Scaling of Error-Field Penetration Threshold in Tokamak Plasmas

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Introduction

- Locked mode formation in low density target plasmas seriously limits available experimental operating space.
- Locked modes thought to be due to *error-field driven magnetic reconnection* triggered when resonant component of error-field (at rational surface) exceeds some *threshold value*.
- Observed threshold in present experiments is small (but manageable): *i.e.*,

$$(\mathrm{b_r}/\mathrm{B_T})_{\mathrm{crit}} > 10^{-4}.$$

- Threshold generally *decreases* with increasing machine size. What is expected penetration threshold for ITER?

Empirical Scaling Laws

- Scaling studies of penetration threshold with "engineering parameters", n_e, B_T, R_0, performed on many tokamaks.[a]
- Writing

$$(b_r/B_T)_{crit} \sim n_e^{\alpha_n} \, B_T^{\alpha_B} \, R_0^{\alpha_R},$$

  all studies agree that $\alpha_n \simeq 1$: *i.e.*, density scaling is *linear*.
- Measured values of α_B lie in range -2.9 to -1.0.
- Value of α_R cannot be directly measured, but can be inferred from dimensionless scaling arguments:

$$\alpha_R = 2\,\alpha_n + 1.25\,\alpha_B.$$

[a] Compass-C/D, Textor, Alcator C-mod, DIII-D, JET

Extrapolation to ITER

- Extrapolation from JET[a] to ITER[b] yields

$$1.3\times 10^{-5} < (b_r/B_T)_{crit} < 2.7\times 10^{-4}.$$

- Proposed ITER error-field correction system designed to reduce resonant error-fields to level

$$b_r/B_T \simeq 5\times 10^{-5}.$$

- Large uncertainty in extrapolation of penetration threshold to ITER (and, hence, in adequacy of error-field correction system). Could reduce uncertainty by developing error-field penetration theory consistent with experimental data.

[a] $n_e = 1.6\times 10^{19}\,\mathrm{m}^{-3}$, $B_T = 3.5\,\mathrm{T}$, $R_0 = 2.95\,\mathrm{m}$, $(b_r/B_T)_{crit} = 1.1\times 10^{-4}$.
[b] $n_e = 2\times 10^{19}\,\mathrm{m}^{-3}$, $B_T = 5.3\,\mathrm{T}$, $R_0 = 6.2\,\mathrm{m}$.

Fitzpatrick (1993) Theory [a]

- Response of plasma governed by *linear resistive-viscous MHD*.
- Scaling of penetration threshold with standard dimensionless parameters:

$$(b_r/B_T)_{crit} \sim \beta^{-1/6}\, \nu_*^{1/6}\, \rho_*^{4/3}.$$

- Scaling with engineering parameters:[b]

$$(b_r/B_T)_{crit} \sim n_e^0\, B_T^{-13/15}\, R_0^{-13/12}.$$

- Predicted scaling highly inconsistent with experimental data, since no density dependence.

[a] Nucl. Fusion **33**, 1049 (1993).

[b] Assuming ohmic power balance, $\tau_M \sim \tau_E$, and (dimensionally consistent) neo-Alcator energy transport: $B_T\, \tau_E \sim n_e\, B_T\, R_0^{13/4} \sim \beta^{5/4}\, \nu_*^{-1/4}\, \rho_*^{-7/2}$.

Cole & Fitzpatrick (2006) Theory [a]

- Response of plasma governed by linear resistive-viscous *drift*-MHD.
- Scaling of penetration threshold with dimensionless parameters:

$$(b_r/B_T)_{crit} \sim \nu_*^{1/4}\, \rho_*^{5/4}.$$

- Scaling with engineering parameters:

$$(b_r/B_T)_{crit} \sim n_e^{1/4}\, B_T^{-23/20}\, R_0^{-15/16}.$$

- Slight improvement in density scaling, but dependence still much too weak.

[a] Plasma Phys. **13**, 032503 (2006).

Cole, Hegna, Callen (2008) Theory [a]

- Like Cole & Fitzpatrick theory, except also takes *neoclassical flow damping* into account.
- Scaling of penetration threshold with dimensionless parameters: [b]

$$(b_r/B_T)_{crit} \sim \beta\, \nu_*^{-1/2}\, \rho_*^{3/2}.$$

- Scaling with engineering parameters:

$$(b_r/B_T)_{crit} \sim n_e^{1/2}\, B_T^{-13/10}\, R_0^{-5/8}.$$

- Further improvement in density scaling, but dependence still too weak.

[a] Plasma Phys. **15**, 056102 (2008).

[b] Assuming flow damping in $1/\nu$ regime.

Fitzpatrick (2011) Theory [a]

- Like Cole, Hegna, Callen theory, except response of plasma in vicinity of rational surface governed by *nonlinear island physics*.
- Scaling of penetration threshold with dimensionless parameters: [b]

$$(b_r/B_T)_{crit} \sim \beta\,\rho_*.$$

- Scaling with engineering parameters:

$$(b_r/B_T)_{crit} \sim n_e\,B_T^{-9/5}\,R_0^{-1/4}.$$

- Scaling fairly consistent with experimental data. Leads to predicted ITER penetration threshold of $(b_r/B_T)_{crit} \sim 5\times 10^{-5}$.

[a] `http://farside.ph.utexas.edu/papers/nonlinear.pdf`

[b] Assuming plasma response in so-called "polarization regime."

Island Width Evolution

- New expression for penetration threshold obtained from recently developed magnetic island model.[a]
- Time evolution of island width, w, governed by[b]

$$4I_1\,\tau_R\,\frac{d}{dt}\left(\frac{w}{r_s}\right) = \Delta' + 2m_\theta\left(\frac{w_v}{w}\right)^2\cos\phi - I_p\,\beta_0\left(\frac{w_0}{r_s}\right)^2\frac{r_s^3}{w^3+\rho_s^3}.$$

- Second term on r.h.s. represents error-field drive. Third term describes stabilizing influence of *ion polarization current*.

[a] R. Fitzpatrick, F.L. Waelbroeck, Phys. Plasmas **17**, 062503 (2010).
[b] See paper for complete list of definitions.

Ion Polarization Stabilization

- Large island limit ($w \gg \rho_s$): Acceleration of ion fluid flowing around curved island separatrix produces perpendicular ion polarization current. Parallel return current has strong *stabilizing* effect on island that scales as w^{-3}.

- Small island limit ($w \simeq \rho_s$): Ion fluid decouples from magnetic flux-surfaces and flows straight through island separatrix. Much smaller ion polarization current generated. Parallel return current has weak *stabilizing* effect on island that scales as w^0.

- Polarization term used in model is interpolation between large and small island limits.

Island Phase Evolution

- Time evolution of island phase, ϕ, governed by [a]

$$-2m_\theta \left(\frac{w_v}{w_0}\right)^2 \left(\frac{w}{r_s}\right)^2 \sin\phi = 4\beta_0 \left(\frac{\nu_{\varphi\, i}}{\tau_M\, \omega_{*\, i}^2}\right)^{1/2} \left(\frac{1}{\omega_{*\, i}}\frac{d\phi}{dt} - \nu_{nc}\right).$$

- Left hand side is electromagnetic *locking torque* due to error-field. Right-hand side is viscous torque due to combination of *anomalous ion perpendicular viscosity* and *neoclassical ion toroidal viscosity*.

[a] See paper for complete list of definitions.

Penetration Regimes

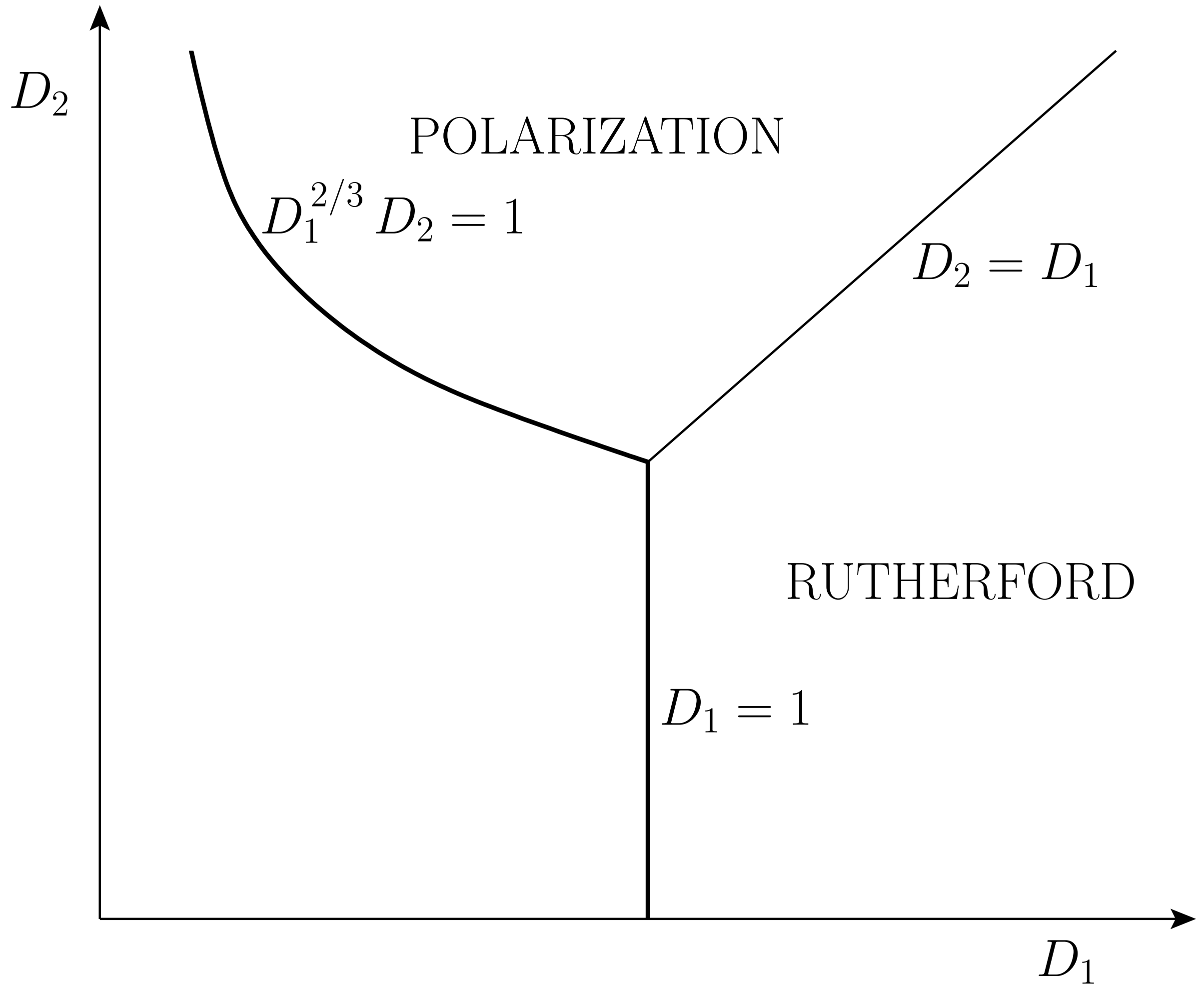
- Island model predicts two distinct regimes for error-field penetration depending on values of

$$D_1 \sim \frac{\tau_R\,\omega_{*\,i}}{(\tau_D\,\tau_M\,\omega_{*\,i}^2)^{1/4}}\,\beta^{1/2},$$

$$D_2 \sim (\tau_D\,\tau_M\,\omega_{*\,i})^{1/2}\,\beta\,\rho_*.$$

- Here, τ_R is resistive t.s., τ_M is momentum confinement t.s., and τ_D is neoclassical toroidal flow damping t.s.
- D_1 and D_2 parameterize relative importance of *neoclassical rotation* and the *ion polarization current*, respectively, in suppressing error-field driven magnetic reconnection.

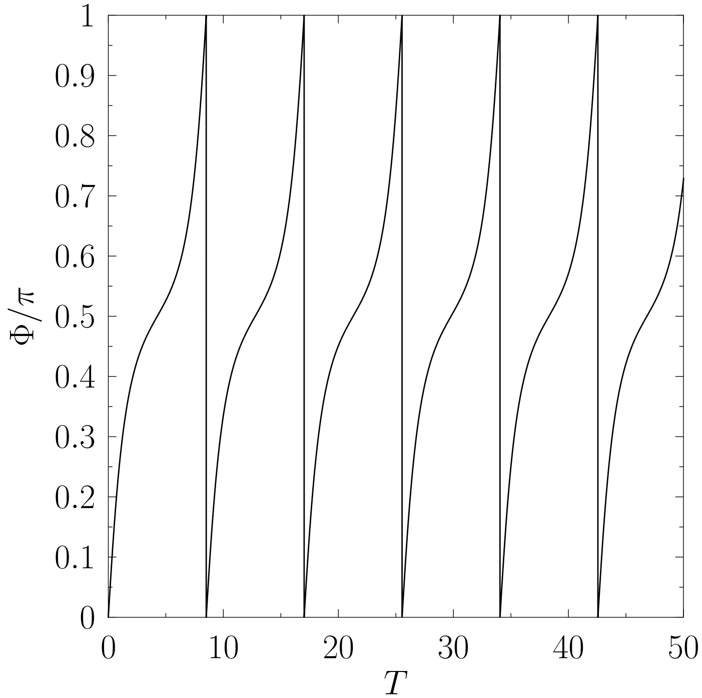
Penetration Regimes

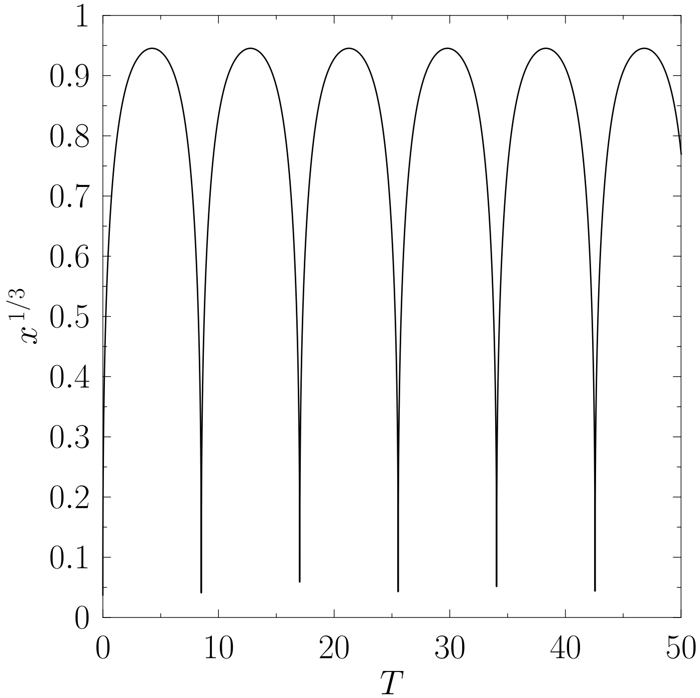


Rutherford Regime

- Suppression of driven magnetic reconnection due to *neoclassical plasma rotation*, which prevents island from locking in phase with resonant error-field. Stabilizing effect of ion polarization current negligible.
- Penetration occurs when electromagnetic locking torque becomes large enough to overwhelm viscous torque, and allows island to lock to resonant error-field.
- Prior to penetration, island width "pulsates", since island spends as much time in stabilizing phase of error-field, as in destabilizing phase.

Rutherford Regime

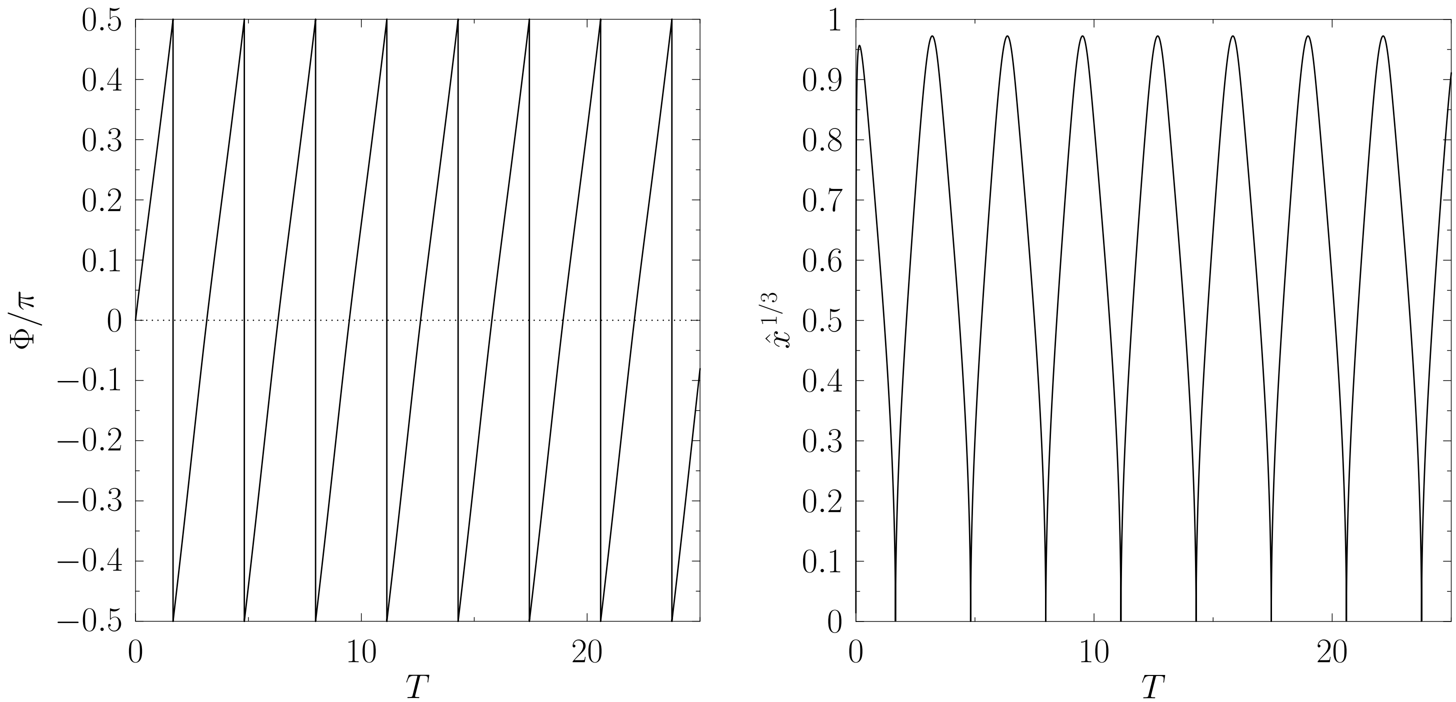




Polarization Regime

- Suppression of driven magnetic reconnection due to *ion polarization current*, which essentially prevents island width from exceeding ρ_s, and, hence, prevents locking to resonant error-field.
- Penetration occurs when error-field drive overwhelms polarization current stabilization, allowing island to grow to large amplitude, and triggering locking of island to resonant error-field.
- Prior to penetration, island width “pulsates” due to balance between error-field destabilization (which varies with island phase) and polarization current stabilization.

Polarization Regime



Scaling of Penetration Threshold

- Rutherford regime:

$$(b_r/B_T)_{crit} \sim \beta\, \nu_*^{-1}\, \rho_*^2 \sim n_e^0\, B_T^{-4/5}\, R_0^{-1}.$$

- Polarization regime:

$$(b_r/B_T)_{crit} \sim \beta\, \rho_* \sim n_e\, B_T^{-9/5}\, R_0^{-1/4}.$$

- Rutherford regime scaling is almost indistinguishable from original Fitzpatrick (1993) scaling, and cannot explain experimental data.
- Polarization regime scaling is consistent with experimental data.

Conclusions

- Observed scaling of error-field penetration threshold in tokamaks with engineering parameters—in particular, *linear* scaling with plasma density—indicates that threshold largely determined by *ion polarization current*.

- Since stabilizing effect of ion polarization current only manifests itself in *nonlinear* island physics, this necessitates nonlinear response model for plasma in immediate vicinity of rational surface. However, in absence of ion polarization current, such a model fails as badly as a linear response model.

- Given that error-field penetration threshold is governed by ion polarization current, seems highly likely that threshold for *neoclassical tearing modes* is also determined by this effect.