Scaling of Error-Field Penetration Threshold in Tokamak Plasmas

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Introduction

- Locked mode formation in low density target plasmas seriously limits available experimental operating space.
- Locked modes thought to be due to error-field driven magnetic reconnection triggered when resonant component of error-field (at rational surface) exceeds some threshold value.
- Observed threshold in present experiments is small (but manageable): i.e.,
  \[(b_r/B_T)_{\text{crit}} > 10^{-4} \]
- Threshold generally decreases with increasing machine size. What is expected penetration threshold for ITER?


Empirical Scaling Laws

- Scaling studies of penetration threshold with “engineering parameters”, $n_e$, $B_T$, $R_0$, performed on many tokamaks.\(^a\)

- Writing

\[
\left( \frac{b_r}{B_T} \right)_{\text{crit}} \sim n_e^{\alpha_n} B_T^{\alpha_B} R_0^{\alpha_R},
\]

all studies agree that $\alpha_n \simeq 1$: i.e., density scaling is linear.

- Measured values of $\alpha_B$ lie in range $-2.9$ to $-1.0$.

- Value of $\alpha_R$ cannot be directly measured, but can be inferred from dimensionless scaling arguments:

\[
\alpha_R = 2 \alpha_n + 1.25 \alpha_B.
\]

\(^a\)Compass-C/D, Textor, Alcator C-mod, DIII-D, JET
Extrapolation to ITER

• Extrapolation from JET\(^a\) to ITER\(^b\) yields

\[ 1.3 \times 10^{-5} < (b_r/B_T)_{\text{crit}} < 2.7 \times 10^{-4}. \]

• Proposed ITER error-field correction system designed to reduce resonant error-fields to level

\[ b_r/B_T \simeq 5 \times 10^{-5}. \]

• Large uncertainty in extrapolation of penetration threshold to ITER (and, hence, in adequacy of error-field correction system). Could reduce uncertainty by developing error-field penetration theory consistent with experimental data.

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\(^a\)\(n_e = 1.6 \times 10^{19} \text{ m}^{-3}, B_T = 3.5 \text{ T}, R_0 = 2.95 \text{ m}, (b_r/B_T)_{\text{crit}} = 1.1 \times 10^{-4}.\)

\(^b\)\(n_e = 2 \times 10^{19} \text{ m}^{-3}, B_T = 5.3 \text{ T}, R_0 = 6.2 \text{ m}.\)
Fitzpatrick (1993) Theory\(^a\)

- Response of plasma governed by *linear resistive-viscous MHD*.
- Scaling of penetration threshold with standard dimensionless parameters:
  \[
  \left(\frac{b_r}{B_T}\right)_{\text{crit}} \sim \beta^{-1/6} \nu_*^{1/6} \rho_*^{4/3}.
  \]

- Scaling with engineering parameters:\(^b\)
  \[
  \left(\frac{b_r}{B_T}\right)_{\text{crit}} \sim n_e^0 B_T^{-13/15} R_0^{-13/12}.
  \]

- Predicted scaling highly inconsistent with experimental data, since no density dependence.

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\(^a\)Nucl. Fusion **33**, 1049 (1993).

\(^b\)Assuming ohmic power balance, \(\tau_M \sim \tau_E\), and (dimensionally consistent) neo-Alcator energy transport: \(B_T \tau_E \sim n_e B_T R_0^{13/4} \sim \beta^{5/4} \nu_*^{-1/4} \rho_*^{-7/2}\).
Cole & Fitzpatrick (2006) Theory\textsuperscript{a}

- Response of plasma governed by linear resistive-viscous drift-MHD.
- Scaling of penetration threshold with dimensionless parameters:
  \[(b_r/B_T)_{\text{crit}} \sim \nu_*^{1/4} \rho_*^{5/4} .\]
- Scaling with engineering parameters:
  \[(b_r/B_T)_{\text{crit}} \sim n_e^{1/4} B_T^{-23/20} R_0^{-15/16} .\]
- Slight improvement in density scaling, but dependence still much too weak.

\textsuperscript{a}Plasma Phys. 13, 032503 (2006).

• Like Cole & Fitzpatrick theory, except also takes *neoclassical flow damping* into account.

• Scaling of penetration threshold with dimensionless parameters: \( (b_r/B_T)_{\text{crit}} \sim \beta \nu^* -1/2 \rho^*_3/2 \).

• Scaling with engineering parameters:

\[
(b_r/B_T)_{\text{crit}} \sim n_e^{1/2} B_T^{-13/10} R_0^{-5/8}.
\]

• Further improvement in density scaling, but dependence still too weak.

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\(^b\) Assuming flow damping in \(1/\nu\) regime.
**Fitzpatrick (2011) Theory**

- Like Cole, Hegna, Callen theory, except response of plasma in vicinity of rational surface governed by *nonlinear island physics*.

- Scaling of penetration threshold with dimensionless parameters:

  \[
  \frac{b_r}{B_T}_{\text{crit}} \sim \beta \rho^*.
  \]

- Scaling with engineering parameters:

  \[
  \frac{b_r}{B_T}_{\text{crit}} \sim n_e B_T^{-9/5} R_0^{-1/4}.
  \]

- Scaling fairly consistent with experimental data. Leads to predicted ITER penetration threshold of

  \[
  \frac{b_r}{B_T}_{\text{crit}} \sim 5 \times 10^{-5}.
  \]

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\(^a\)http://farside.ph.utexas.edu/papers/nonlinear.pdf

\(^b\)Assuming plasma response in so-called “polarization regime.”
Island Width Evolution

• New expression for penetration threshold obtained from recently developed magnetic island model.\(^a\)

• Time evolution of island width, \(w\), governed by \(^b\)

\[
4I_1 \tau_R \frac{d}{dt} \left( \frac{w}{r_s} \right) = \Delta' + 2m_\theta \left( \frac{w_v}{w} \right)^2 \cos \phi \\
- I_p \beta_0 \left( \frac{w_0}{r_s} \right)^2 \frac{r_s^3}{w^3 + \rho_s^3}.
\]

• Second term on r.h.s. represents error-field drive. Third term describes stabilizing influence of \textit{ion polarization current}.


\(^b\)See paper for complete list of definitions.
Ion Polarization Stabilization

- **Large island limit** ($w \gg \rho_s$): Acceleration of ion fluid flowing around curved island separatrix produces perpendicular ion polarization current. Parallel return current has strong *stabilizing* effect on island that scales as $w^{-3}$.

- **Small island limit** ($w \approx \rho_s$): Ion fluid decouples from magnetic flux-surfaces and flows straight through island separatrix. Much smaller ion polarization current generated. Parallel return current has weak *stabilizing* effect on island that scales as $w^0$.

- Polarization term used in model is interpolation between large and small island limits.
Island Phase Evolution

- Time evolution of island phase, $\phi$, governed by

$$-2m_\theta \left( \frac{w_v}{w_0} \right)^2 \left( \frac{w}{r_s} \right)^2 \sin \phi = 4\beta_0 \left( \frac{\nu_{\phi i}}{\tau_M \omega_{*i}^2} \right)^{1/2} \left( \frac{1}{\omega_{*i}} \frac{d\phi}{dt} - \nu_{nc} \right).$$

- Left hand side is electromagnetic locking torque due to error-field. Right-hand side is viscous torque due to combination of anomalous ion perpendicular viscosity and neoclassical ion toroidal viscosity.

$^a$See paper for complete list of definitions.
Penetration Regimes

- Island model predicts two distinct regimes for error-field penetration depending on values of

\[ D_1 \sim \frac{\tau_R \omega_{*i}}{(\tau_D \tau_M \omega_{*i}^2)^{1/4}} \beta^{1/2}, \]

\[ D_2 \sim (\tau_D \tau_M \omega_{*i})^{1/2} \beta \rho_{*}. \]

- Here, \( \tau_R \) is resistive t.s., \( \tau_M \) is momentum confinement t.s., and \( \tau_D \) is neoclassical toroidal flow damping t.s.

- \( D_1 \) and \( D_2 \) parameterize relative importance of \textit{neoclassical rotation} and the \textit{ion polarization current}, respectively, in suppressing error-field driven magnetic reconnection.
Penetration Regimes

\[ D_1^{2/3} D_2 = 1 \]

POLARIZATION

\[ D_2 = D_1 \]

RUTHERFORD

\[ D_1 = 1 \]
Rutherford Regime

- Suppression of driven magnetic reconnection due to *neoclassical plasma rotation*, which prevents island from locking in phase with resonant error-field. Stabilizing effect of ion polarization current negligible.

- Penetration occurs when electromagnetic locking torque becomes large enough to overwhelm viscous torque, and allows island to lock to resonant error-field.

- Prior to penetration, island width “pulsates”, since island spends as much time in stabilizing phase of error-field, as in destabilizing phase.
Rutherford Regime

\[
\frac{\phi}{\pi} \quad 0 \quad 10 \quad 20 \quad 30 \quad 40 \quad 50
\]

\[
x^{1/3} \quad 0 \quad 10 \quad 20 \quad 30 \quad 40 \quad 50
\]
**Polarization Regime**

- Suppression of driven magnetic reconnection due to *ion polarization current*, which essentially prevents island width from exceeding $\rho_s$, and, hence, prevents locking to resonant error-field.

- Penetration occurs when error-field drive overwhelms polarization current stabilization, allowing island to grow to large amplitude, and triggering locking of island to resonant error-field.

- Prior to penetration, island width “pulsates” due to balance between error-field destabilization (which varies with island phase) and polarization current stabilization.
Polarization Regime

\[ \Phi / \pi \]

\[ \hat{x} / \beta \]

\[ T = \{0, 10, 20\} \]
Scaling of Penetration Threshold

- **Rutherford regime:**
  \[
  \left( \frac{b_r}{B_T} \right)_{\text{crit}} \sim \beta \nu_\ast^{-1} \rho_\ast^2 \sim n_e^0 B_T^{-4/5} R_0^{-1}.
  \]

- **Polarization regime:**
  \[
  \left( \frac{b_r}{B_T} \right)_{\text{crit}} \sim \beta \rho_\ast \sim n_e B_T^{-9/5} R_0^{-1/4}.
  \]

- Rutherford regime scaling is almost indistinguishable from original Fitzpatrick (1993) scaling, and cannot explain experimental data.

- Polarization regime scaling is consistent with experimental data.
Conclusions

• Observed scaling of error-field penetration threshold in tokamaks with engineering parameters—in particular, linear scaling with plasma density—indicates that threshold largely determined by ion polarization current.

• Since stabilizing effect of ion polarization current only manifests itself in nonlinear island physics, this necessitates nonlinear response model for plasma in immediate vicinity of rational surface. However, in absence of ion polarization current, such a model fails as badly as a linear response model.

• Given that error-field penetration threshold is governed by ion polarization current, seems highly likely that threshold for neoclassical tearing modes is also determined by this effect.