Phase Locking of Multihelicity Neoclassical Tearing Modes in Tokamak Plasmas[a]

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[a] `http://farside/ph.utexas.edu/Preprints/threewave.pdf`

Introduction and Motivation

- Conventional high-β tokamak discharges exhibit single neoclassical tearing mode (NTM)—typically, $m = 3, n = 2$ mode.
- In many (40/100 shots) JET and DIII-D "hybrid" discharges, second NTM appears later—typically, $2, 1$ mode.[a]
- 3,2 and 2,1 modes subsequently phase lock (30/40 shots), such that

$$\omega^{3,2} = 2\,\omega^{2,1}.$$

- Phase-locked configuration such that 3,2 and 2,1 X-points align on outboard mid-plane.[b]
- Phase locking leads to flattening of core toroidal rotation profile.

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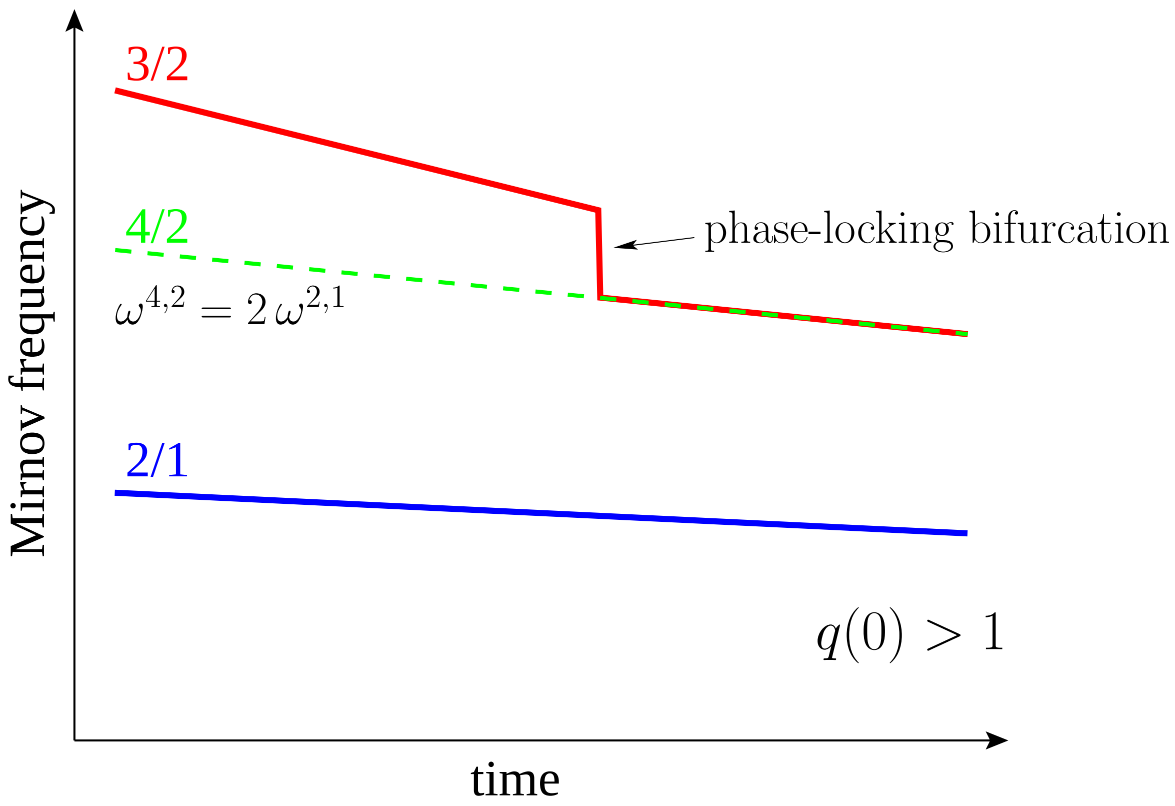
[a] B. Tobais, et al., Rev. Sci. Instr. **85**, 11D847 (2014). E. Alessi, et al., Proc. 41st EPS Conf. on Plasma Phys. (2014).

[b] T. Hender, et al., Nucl. Fusion **44**, 788 (2004).

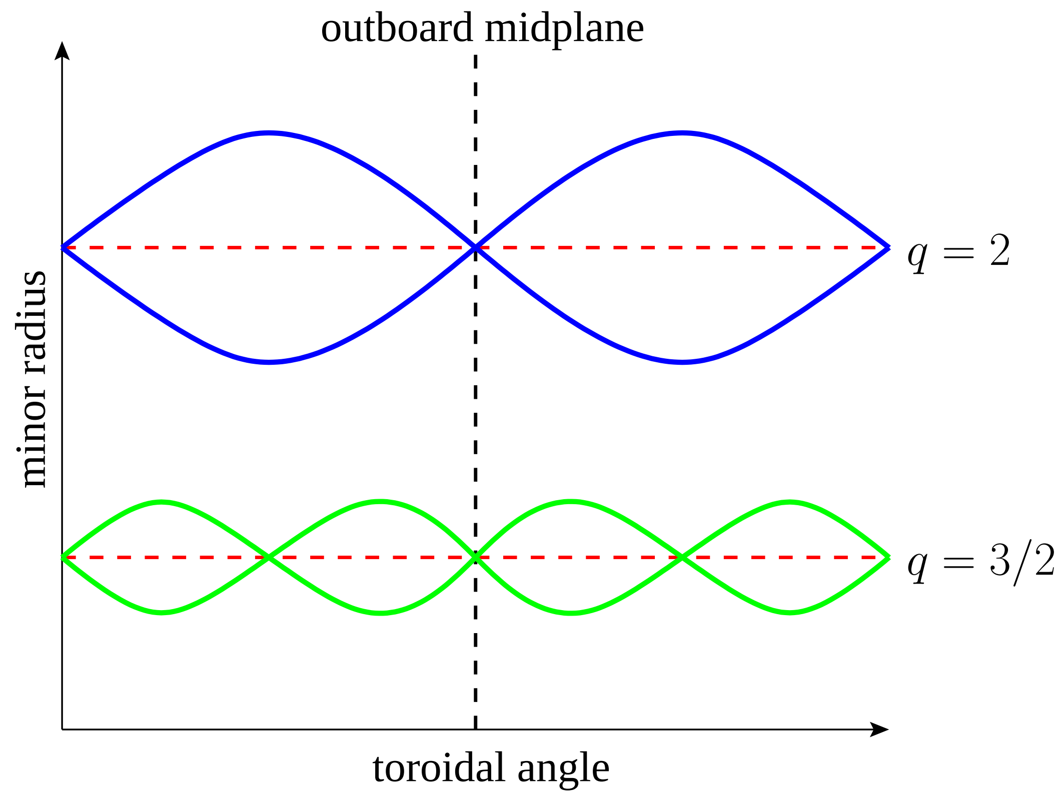
Schematic Phase Locking Data



DIII-D hybrid discharge in absence of $q = 1$ surface. Phase locking occurs when $\omega^{3,2} - \omega^{4,2}$ reduced to about half original value.[a]

[a] B. Tobias, et al., to appear.

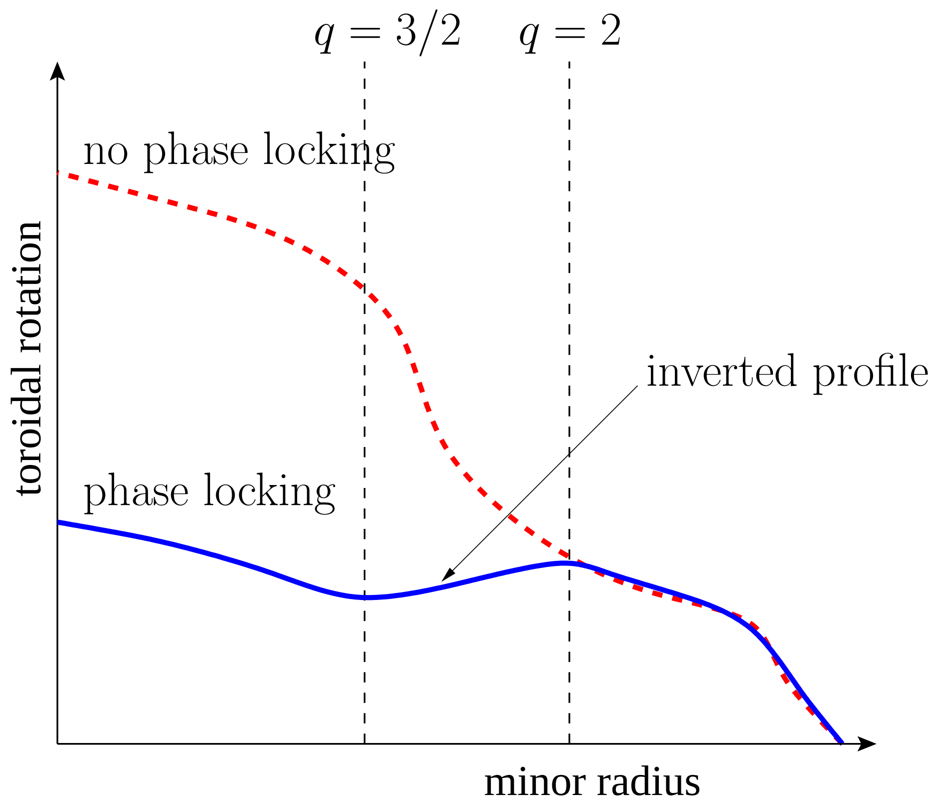
Configuration of Phase-Locked Islands



Seen in both JET and DIII-D hybrid discharges.[a]

[a] T. Hender, et al., Nucl. Fusion **44**, 788 (2004). B. Tobias, et al., to appear.

Effect on Toroidal Rotation Profile



Seen in both JET and DIII-D hybrid discharges.[a]

[a]E. Alessi, et al., Proc. 41st EPS Conf. on Plasma Phys. (2014). B. Tobias, et al., to appear.

Helical Phase

- In cylindrical geometry, magnetic perturbation associated with m, n tearing mode:

$$\delta\mathbf{B}(\mathbf{r}, t) = \nabla\psi \times \mathbf{e}_\phi,$$

where

$$\psi(\mathbf{r}, t) = \psi^{m,n}(r, t)\, e^{\,i\,[m\,\theta - n\,\phi + \varphi^{\,m,n}\,(t)]}.$$

- Magnetic island entrained by plasma at rational surface:

$$\frac{d\varphi^{m,n}}{dt} \equiv \omega^{m,n} = -m\,\Omega_\theta(r_s^{m,n}) + n\,\Omega_\phi(r_s^{m,n}).$$

- Differential rotation causes de-corellation of helical phases of different tearing modes: i.e., no particular relation between φ^{m_1,n_1} and φ^{m_2,n_2}.

Three-Wave Coupling - I

- Three-wave coupling generated by $\delta\mathbf{J} \times \delta\mathbf{B}$ term in ideal-MHD force balance equation.
- Couples three tearing modes whose mode numbers satisfy:

$$m_3 = m_1 + m_2, \qquad n_3 = n_1 + n_2.$$

- Coupling generates term[a]

$$\frac{(W^{m_2,n_2})^2\,(W^{m_3,n_3})^2}{(W^{m_1,n_1})^2}\,\cos\varphi$$

in Rutherford island width evolution equation of m_1, n_1 mode, with analogous terms in Rutherford equations of other two modes.

[a] C.C. Hegna, PoP **3**, 4646 (1996). R. Fitzpatrick, PoP **6**, 1168 (1999). R.M. Coelho, et al. PoP **6**, 1194 (1999).

Three-Wave Coupling - II

- Here,

$$\varphi = \varphi^{m_1,n_1} + \varphi^{m_2,n_2} - \varphi^{m_3,n_3}.$$

- Coupling generates electromagnetic locking torque at each coupled rational surface

$$T \propto (W^{m_1,n_1})^2 (W^{m_2,n_2})^2 (W^{m_3,n_3})^2 \sin\varphi.$$

- Torques act to reduce “slip frequency”:

$$\frac{d\varphi}{dt} \equiv \omega = \omega^{m_1,n_1} + \omega^{m_2,n_2} - \omega^{m_3,n_3}.$$

Three-Wave Coupling - III

- If slip frequency non-zero then, to lowest order, coupling terms in Rutherford equations and electromagnetic locking torques average to zero, because $\langle\cos\varphi\rangle = \langle\sin\varphi\rangle \simeq 0$.
- Coupling terms in Rutherford equations give rise to small modulation in island widths: $\delta W^{m_1,n_1} \propto \sin\varphi$, etc.
- Modulation in island widths generates small DC locking torques at three coupled rational surfaces. Torques modify toroidal rotation profile in such a way as to reduce slip frequency. Modification localized to region internal to outermost rational surface. Torques opposed by viscous restoring torques. Torque balance fails when slip frequency reduced to half its original value, at which point there is bifurcation to phase-locked state characterized by $\omega = 0$.

Three-Wave Coupling - IV

- Phase-locked state such that X-points of three coupled islands coincide permanently at particular angular location that rotates both poloidally and toroidally.
- With three coupled rational surfaces, condition $\omega = 0$ sufficiently unconstrained that cannot make definite prediction about final toroidal rotation profile.
- Three-wave coupling fairly unlikely scenario in tokamak, because requires three simultaneously unstable NTMs that satisfy three-wave coupling constraint.

Toroidal Coupling - I

- Toroidal coupling is form of three-wave coupling in which one of modes is $1, 0$ component of equilibrium: i.e., Shafranov shift. So, $m_1, n_1 = m, n$, $m_2, n_2 = 1, 0$, and $m_3, n_3 = m + 1, n$.
- $1, 0$ mode is non-rotating, and such that associated plasma displacement in direction of outboard mid-plane.
- Slip frequency:
$$\omega = \omega^{m,n} - \omega^{m+1,n}.$$
- As before, modulation in island widths gives rise to DC electromagnetic locking torques. Torques modify toroidal rotation profile interior to $m + 1, n$ rational surface so as to reduce slip frequency. Bifurcation to phase-locked state with $\omega = 0$ when slip frequency reduced to half original value.

Toroidal Coupling - II

- Phase-locked state such that X-points of m, n and $m+1, n$ island chains coincide permanently on outboard mid-plane. X-points rotate toroidally.

- Condition $\omega = 0$ sufficiently constraining that can make prediction about final rotation profile:

$$\Omega_\phi^{m,n} = \Omega_\phi^{m+1,n} - \frac{\Omega_\theta}{n}.$$

  Here, assuming (for sake of simplicity) solid-body poloidal rotation. Toroidal rotation profile flattened (if Ω_θ negligible), or even inverted (if Ω_θ non-negligible).

Nonlinear Toroidal Coupling - I

- $3,2$ and $2,1$ modes can couple via a combination of three-wave coupling and toroidal coupling:

$$\begin{aligned} 2,1+2,1 &= 4,2, \\ 3,2+1,0 &= 4,2. \end{aligned}$$

- Effective slip frequency is

$$\omega = \omega^{3,2} - 2\,\omega^{2,1}.$$

- Electromagnetic torques modify toroidal rotation profile interior to $2,1$ rational surface so as to reduce slip frequency. Bifurcation to phase-locked state when slip frequency reduced to half original value.

Nonlinear Toroidal Coupling - II

- Phase-locked state such that X-points of $3, 2$ and $2, 1$ island chains coincide permanently on outboard mid-plane. X-points rotate toroidally.

- Condition $\omega = 0$ sufficiently containing that can make prediction about final rotation profile:

$$\Omega_\phi^{3,2} = \Omega_\phi^{2,1} - \frac{\Omega_\theta}{2}.$$

  Here, assuming (for sake of simplicity) solid-body poloidal rotation. Toroidal rotation profile flattened (if Ω_θ negligible), or even inverted (if Ω_θ non-negligible).

Summary

- Quasi-cylindrical model accounts for (nearly) all salient features of phase locking of $3, 2$ and $2, 1$ NTMs in DIII-D and JET hybrid discharges.
- In addition, model can predict critical $3, 2$ and $2, 1$ island widths at which phase locking occurs. Prediction consistent with experimental observations.
- Model can be scaled to ITER. Phase locking much more likely to occur in ITER, compared to DIII-D and JET, given much smaller differential plasma rotation expected in ITER.

Future Work

- Incorporate $1, 1$ perturbation into analysis. Plenty of examples, in JET, of nonlinear phase locking of $m = 1$, $m = 2$, and $m = 3$ NTMs mediated by persistent $1, 1$ (fishbone?).
- In small number (about 8) of DIII-D discharges, $2, 1$ and $3, 2$ NTMs exhibit strong mutual locking torques, despite fact that $2\,\omega^{2,1} - \omega^{3,2} \simeq 3\,\mathrm{kHz}$. Inexplicable within context of cylindrical model. Can possibly be explained by toroidal model.[a]
- Comparison of toroidal rotation profile in DIII-D with TRANSP predictions allows identification of regions where locking torques develop. Shows, quite clearly, that torques localized in vicinities of $2, 1$ and $3, 2$ rational surfaces. First direct experimental verification of this long-standing theoretical prediction. More detailed comparison between experiment and theory worthwhile.

[a] B. Tobias, et al., to appear.