

Mode Expulsion with Toroidal Flow Damping

- In presence of toroidal flow damping, ion poloidal and toroidal velocities at rational surface both *fixed*.
- Local ion toroidal velocity *cannot* adjust itself such that island chain presents no obstacle to neoclassical flow.
- Intense velocity gradients develop around magnetic separatrix, generating large *drag torque* and *ion polarization current*.
- *Much stronger* island/flow interaction in toroidally flow-damped plasma than in non-flow-damped plasma. Mode expulsion threshold likely to be *significantly different*. Can we quantify this?

Width Evolution Equation

- Radial width of locked island chain governed by:

$$\frac{dw}{dt} \propto \Delta' \, r_s + 2 m_\theta \left(\frac{w_v}{w}\right)^2 \cos\phi + J_c \left(\frac{r_s}{w}\right)^3.$$

- Δ' - linear stability index; r_s - rational surface radius; m_θ - poloidal mode number; w - island width; w_v - vacuum island width; ϕ - helical phase of island chain w.r.t. vacuum island chain;

$$J_c \equiv \frac{\mu_0 \, L_s \, w}{\pi \, B_0 \, r_s^2} \int_{r_{s-}}^{r_{s+}} \oint \delta j_\parallel \, \cos\zeta \, d\zeta \, dr;$$

  B_0 - toroidal field-strength; L_s - magnetic shear-length; $\delta j_\parallel$ - perturbed parallel current density; ζ - helical angle.

- J_c parameterizes *ion polarization current*.

Phase Evolution Equation

- Helical phase of locked island chain governed by:

$$\frac{d^2\phi}{dt^2} \propto 2m_\theta \left(\frac{w_v}{r_s}\right)^2 \left(\frac{w}{r_s}\right)^2 \sin\phi + J_s \left(\frac{w}{r_s}\right).$$

- First term on r.h.s. is *EM locking torque*, and

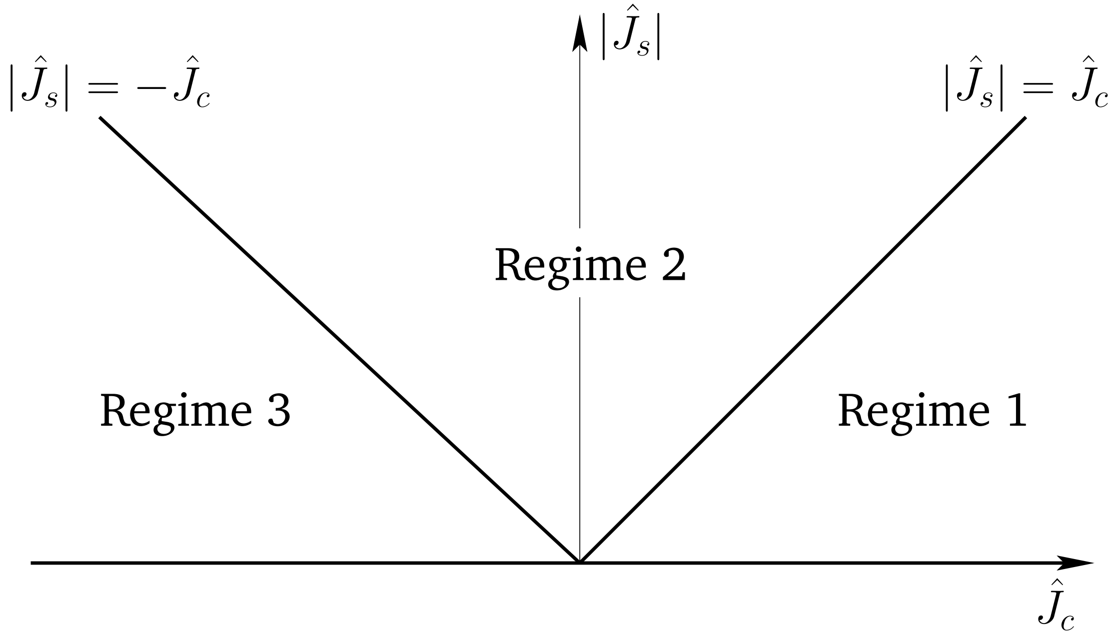
$$J_s \equiv \frac{\mu_0\, L_s\, w}{\pi\, B_0\, r_s^2} \int_{r_{s-}}^{r_{s+}} \oint \delta j_{\|}\, \sin\zeta\, d\zeta\, dr,$$

parameterizes *neoclassical drag torque*.

Steady-State Island Equilibria

- Assuming that $\hat{J}_c \equiv J_c/(-\Delta' r_s)$ and $\hat{J}_s \equiv J_s/(-\Delta' r_s)$ both *constants*, search for *steady-state* solutions of island width and phase evolution equations.

- *Three* different regimes found.



Regime 1: $\hat{J}_c > 0$, $|\hat{J}_s| \ll \hat{J}_c$

- Island chain maintained in plasma by *destabilizing* polarization current.
- Chain *unlocks* from resonant harmonic of error-field, spins-up, and is expelled, when $\hat{b} \equiv (2m_\theta/-\Delta'\, r_s)\,(w_v/r_s)^2$ falls below

$$\hat{b}_{cr} = |\hat{J}_s|/\hat{J}_c^{\,1/3}.$$

Here, $\hat{b}$ parameterizes amplitude of resonant harmonic.

- Island chain *strongly amplified* by plasma.

Regime 2: $|\hat{J}_s| \gg |\hat{J}_c|$

- Island chain maintained in plasma by resonant harmonic of error-field.
- Chain *unlocks* from resonant harmonic, spins-up, and is expelled, when $\hat{b}$ falls below

$$\hat{b}_{cr} = (27/4)^{1/6}\,|\hat{J}_s|^{2/3}.$$

- Island chain *weakly suppressed* by plasma.

Regime 3: $\hat{J}_c < 0,\ |\hat{J}_s| \ll |\hat{J}_c|$

- Island chain maintained in plasma by resonant harmonic of error-field.

- Stabilizing polarization current causes island chain width to monotonically *decay* when $\hat{b}$ falls below

$$\hat{b}_{cr} = (3/2^{2/3})\,|\hat{J}_c|^{2/3}.$$

- Island chain *moderately suppressed* by plasma.

Calculation of $\hat{J}_c$ and $\hat{J}_s$

- J_c and J_s calculated from *neoclassical drift-MHD fluid model*.[a]
- Adopt so-called *intermediate poloidal flow damping ordering*:

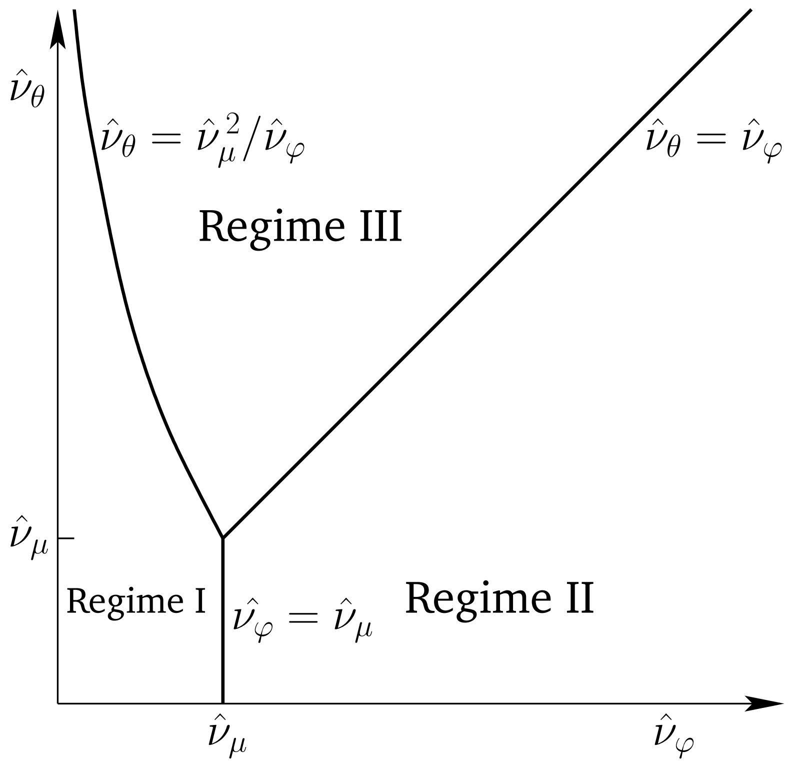
$$\omega_{*\,i} \gg \nu_\theta \gg (\epsilon/q)^2\,\nu_\theta,\,\nu_\varphi,\,\nu_\mu.$$

$\omega_{*\,i}$ - ion diamagnetic frequency; ν_θ - poloidal flow damping rate; ν_φ - toroidal flow damping rate; ν_μ - perpendicular viscous diffusion rate across island chain.

[a] R. Fitzpatrick, and F.L. Waelbroeck, Phys. Plasmas **16**, 072507 (2009).

Flow Damping Regimes

- *Three* different regimes found, depending on relative sizes of $\hat{\nu}_\theta \equiv \nu_\theta/\omega_{*\,i}$, $\hat{\nu}_\varphi \equiv \nu_\varphi/[(\epsilon/q)^2\,\omega_{*\,i}]$, $\hat{\nu}_\mu \equiv \nu_\mu/[4\,(\epsilon/q)^2\,\omega_{*\,i}]$.



Regime I: $\hat{\nu}_\mu \gg \hat{\nu}_\varphi^{1/2}\,(\hat{\nu}_\theta + \hat{\nu}_\varphi)^{1/2}$

- Cosine integral:

$$\hat{J}_c = 1.38\,\hat{\beta}\,\hat{U}_p^{nc}\,(\hat{U}_p^{nc} + 1),$$

where $\hat{\beta} \equiv \beta_i\,\rho_i^2\,L_s^2/[r_s^2\,L_n^2\,(-\Delta'\,r_s)]$, $\hat{U}_p^{nc} \equiv [\hat{\nu}_\varphi/(\hat{\nu}_\theta + \hat{\nu}_\varphi)]\,\hat{V}_p^{nc}$, $\hat{V}_p^{nc} \equiv V_p^{nc}/V_{*\,i}$. β_i - ion beta; ρ_i - ion gyroradius; L_n - density gradient length; $V_{*\,i}$ - ion diamagnetic velocity.

- Sine integral:

$$\hat{J}_s = 5.51\,\hat{\beta}\,\hat{\nu}_\theta\,\hat{U}_p^{nc}.$$

- Note that $\hat{J}_c \to 0$, $\hat{J}_s \to 0$ as $\hat{\nu}_\varphi \to 0$: *i.e.*, polarization current and drag torque negligible in absence of toroidal flow damping.

- Polarization current *stabilizing* when $0 > V_p^{nc} > -(1 + \hat{\nu}_\theta/\hat{\nu}_\varphi)\,V_{*\,i}$. ($V_p^{nc} < 0$ - neoclassical flow in *ion* diamagnetic direction.)

Regime II: $\hat{\nu}_\varphi \gg \hat{\nu}_\theta,\, \hat{\nu}_\mu$

- Cosine integral:

$$\hat{J}_c = 1.38\,\hat{\beta}\,\hat{V}_p^{nc}\,(\hat{V}_p^{nc} + 1).$$

- Sine integral:

$$\hat{J}_s = 5.51\,\hat{\beta}\,\hat{\nu}_\theta\,\hat{V}_p^{nc}.$$

- Polarization current *stabilizing* when $0 > V_p^{nc} > -V_{*\,i}$.

Regime III: $\hat{\nu}_\theta \gg \hat{\nu}_\varphi,\, \hat{\nu}_\mu^{\,2}/\hat{\nu}_\varphi$

- Cosine integral:

$$\hat{J}_c = 0.617\,\hat{\beta}\,(\hat{\nu}_\varphi/\hat{\nu}_\theta)^{3/4}\,\hat{V}_p^{\,nc}\,(\hat{V}_p^{\,nc}/4+1).$$

- Sine integral:

$$\hat{J}_s = 0.617\,\hat{\beta}\,\hat{\nu}_\theta^{\,1/4}\,\hat{\nu}_\varphi^{\,3/4}\,\hat{V}_p^{\,nc}.$$

- Polarization current ***stabilizing*** when $0 > V_p^{\,nc} > -4\,V_{\ast\,i}$.

- Polarization effect ***zero*** (*i.e.*, $\hat{J}_c = 0$) for freely rotating (*i.e.*, $\hat{J}_s = 0$) island chain. (Same is true in other two regimes.) So, in toroidally flow-damped plasma, locked island chain experiences polarization effect that freely rotating island chain does not. Possible explanation for anomalous growth of locked chains.

Summary

- *Locked* island chain subject to significant *ion polarization current* and *neoclassical drag* to which freely rotating island is not.
- Polarization current and neoclassical drag can cause *expulsion* of island chain from plasma.
- Polarization current can also greatly *amplify* island chain, depending on direction of neoclassical flow at rational surface.
- Have calculated expulsion threshold for locked island chain as function of neoclassical flows, flow damping rates, viscosity, plasma beta, *etc.*, in so-called *intermediate poloidal flow damping regime*.

Future Work

- Extend calculation to other flow damping regimes (*i.e.*, strong, weak, and negligible poloidal flow damping regimes).
- Use neoclassical drift-MHD fluid model to calculate *mode penetration threshold* as function of plasma parameters.
- Use neoclassical drift-MHD fluid model to determine effect of poloidal and toroidal *velocity shear* on island stability.