

# Scaling of Error-Field Penetration Threshold in Tokamak Plasmas

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## Introduction

- Locked mode formation in low density target plasmas seriously limits available experimental operating space.
- Locked modes thought to be due to *error-field driven magnetic reconnection* triggered when resonant component of error-field (at rational surface) exceeds some *threshold value*.
- Observed threshold in present experiments is small (but manageable): *i.e.*,

$$(b_r/B_T)_{\text{crit}} > 10^{-4}.$$

- Threshold generally *decreases* with increasing machine size. What is expected penetration threshold for ITER?

## Empirical Scaling Laws

- Scaling studies of penetration threshold with “engineering parameters”,  $n_e$ ,  $B_T$ ,  $R_0$ , performed on many tokamaks.<sup>a</sup>
- Writing

$$(b_r/B_T)_{\text{crit}} \sim n_e^{\alpha_n} B_T^{\alpha_B} R_0^{\alpha_R},$$

all studies agree that  $\alpha_n \simeq 1$ : *i.e.*, density scaling is *linear*.

- Measured values of  $\alpha_B$  lie in range  $-2.9$  to  $-1.0$ .
- Value of  $\alpha_R$  cannot be directly measured, but can be inferred from dimensionless scaling arguments:

$$\alpha_R = 2 \alpha_n + 1.25 \alpha_B.$$

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<sup>a</sup>Compass-C/D, Textor, Alcator C-mod, DIII-D, JET

## Extrapolation to ITER

- Extrapolation from JET<sup>a</sup> to ITER<sup>b</sup> yields

$$1.3 \times 10^{-5} < (b_r/B_T)_{\text{crit}} < 2.7 \times 10^{-4}.$$

- Proposed ITER error-field correction system designed to reduce resonant error-fields to level

$$b_r/B_T \simeq 5 \times 10^{-5}.$$

- Large uncertainty in extrapolation of penetration threshold to ITER (and, hence, in adequacy of error-field correction system).  
Could reduce uncertainty by developing error-field penetration theory consistent with experimental data.

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<sup>a</sup> $n_e = 1.6 \times 10^{19} \text{ m}^{-3}$ ,  $B_T = 3.5 \text{ T}$ ,  $R_0 = 2.95 \text{ m}$ ,  $(b_r/B_T)_{\text{crit}} = 1.1 \times 10^{-4}$ .

<sup>b</sup> $n_e = 2 \times 10^{19} \text{ m}^{-3}$ ,  $B_T = 5.3 \text{ T}$ ,  $R_0 = 6.2 \text{ m}$ .

## Fitzpatrick (1993) Theory<sup>a</sup>

- Response of plasma governed by *linear resistive-viscous MHD*.
- Scaling of penetration threshold with standard dimensionless parameters:

$$(b_r/B_T)_{\text{crit}} \sim \beta^{-1/6} \nu_*^{1/6} \rho_*^{4/3}.$$

- Scaling with engineering parameters:<sup>b</sup>

$$(b_r/B_T)_{\text{crit}} \sim n_e^0 B_T^{-13/15} R_0^{-13/12}.$$

- Predicted scaling highly inconsistent with experimental data, since no density dependence.

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<sup>a</sup>Nucl. Fusion **33**, 1049 (1993).

<sup>b</sup>Assuming ohmic power balance,  $\tau_M \sim \tau_E$ , and (dimensionally consistent) neo-Alcator energy transport:  $B_T \tau_E \sim n_e B_T R_0^{13/4} \sim \beta^{5/4} \nu_*^{-1/4} \rho_*^{-7/2}$ .

## Cole & Fitzpatrick (2006) Theory<sup>a</sup>

- Response of plasma governed by linear resistive-viscous *drift*-MHD.
- Scaling of penetration threshold with dimensionless parameters:

$$(b_r/B_T)_{\text{crit}} \sim \nu_*^{1/4} \rho_*^{5/4}.$$

- Scaling with engineering parameters:

$$(b_r/B_T)_{\text{crit}} \sim n_e^{1/4} B_T^{-23/20} R_0^{-15/16}.$$

- Slight improvement in density scaling, but dependence still much too weak.

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<sup>a</sup>Plasma Phys. **13**, 032503 (2006).

## Cole, Hegna, Callen (2008) Theory<sup>a</sup>

- Like Cole & Fitzpatrick theory, except also takes *neoclassical flow damping* into account.
- Scaling of penetration threshold with dimensionless parameters:<sup>b</sup>

$$(b_r/B_T)_{\text{crit}} \sim \beta \nu_*^{-1/2} \rho_*^{3/2}.$$

- Scaling with engineering parameters:

$$(b_r/B_T)_{\text{crit}} \sim n_e^{1/2} B_T^{-13/10} R_0^{-5/8}.$$

- Further improvement in density scaling, but dependence still too weak.

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<sup>a</sup>Plasma Phys. **15**, 056102 (2008).

<sup>b</sup>Assuming flow damping in  $1/\nu$  regime.

## Fitzpatrick (2011) Theory<sup>a</sup>

- Like Cole, Hegna, Callen theory, except response of plasma in vicinity of rational surface governed by *nonlinear island physics*.
- Scaling of penetration threshold with dimensionless parameters:<sup>b</sup>

$$(b_r/B_T)_{\text{crit}} \sim \beta \rho_*.$$

- Scaling with engineering parameters:

$$(b_r/B_T)_{\text{crit}} \sim n_e B_T^{-9/5} R_0^{-1/4}.$$

- Scaling fairly consistent with experimental data. Leads to predicted ITER penetration threshold of  $(b_r/B_T)_{\text{crit}} \sim 5 \times 10^{-5}$ .

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<sup>a</sup><http://farside.ph.utexas.edu/papers/nonlinear.pdf>

<sup>b</sup>Assuming plasma response in so-called “polarization regime.”



## Island Width Evolution

- New expression for penetration threshold obtained from recently developed magnetic island model.<sup>a</sup>
- Time evolution of island width,  $w$ , governed by<sup>b</sup>

$$4I_1 \tau_R \frac{d}{dt} \left( \frac{w}{r_s} \right) = \Delta' + 2m_\theta \left( \frac{w_v}{w} \right)^2 \cos \phi - I_p \beta_0 \left( \frac{w_0}{r_s} \right)^2 \frac{r_s^3}{w^3 + \rho_s^3}.$$

- Second term on r.h.s. represents error-field drive. Third term describes stabilizing influence of *ion polarization current*.

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<sup>a</sup>R. Fitzpatrick, F.L. Waelbroeck, Phys. Plasmas **17**, 062503 (2010).

<sup>b</sup>See paper for complete list of definitions.

## Ion Polarization Stabilization

- Large island limit ( $w \gg \rho_s$ ): Acceleration of ion fluid flowing around curved island separatrix produces perpendicular ion polarization current. Parallel return current has strong *stabilizing* effect on island that scales as  $w^{-3}$ .
- Small island limit ( $w \simeq \rho_s$ ): Ion fluid decouples from magnetic flux-surfaces and flows straight through island separatrix. Much smaller ion polarization current generated. Parallel return current has weak *stabilizing* effect on island that scales as  $w^0$ .
- Polarization term used in model is interpolation between large and small island limits.

## Island Phase Evolution

- Time evolution of island phase,  $\phi$ , governed by <sup>a</sup>

$$-2m_{\theta} \left( \frac{w_v}{w_0} \right)^2 \left( \frac{w}{r_s} \right)^2 \sin \phi = 4\beta_0 \left( \frac{\nu_{\phi i}}{\tau_M \omega_{*i}^2} \right)^{1/2} \left( \frac{1}{\omega_{*i}} \frac{d\phi}{dt} - \nu_{nc} \right).$$

- Left hand side is electromagnetic *locking torque* due to error-field. Right-hand side is viscous torque due to combination of *anomalous ion perpendicular viscosity* and *neoclassical ion toroidal viscosity*.

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<sup>a</sup>See paper for complete list of definitions.

## Penetration Regimes

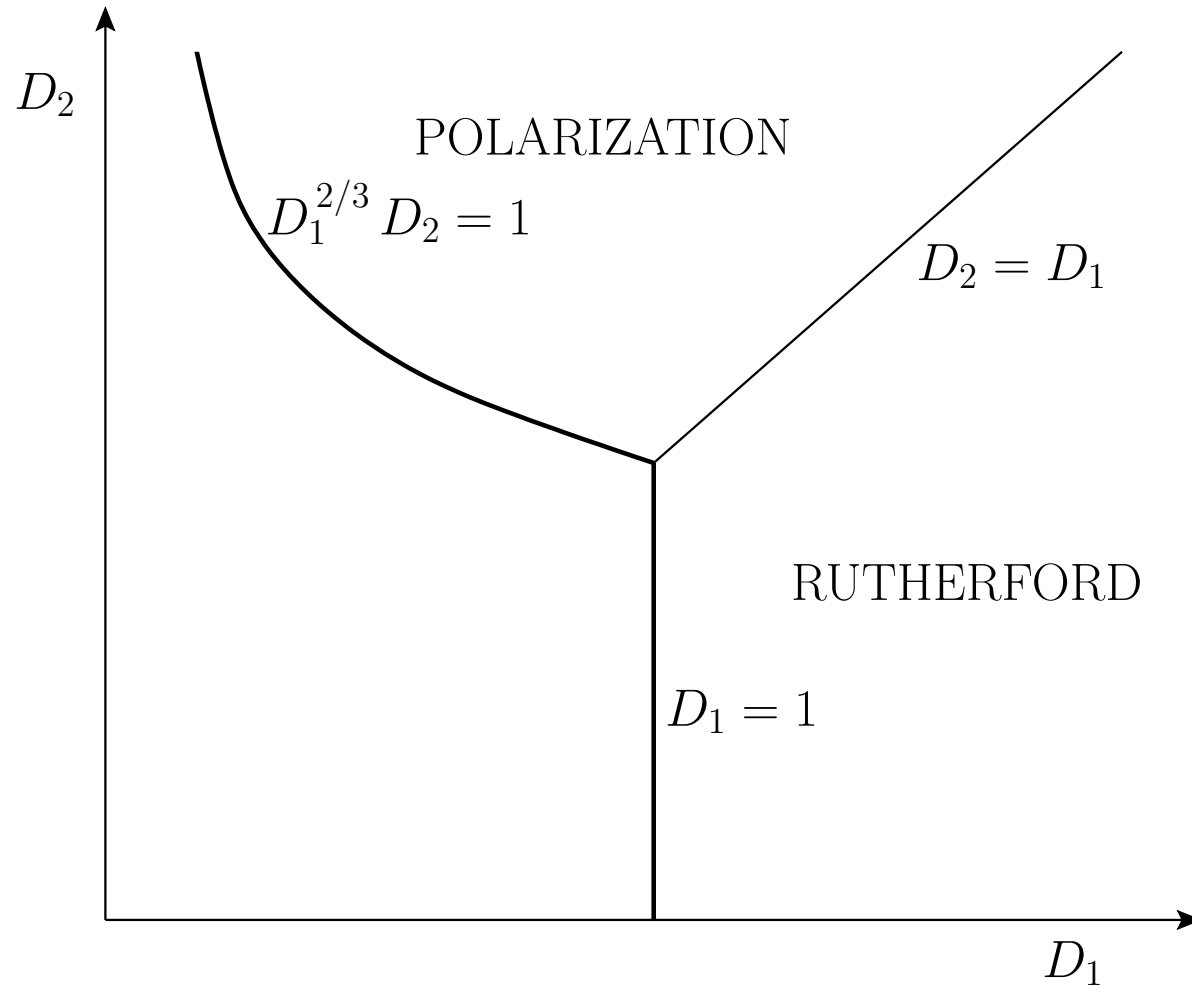
- Island model predicts two distinct regimes for error-field penetration depending on values of

$$D_1 \sim \frac{\tau_R \omega_{*i}}{(\tau_D \tau_M \omega_{*i}^2)^{1/4}} \beta^{1/2},$$

$$D_2 \sim (\tau_D \tau_M \omega_{*i})^{1/2} \beta \rho_*.$$

- Here,  $\tau_R$  is resistive t.s.,  $\tau_M$  is momentum confinement t.s., and  $\tau_D$  is neoclassical toroidal flow damping t.s.
- $D_1$  and  $D_2$  parameterize relative importance of *neoclassical rotation* and the *ion polarization current*, respectively, in suppressing error-field driven magnetic reconnection.

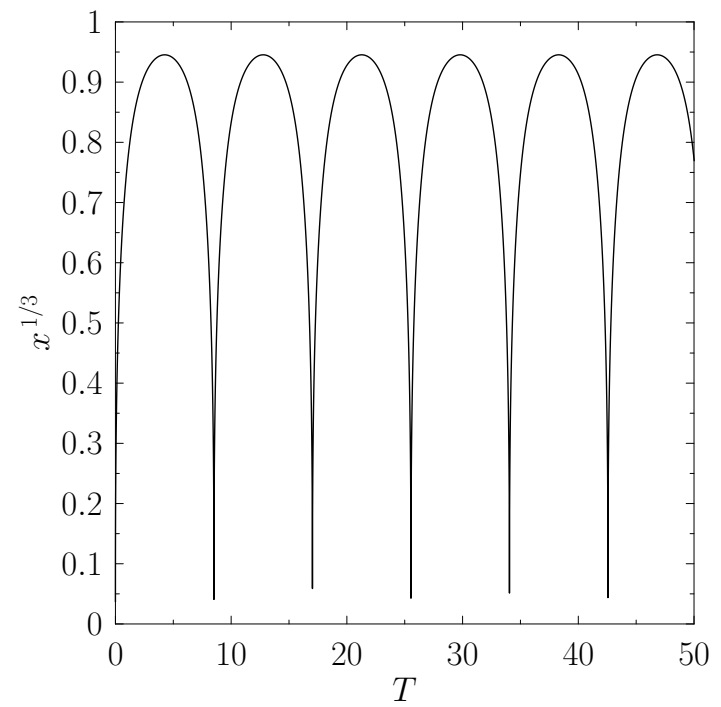
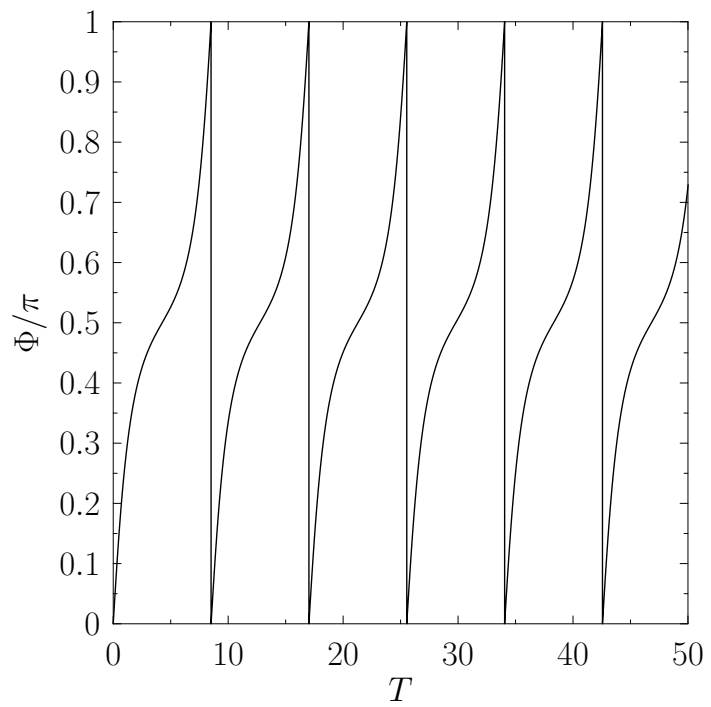
## Penetration Regimes



## Rutherford Regime

- Suppression of driven magnetic reconnection due to *neoclassical plasma rotation*, which prevents island from locking in phase with resonant error-field. Stabilizing effect of ion polarization current negligible.
- Penetration occurs when electromagnetic locking torque becomes large enough to overwhelm viscous torque, and allows island to lock to resonant error-field.
- Prior to penetration, island width “pulsates”, since island spends as much time in stabilizing phase of error-field, as in destabilizing phase.

## Rutherford Regime

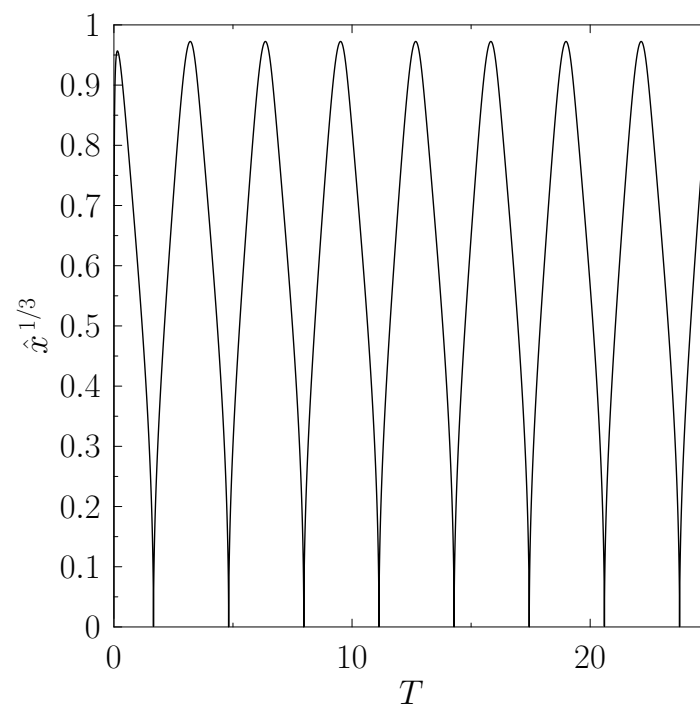
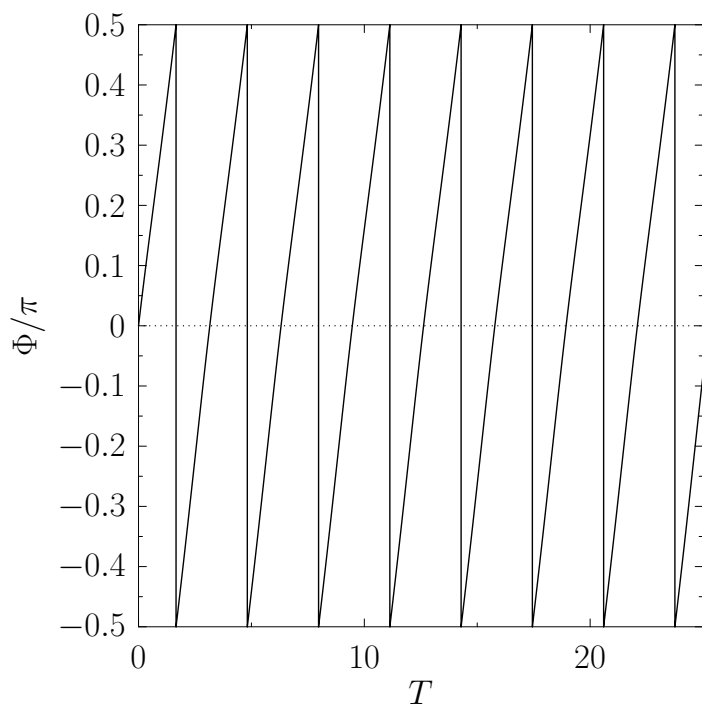


## Polarization Regime

- Suppression of driven magnetic reconnection due to *ion polarization current*, which essentially prevents island width from exceeding  $\rho_s$ , and, hence, prevents locking to resonant error-field.
- Penetration occurs when error-field drive overwhelms polarization current stabilization, allowing island to grow to large amplitude, and triggering locking of island to resonant error-field.
- Prior to penetration, island width “pulsates” due to balance between error-field destabilization (which varies with island phase) and polarization current stabilization.



## Polarization Regime



## Scaling of Penetration Threshold

- Rutherford regime:

$$(b_r/B_T)_{\text{crit}} \sim \beta v_*^{-1} \rho_*^2 \sim n_e^0 B_T^{-4/5} R_0^{-1}.$$

- Polarization regime:

$$(b_r/B_T)_{\text{crit}} \sim \beta \rho_* \sim n_e B_T^{-9/5} R_0^{-1/4}.$$

- Rutherford regime scaling is almost indistinguishable from original Fitzpatrick (1993) scaling, and cannot explain experimental data.
- Polarization regime scaling is consistent with experimental data.

## Conclusions

- Observed scaling of error-field penetration threshold in tokamaks with engineering parameters—in particular, *linear* scaling with plasma density—indicates that threshold largely determined by *ion polarization current*.
- Since stabilizing effect of ion polarization current only manifests itself in *nonlinear* island physics, this necessitates nonlinear response model for plasma in immediate vicinity of rational surface. However, in absence of ion polarization current, such a model fails as badly as a linear response model.
- Given that error-field penetration threshold is governed by ion polarization current, seems highly likely that threshold for *neoclassical tearing modes* is also determined by this effect.