

**Effect of an Error-Field on the
Stability of the Resistive Wall Mode
in DIII-D**

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Introduction

- Recent reconfiguration of neutral beams on DIII-D allows, for first time, decoupling of plasma β from plasma rotation.
- First balanced beam experiments reveal critical rotation rate to stabilize RMW which is **significantly lower** ($\sim 0.5\%$) than critical rate previously observed in strongly rotating plasmas ($\sim 2\%$).
- Old RWM experiments used **error-fields** to control plasma rotation.
- Speculation that critical rotation rate in new experiments represents **true** RWM stability boundary. Critical rate measured in old experiments corresponds to sudden collapse in plasma rotation due to loss of force balance with error-field.
- Does this explanation hold water?

Cylindrical Theory

- Treat plasma as periodic cylinder of radius a (minor radius) and periodicity length $2\pi R_0$ (major radius).
- Adopt cylindrical polar coordinates (r, θ, z) .
- Equilibrium magnetic field: $\mathbf{B} = (0, B_\theta(r), B_\phi)$.
- Normalize all lengths to a , all magnetic fields to B_ϕ , all mass densities to central plasma mass density ρ_0 , all times to hydromagnetic time $\tau_H = R_0 \sqrt{\mu_0 \rho_0} / B_\phi$.

Plasma Equilibrium

- Density profile:

$$\rho = (1 - r^2)^{1/2}.$$

- Safety-factor profile:^a

$$q(r) = \frac{q_a}{1 - (1 - r^2) q_a/q_0},$$

where q_0 is central- q and q_a is edge- q .

- Toroidal plasma current:

$$J_\phi = J_0 (1 - r^2)^{q_a/q_0 - 1}.$$

^aJ.A. Wesson, Nucl. Fusion **18**, 87 (1978).

Ideal Magnetohydrodynamics

- Consider external-kink mode with m periods in poloidal direction and n periods in toroidal direction.
- According to ideal-MHD, radial displacement, ξ , in plasma satisfies eigenmode equation

$$\frac{d}{dr} \left[r (\rho \gamma^2 + Q^2) \frac{d(r \xi)}{dr} \right] - \left[m^2 (\rho \gamma^2 + Q^2) + r \frac{dQ^2}{dr} \right] \xi = 0.$$

- Here, γ is growth-rate (in plasma frame), and

$$Q = \frac{m}{q} - n.$$

- No resonant surface (where $Q = 0$) in plasma for external-kink mode.

Boozer Stability Parameter

- Launch well-behaved solution of ideal eigenmode equation from magnetic axis ($r = 0$). Integrate to edge ($r = 1$).
- Boozer stability parameter defined^a

$$s = -\frac{1}{2} \left(1 + m^{-1} \frac{d[\ln(r Q \xi)]}{d \ln r} \Big|_{r=1} \right).$$

- If plasma surrounded by single concentric thin resistive wall of radius r_w and time-constant τ_w then RWM dispersion relation written

$$\gamma [c - (1 - c) s] - \gamma_w s = 0,$$

where $\gamma_w = 2m/\tau_w$ and $c = r_w^{-2m}$.

^aA.H. Boozer, Phys. Plasmas **5**, 3350 (1998).

RWM Stability Parameter

- No-wall external-kink stability boundary: $s = 0$.
- Ideal-wall external-kink stability boundary: $s = s_c \equiv c/(1 - c)$.
- Can define

$$\bar{s} = \frac{s}{s_c}.$$

No-wall / perfect wall stability boundaries: $\bar{s} = 0$ / $\bar{s} = 1$.

- Parameter \bar{s} equivalent to familiar experimental parameter

$$\frac{\beta - \beta_{nw}}{\beta_{pw} - \beta_{nw}}.$$

Plasma Rotation and Dissipation

- In absence of plasma rotation and dissipation, RWM stability boundary is $\bar{s} = 0$. In presence of rotation and dissipation, RWM stability boundary can, in principle, be raised to $\bar{s} = 1$.
- Assume that plasma rotates **uniformly** in poloidal and toroidal directions with angular velocities Ω_θ and Ω_ϕ , respectively.
- Plasma dissipation provided by edge **neoclassical flow-damping**.

Dissipation via Neoclassical Flow-Damping

- Dissipation via neoclassical flow-damping can be incorporated into ideal-MHD eigenmode equation by making substitution:^a

$$\rho \gamma^2 \rightarrow \rho \gamma'^2 \left(1 + \frac{q^2}{\epsilon_0^2} \frac{\mu_\theta}{\gamma' + r^2 \mu_\theta} \right),$$

where $\gamma' = \gamma - i m \Omega_\theta + i n \Omega_\phi$, $\epsilon_0 = a/R_0$, and μ_θ is (edge) neoclassical poloidal flow-damping parameter.

^aK.C. Shiang, Phys. Plasmas **11**, 5525 (2004).

Neoclassical Flow-Damping

- Neoclassical poloidal flow-damping parameter given by ^a

$$\mu_{\theta} = \frac{3}{2} \frac{\epsilon_0^4}{q_a^2} \frac{\eta_0}{\rho} \frac{\tau_H}{a^2} F(\nu_*),$$

where

$$F(\nu_*) = \frac{\nu_*}{(3.23 \epsilon_0^{3/2} + 3.15 \epsilon_0^{3/4} \nu_*^{1/2} + \nu_*)} \frac{\nu_*}{(1.52 + \nu_*)},$$

and

$$\nu_* = \frac{R_0 q_a}{v_i \tau_i}.$$

Here, η_0 is (edge) Braginskii parallel viscosity, v_i is (edge) ion thermal velocity, and τ_i (edge) ion collision time.

^aJ.D. Callen, *et al.*, Kyoto IAEA mtg., 1986.

RWM Dispersion Relation

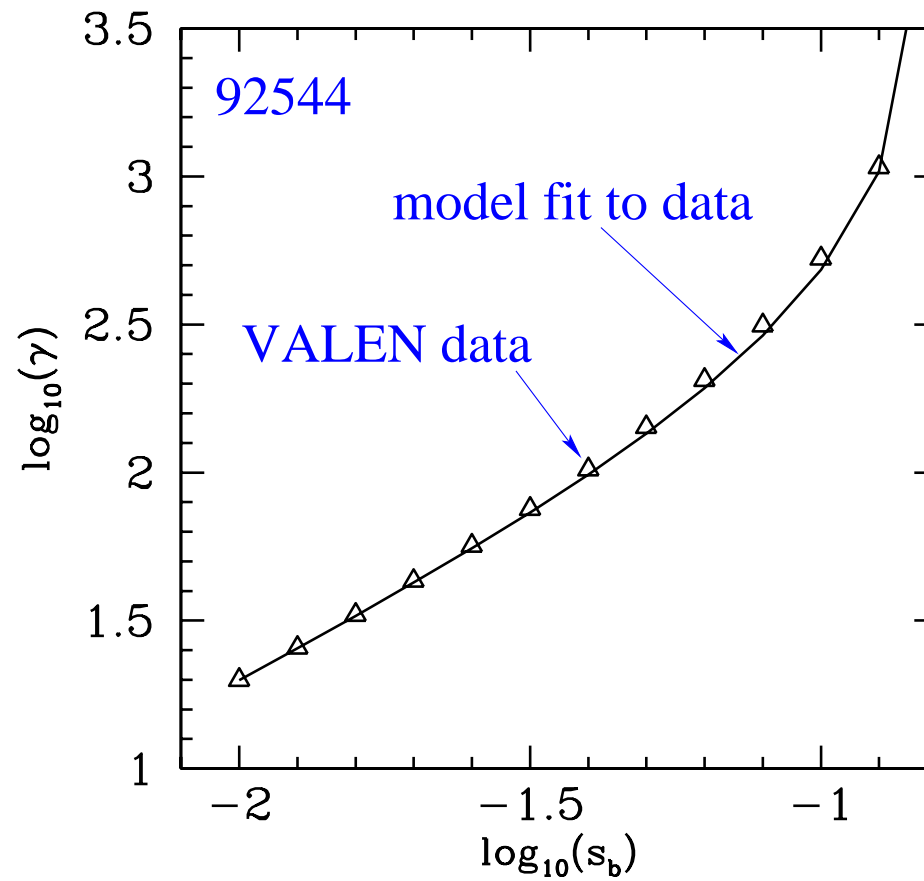
- In presence of plasma rotation and dissipation, numerically calculated stability parameter s becomes complex.
- Can substitute into RWM dispersion relation,

$$\gamma [c - (1 - c) s] - \gamma_w s = 0,$$

to obtain RWM growth-rate.

- Can determine wall parameters, c and γ_w , from cylindrical theory.
- More accurate to determine wall parameters by fitting above dispersion relation to output from VALEN (for real s).

DIII-D Wall Parameters

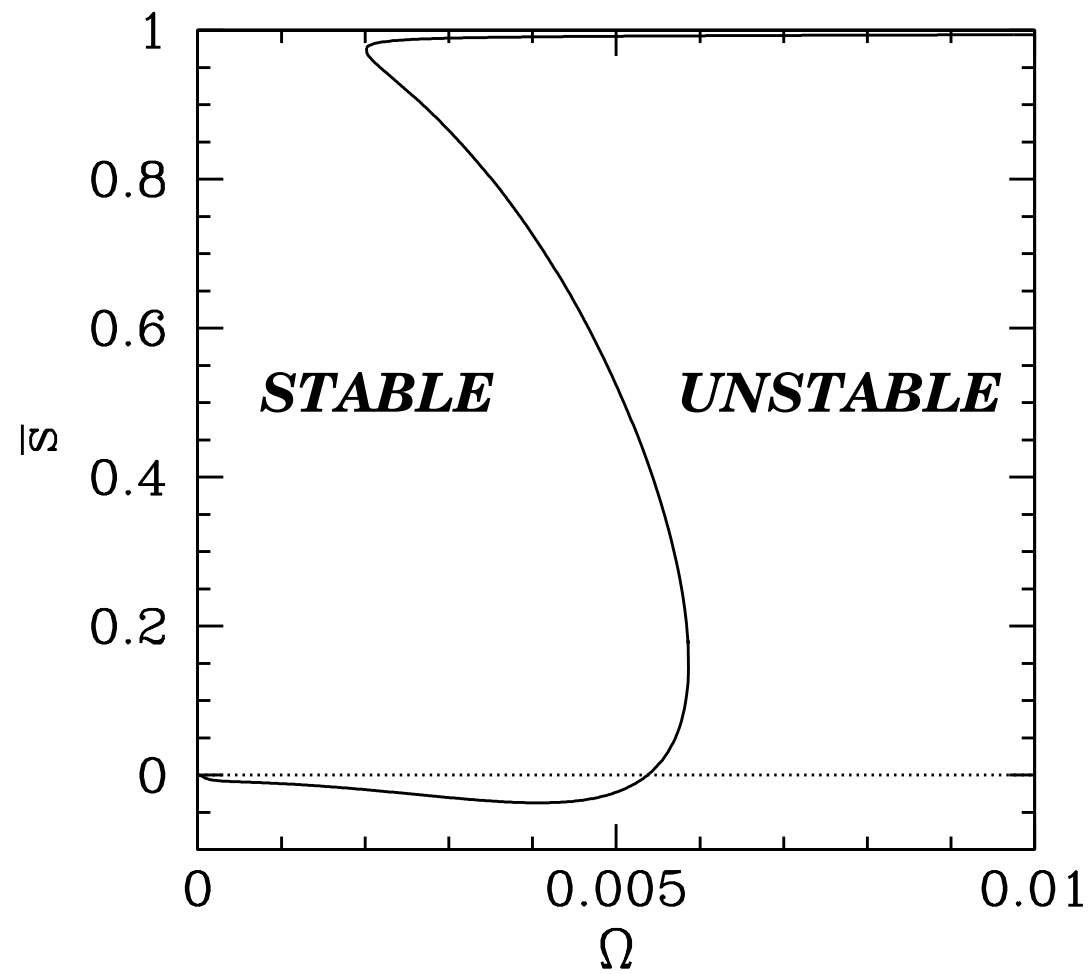


$$c = 0.14 \quad \gamma_w = 262 \text{ s}^{-1}$$

DIII-D RWM Stability Calculation

- Physics parameters: $R_0 = 1.69$ m, $a = 0.54$ m, $B_\phi = 2.1$ T, $n(0) = 6 \times 10^{19} \text{ m}^{-3}$, $n(a) = 2 \times 10^{19} \text{ m}^{-3}$, $T_e(a) = 100$ eV, $T_i(a) = 100$ eV.
- Derived parameters: $\tau_H = 3 \times 10^{-7}$ s, $\nu_* = 0.82$, $\mu_\theta = 1 \times 10^{-4}$.
- Equilibrium parameters: $q_a = 2.95$, $\alpha = 0.5$.
- Mode parameters: $m = 3$, $n = 1$.
- Plasma stability varied by varying q_0 .

DIII-D RWM Stability Boundaries



Effect of Resonant Error-Field

- Resonant error-field does not interact directly with RWM (because of frequency mismatch).
- Error-field interacts indirectly with RWM by **braking** plasma rotation.
- Using same plasma eigenmode equation as that used to calculate complex RWM stability parameter, $s(\gamma)$, can calculate complex error-field response parameter, $s' \equiv s(0)$.
- Plasma amplifies error-field by factor $1/(-s')$.

Torque Balance - I

- Let

$$\Omega = m \Omega_\theta - n \Omega_\phi$$

be effective plasma rotation frequency.

- Plasma torque balance equation takes form

$$\Omega^{(0)} = \Omega + G \frac{\text{Im}(s')}{|s'|^2} b_r^2,$$

where $\Omega^{(0)}$ is plasma rotation frequency in absence of error-field, and b_r is vacuum edge radial error-field strength.

Torque Balance - II

- Have

$$G = 2(1 + \alpha) \left[m \frac{(2 + \alpha)}{\tau_\theta^{-1} + \tau_M^{-1}} + \frac{n^2 \epsilon_0^2}{m \tau_M^{-1}} \right].$$

where $\tau_\theta^{-1} = (q_a/\epsilon_0)^2 \mu_\theta$ is neoclassical poloidal flow-damping time, and τ_M is phenomenological toroidal momentum confinement time.

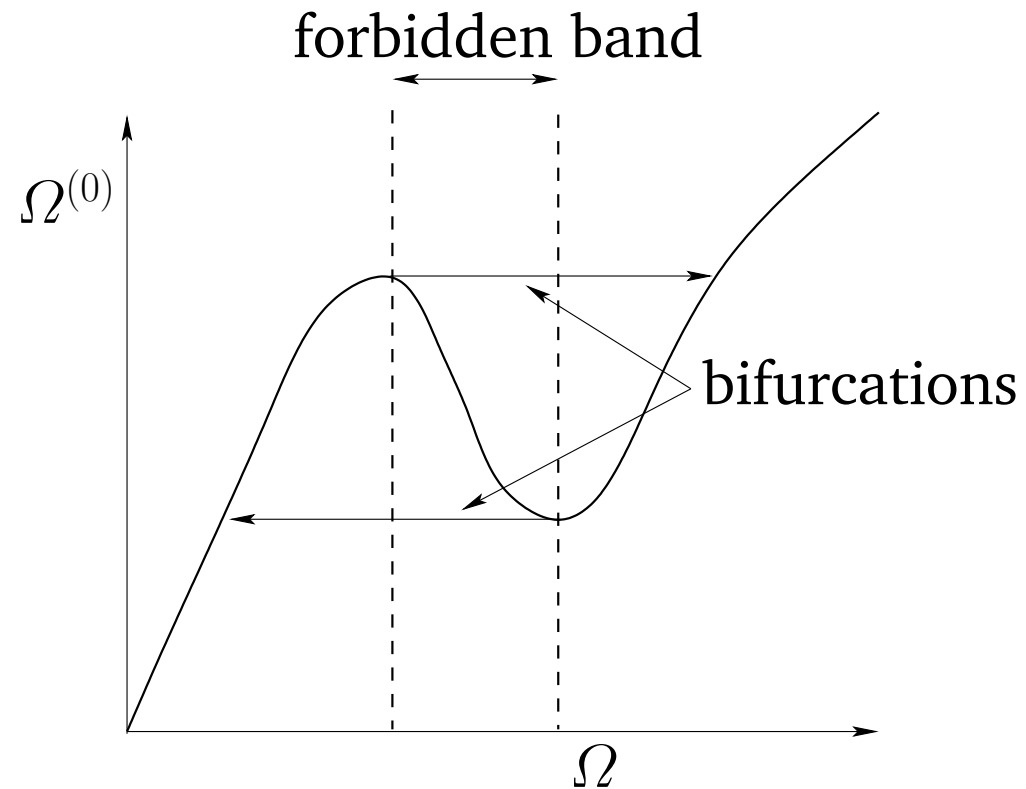
- For DIII-D, $\tau_M \sim 60 \text{ ms}$ (2×10^5 in normalized units), and $\tau_\theta = 10^2$ (in normalized units).
- Require

$$\frac{d\Omega^{(0)}}{d\Omega} > 0$$

for dynamical stability.

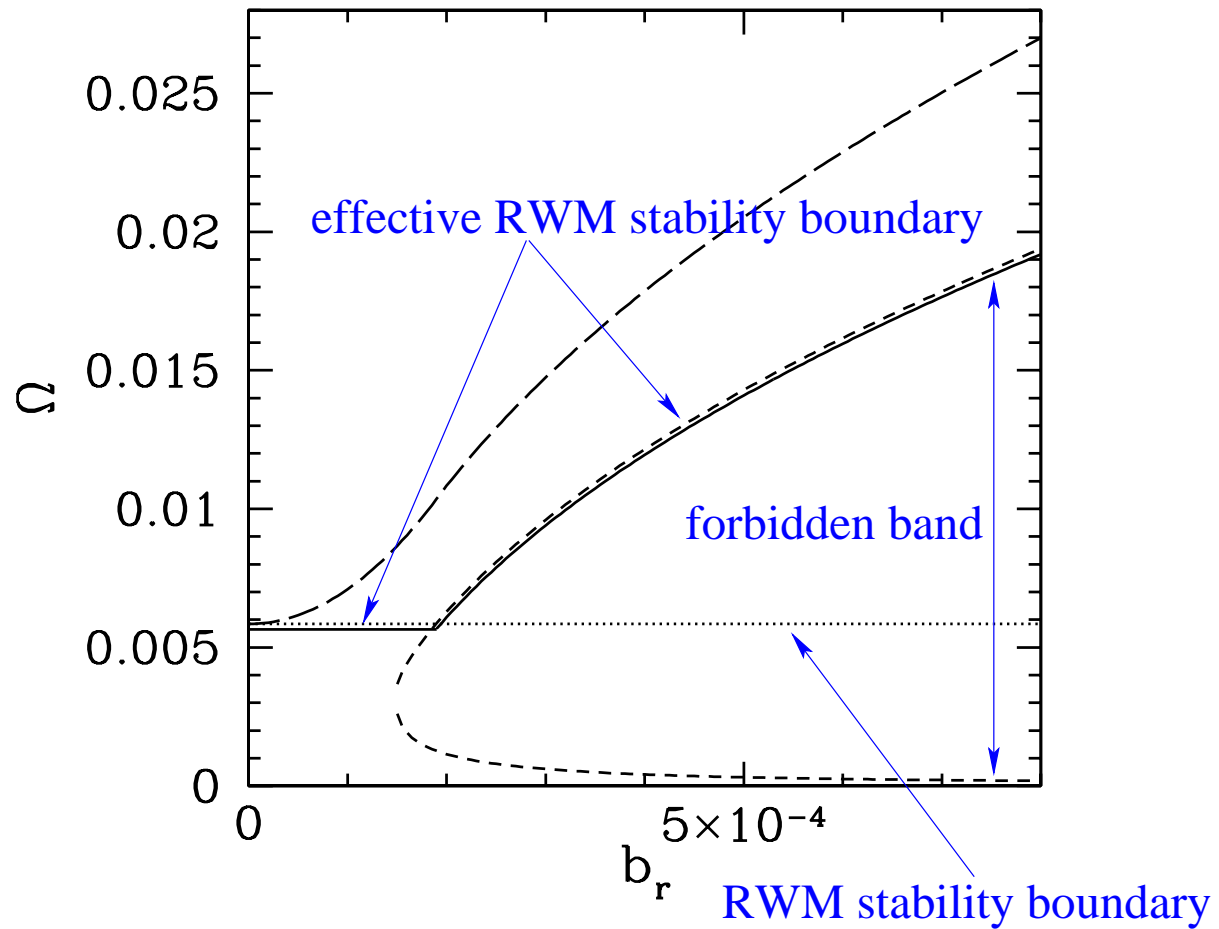
Torque Balance - III

- Calculated torque balance curves exhibit characteristic **induction motor** bifurcations.



Modified RWM Stability Boundary

$$\bar{s} = 0.2$$



Summary

- Have constructed simple model of RWM in DIII-D. All parameters in model can be related to directly measurable quantities.
- Model predicts that critical rotation rate in DIII-D required to stabilize RWM is **0.6%**, which is similar to critical rotation rate observed in balanced beam experiments.
- In presence of resonant error-field of magnitude about **5 gauss**, model predicts that forbidden band of plasma rotation rates engulfs critical rate. Effective critical rotation rate increases to **2%**. Similar to critical rate observed in old RWM experiments.
- Proposed explanation of lower critical rotation seen in new RWM experiments seems highly plausible.