Effect of an Error-Field on the Stability of the Resistive Wall Mode in DIII-D

Richard Fitzpatrick

Institute for Fusion Studies

University of Texas at Austin

Introduction

- Recent reconfiguration of neutral beams on DIII-D allows, for first time, decoupling of plasma β from plasma rotation.
- First balanced beam experiments reveal critical rotation rate to stabilize RMW which is significantly lower ($\sim 0.5\%$) than critical rate previously observed in strongly rotating plasmas ($\sim 2\%$).
- Old RWM experiments used error-fields to control plasma rotation.
- Speculation that critical rotation rate in new experiments represents true RWM stability boundary. Critical rate measured in old experiments corresponds to sudden collapse in plasma rotation due to loss of force balance with error-field.
- Does this explanation hold water?

Cylindrical Theory

- Treat plasma as periodic cylinder of radius a (minor radius) and periodicity length $2\pi\,R_0$ (major radius).
- Adopt cylindrical polar coordinates (r, θ, z).
- Equilibrium magnetic field: $\mathbf{B} = (0, B_\theta(r), B_\phi)$.
- Normalize all lengths to a, all magnetic fields to B_ϕ, all mass densities to central plasma mass density ρ_0, all times to hydromagnetic time $\tau_H = R_0\,\sqrt{\mu_0\,\rho_0}/B_\phi$.

Plasma Equilibrium

- Density profile:

$$\rho = (1 - r^2)^{1/2}.$$

- Safety-factor profile: [a]

$$q(r) = \frac{q_a}{1 - (1 - r^2)^{q_a/q_0}},$$

where q_0 is central-q and q_a is edge-q.

- Toroidal plasma current:

$$J_\phi = J_0\,(1 - r^2)^{q_a/q_0 - 1}.$$

[a] J.A. Wesson, Nucl. Fusion **18**, 87 (1978).

Ideal Magnetohydrodynamics

- Consider external-kink mode with m periods in poloidal direction and n periods in toroidal direction.

- According to ideal-MHD, radial displacement, ξ, in plasma satisfies eigenmode equation

$$\frac{d}{dr}\left[r\,(\rho\,\gamma^2+Q^2)\,\frac{d(r\,\xi)}{dr}\right]-\left[m^2\,(\rho\,\gamma^2+Q^2)+r\,\frac{dQ^2}{dr}\right]\xi=0.$$

- Here, γ is growth-rate (in plasma frame), and

$$Q=\frac{m}{q}-n.$$

- No resonant surface (where $Q=0$) in plasma for external-kink mode.

Boozer Stability Parameter

- Launch well-behaved solution of ideal eigenmode equation from magnetic axis ($r = 0$). Integrate to edge ($r = 1$).
- Boozer stability parameter defined[a]

$$s = -\frac{1}{2}\left(1 + m^{-1}\,\left.\frac{d[\ln(r\,Q\,\xi)]}{d\ln r}\right|_{r=1}\right).$$

- If plasma surrounded by single concentric thin resistive wall of radius r_w and time-constant τ_w then RWM dispersion relation written

$$\gamma\,[c-(1-c)\,s]-\gamma_w\,s=0,$$

where $\gamma_w = 2\,m/\tau_w$ and $c = r_w^{-2\,m}$.

[a] A.H. Boozer, Phys. Plasmas **5**, 3350 (1998).

RWM Stability Parameter

- No-wall external-kink stability boundary: $s = 0$.
- Ideal-wall external-kink stability boundary: $s = s_c \equiv c/(1-c)$.
- Can define

$$\bar{s} = \frac{s}{s_c}.$$

No-wall / perfect wall stability boundaries: $\bar{s} = 0$ / $\bar{s} = 1$.

- Parameter $\bar{s}$ equivalent to familiar experimental parameter

$$\frac{\beta - \beta_{nw}}{\beta_{pw} - \beta_{nw}}.$$

Plasma Rotation and Dissipation

- In absence of plasma rotation and dissipation, RWM stability boundary is $\bar{s} = 0$. In presence of rotation and dissipation, RWM stability boundary can, in principle, be raised to $\bar{s} = 1$.
- Assume that plasma rotates uniformly in poloidal and toroidal directions with angular velocities Ω_θ and Ω_ϕ, respectively.
- Plasma dissipation provided by edge neoclassical flow-damping.

Dissipation via Neoclassical Flow-Damping

- Dissipation via neoclassical flow-damping can be incorporated into ideal-MHD eigenmode equation by making substitution: [a]

$$\rho\,\gamma^{2} \rightarrow \rho\,\gamma'^{\,2}\left(1+\frac{q^2}{\epsilon_0^2}\,\frac{\mu_\theta}{\gamma' + r^2\,\mu_\theta}\right),$$

where $\gamma' = \gamma - \mathrm{i}\,m\,\Omega_\theta + \mathrm{i}\,n\,\Omega_\phi$, $\epsilon_0 = a/R_0$, and μ_θ is (edge) neoclassical poloidal flow-damping parameter.

[a] K.C. Shiang, Phys. Plasmas **11**, 5525 (2004).

Neoclassical Flow-Damping

- Neoclassical poloidal flow-damping parameter given by [a]

$$\mu_\theta = \frac{3}{2}\,\frac{\epsilon_0^4}{q_a^2}\,\frac{\eta_0}{\rho}\,\frac{\tau_H}{a^2}\,F(\nu_*),$$

where

$$F(\nu_*) = \frac{\nu_*}{(3.23\,\epsilon_0^{3/2} + 3.15\,\epsilon_0^{3/4}\,\nu_*^{1/2} + \nu_*)}\,\frac{\nu_*}{(1.52 + \nu_*)},$$

and

$$\nu_* = \frac{R_0\,q_a}{v_i\,\tau_i}.$$

Here, η_0 is (edge) Braginskii parallel viscosity, v_i is (edge) ion thermal velocity, and τ_i (edge) ion collision time.

[a] J.D. Callen, *et al.*, Kyoto IAEA mtg., 1986.

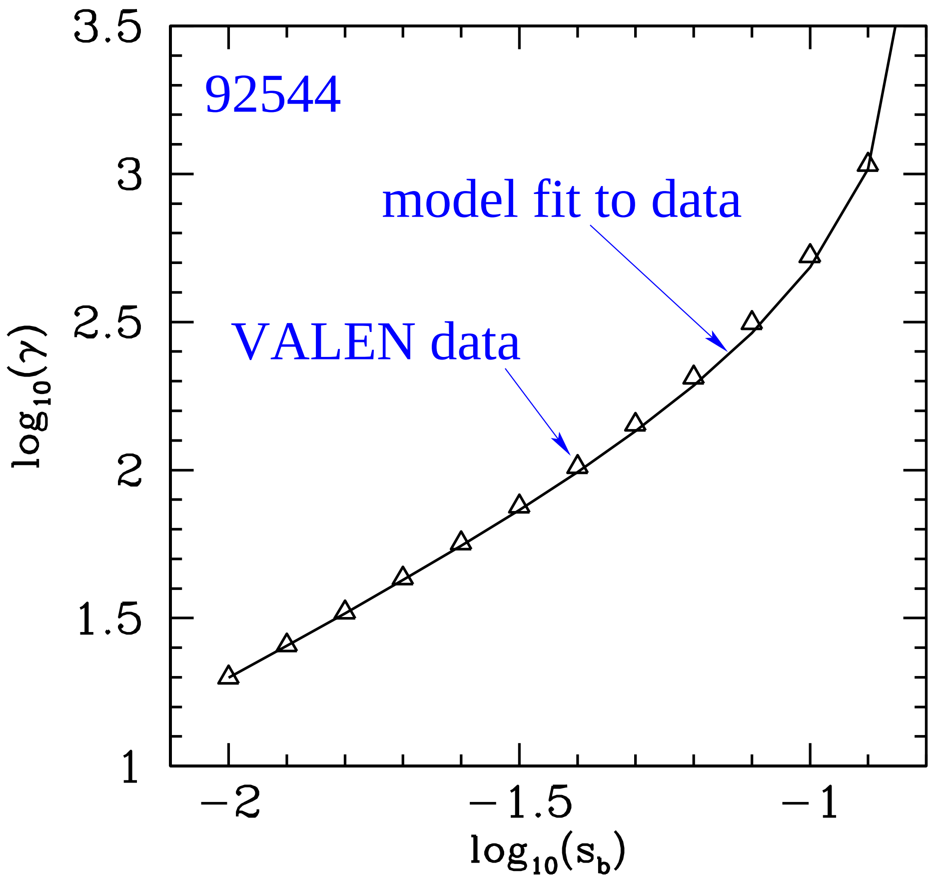
RWM Dispersion Relation

- In presence of plasma rotation and dissipation, numerically calculated stability parameter s becomes complex.
- Can substitute into RWM dispersion relation,

$$\gamma\,[c-(1-c)\,s]-\gamma_w\,s=0,$$

  to obtain RWM growth-rate.
- Can determine wall parameters, c and γ_w, from cylindrical theory.
- More accurate to determine wall parameters by fitting above dispersion relation to output from VALEN (for real s).

DIII-D Wall Parameters

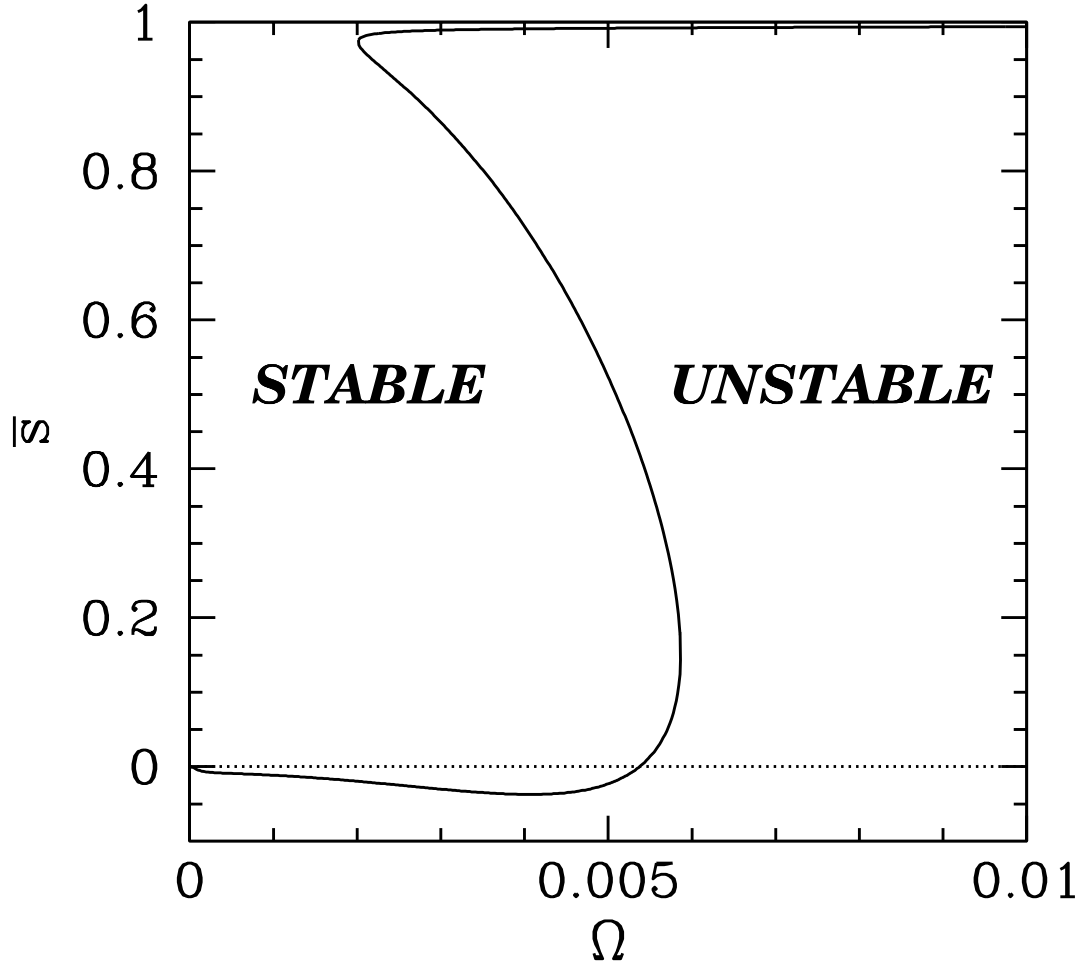


$$c = 0.14 \quad \gamma_w = 262\ \mathrm{s}^{-1}$$

DIII-D RWM Stability Calculation

- Physics parameters: $R_0 = 1.69$ m, $a = 0.54$ m, $B_\phi = 2.1$ T, $n(0) = 6 \times 10^{19}\,\mathrm{m}^{-3}$, $n(a) = 2 \times 10^{19}\,\mathrm{m}^{-3}$, $T_e(a) = 100$ eV, $T_i(a) = 100$ eV.
- Derived parameters: $\tau_H = 3 \times 10^{-7}$ s, $\nu_* = 0.82$, $\mu_\theta = 1 \times 10^{-4}$.
- Equilibrium parameters: $q_a = 2.95$, $\alpha = 0.5$.
- Mode parameters: $m = 3$, $n = 1$.
- Plasma stability varied by varying q_0.

DIII-D RWM Stability Boundaries



Effect of Resonant Error-Field

- Resonant error-field does not interact directly with RWM (because of frequency mismatch).
- Error-field interacts indirectly with RWM by braking plasma rotation.
- Using same plasma eigenmode equation as that used to calculate complex RWM stability parameter, $s(\gamma)$, can calculate complex error-field response parameter, $s' \equiv s(0)$.
- Plasma amplifies error-field by factor $1/(-s')$.

Torque Balance - I

- Let

$$\Omega = m\,\Omega_\theta - n\,\Omega_\phi$$

be effective plasma rotation frequency.

- Plasma torque balance equation takes form

$$\Omega^{(0)} = \Omega + G\,\frac{\mathrm{Im}(s')}{|s'|^2}\,b_r^{\,2},$$

where $\Omega^{(0)}$ is plasma rotation frequency in absence of error-field, and b_r is vacuum edge radial error-field strength.

Torque Balance - II

- Have

$$G = 2\,(1+\alpha)\left[m\,\frac{(2+\alpha)}{\tau_\theta^{-1}+\tau_M^{-1}} + \frac{n^2}{m}\,\frac{\epsilon_0^2}{\tau_M^{-1}}\right].$$

  where $\tau_\theta^{-1} = (q_a/\epsilon_0)^2\,\mu_\theta$ is neoclassical poloidal flow-damping time, and τ_M is phenomenological toroidal momentum confinement time.

- For DIII-D, $\tau_M \sim 60\,\mathrm{ms}$ (2×10^5 in normalized units), and $\tau_\theta = 10^2$ (in normalized units).

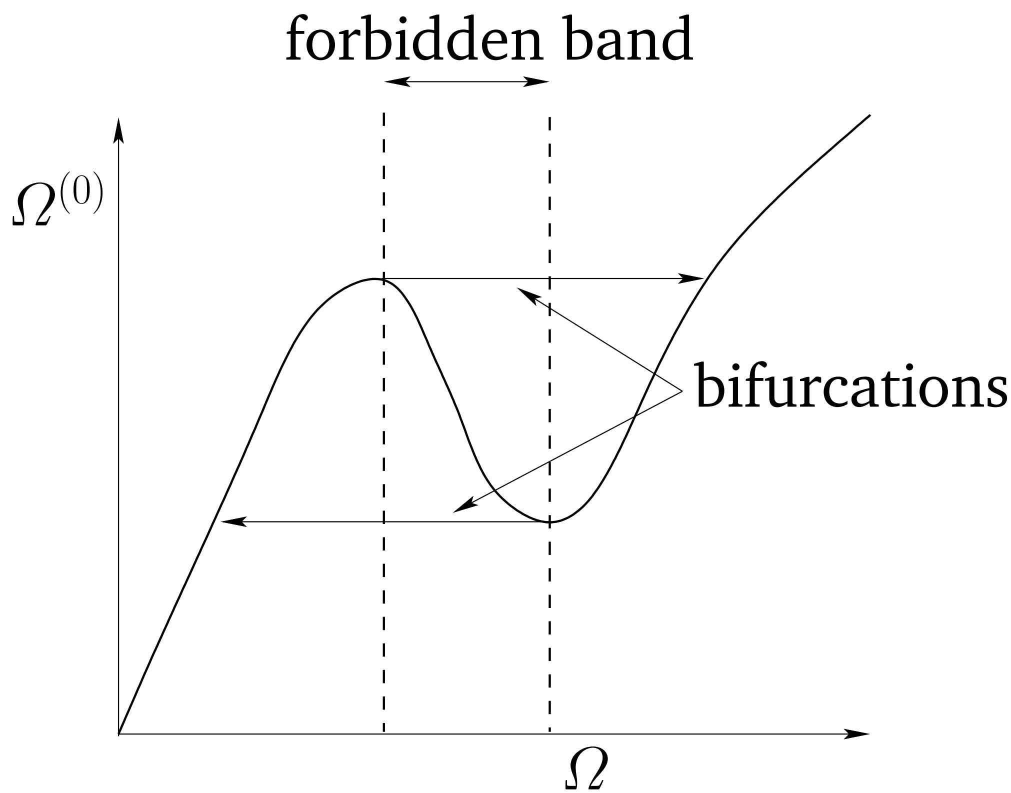
- Require

$$\frac{d\Omega^{(0)}}{d\Omega} > 0$$

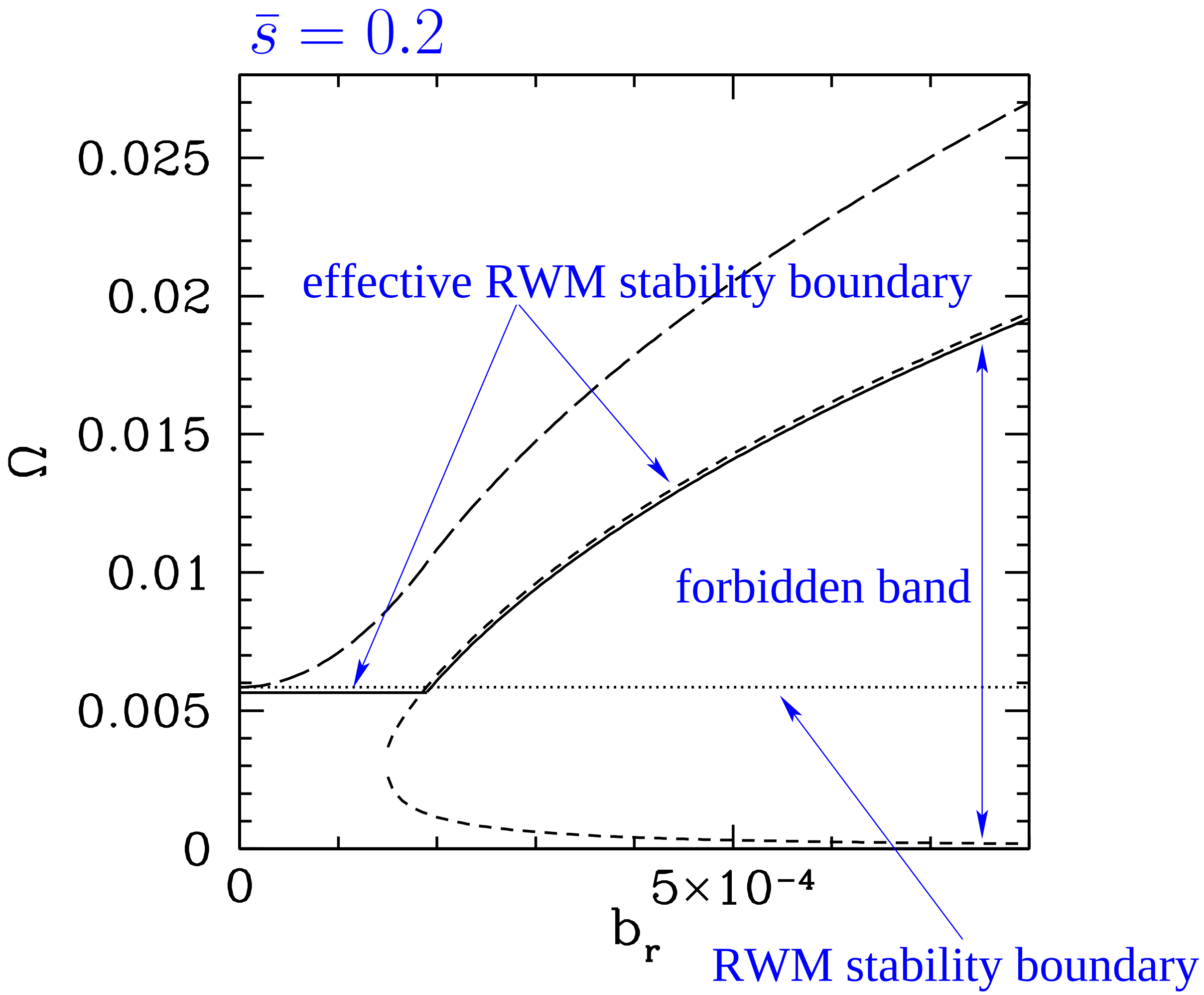
  for dynamical stability.

Torque Balance - III

- Calculated torque balance curves exhibit characteristic induction motor bifurcations.



Modified RWM Stability Boundary



Summary

- Have constructed simple model of RWM in DIII-D. All parameters in model can be related to directly measurable quantities.
- Model predicts that critical rotation rate in DIII-D required to stabilize RWM is $0.6\,\%$, which is similar to critical rotation rate observed in balanced beam experiments.
- In presence of resonant error-field of magnitude about 5 gauss, model predicts that forbidden band of plasma rotation rates engulfs critical rate. Effective critical rotation rate increases to $2\,\%$. Similar to critical rate observed in old RWM experiments.
- Proposed explanation of lower critical rotation seen in new RWM experiments seems highly plausible.