# External Modes and Resistive Wall Modes

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# External Kink Modes

- Principal figure of merit in tokamak plasma is

$$\beta = \frac{\text{plasma energy density}}{\text{magnetic energy density}}.$$

- Main $\beta$-limiting instability is *external-kink mode*. This is *ideal* instability driven by radial *pressure and current gradients*.
- Talk will concentrate on current driven instabilities, since these can be described by *cylindrical theory*.
- Hope is that physics of pressure (*i.e.*, $\beta$) driven instabilities is analogous to that of current driven instabilities.

# Cylindrical Theory

- Treat plasma as periodic cylinder of radius $a$ (minor radius) and periodicity length $2\,\pi\,R_0$ (major radius).
- Adopt cylindrical polar coordinates ($r$, $\theta$, $z$).
- Equilibrium magnetic field: $\mathbf{B} = (0, B_\theta(r), B_z)$.
- Normalize all lengths to $a$, all magnetic fields to $B_z$, all mass densities to central plasma mass density $\rho_0$, all times to hydromagnetic time $\tau_H = R_0\,\sqrt{\mu_0\,\rho_0}/B_z$.

## Plasma Equilibrium

- Density profile:

$$\rho = (1 - r^2)^\alpha.$$

- Safety-factor ($q = r\, B_z/R_0\, B_\theta$) profile: [a]

$$q(r) = \frac{q_a}{1 - (1 - r^2)^{q_a/q_0}},$$

where $q_0$ is central-$q$ and $q_a$ is edge-$q$.

- Toroidal plasma current:

$$J_z = J_0\,(1 - r^2)^{q_a/q_0 - 1}.$$

[a] J.A. Wesson, Nucl. Fusion **18**, 87 (1978).

## Ideal Magnetohydrodynamics

- Consider external-kink mode with $m$ periods in poloidal direction and $n$ periods in toroidal direction.

- According to ideal-MHD, radial displacement in plasma $\xi$ satisfies

$$\frac{d}{dr}\left[r\,(\rho\,\gamma^2+Q^2)\,\frac{d(r\,\xi)}{dr}\right]-\left[m^2\,(\rho\,\gamma^2+Q^2)+r\,\frac{dQ^2}{dr}\right]\xi=0.$$

- Here, $\gamma$ is growth-rate (in plasma frame), and

$$Q=\frac{m}{q}-n.$$

- No resonant surface (where $Q=0$) in plasma for external-kink mode.

## Edge Boundary Condition

- Let

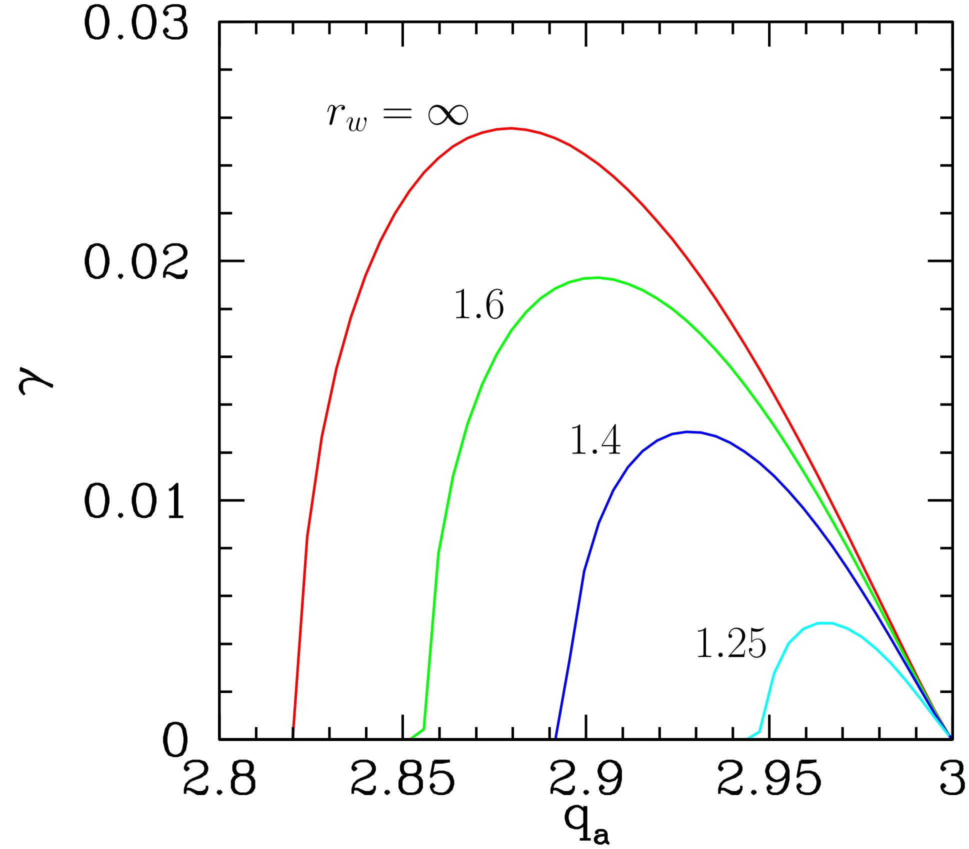
$$\lambda(\gamma) = -\left(\frac{d[\ln(r\,Q\,\xi)]}{m\,dr}\right)_{r=1}.$$

- If plasma surrounded by thin resistive wall of radius $r_w$ and time-constant $\tau_w$ then

$$\lambda(\gamma) = \frac{1 + (\gamma\,\tau_w/2\,m)\,(1 + r_w^{-2m})}{1 + (\gamma\,\tau_w/2\,m)\,(1 - r_w^{-2m})}.$$

- Above dispersion relation can be solved (*e.g.*, via Newton iteration) to give growth-rate $\gamma$ of external-kink mode.

# Effect of Ideal Wall on 3/1 External Kink Mode

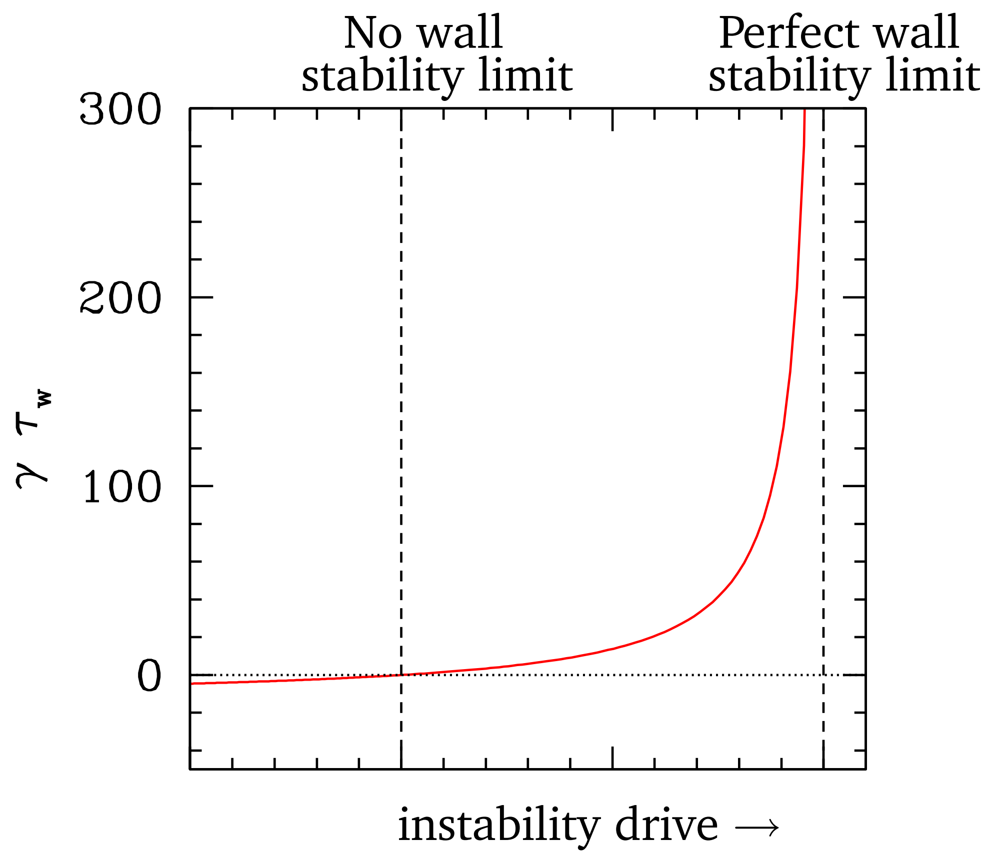



$\alpha = 0.5,\ q_0 = 1.3$

## Resistive Wall Mode

- External kink mode stabilized by close-fitting perfectly conducting wall (*i.e.*, $\tau_w \rightarrow \infty$).
- Suggests that close-fitting conducting wall (*e.g.*, vacuum vessel) might increase $\beta$-limit.
- Unfortunately, conducting walls posses finite *resistivity*. Resistive wall has finite time-constant $\tau_w$ which is generally much greater than $\tau_H$ but still much less than pulse length of discharge.
- Resistive wall *does not* stabilize external kink mode. Instead, converts mode into slowly growing ($\gamma \sim \tau_w^{-1}$) *resistive wall mode*.

# Effect of Resistive Wall on 3/1 External Kink Mode




$\alpha = 0.5,\ q_a = 2.95,\ r_w = 1.1,\ \tau_w = 10^6.$

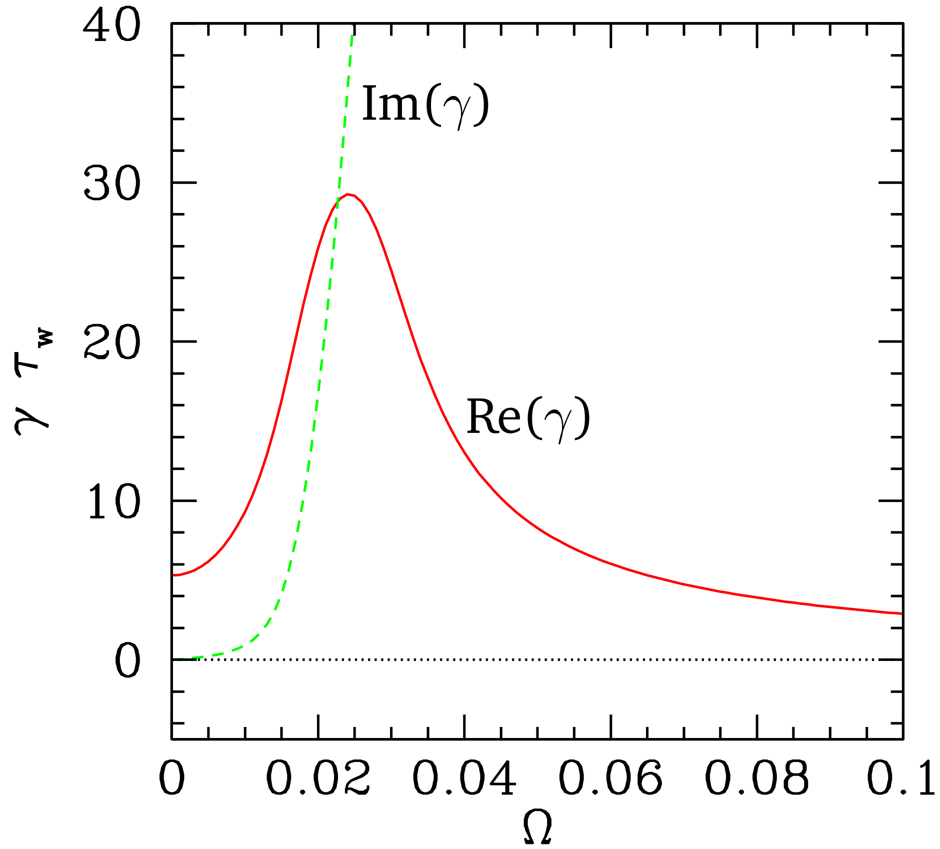
## Plasma Rotation

- Tokamak plasmas rotate toroidally at few percent of Alfvèn speed.
- Plasma rotation either intrinsic (not well understood) or due to unbalanced neutral beam injection.
- Since resistive wall mode is plasma instability, would expect plasma rotation to give mode *real frequency*.
- If real frequency exceeds $\tau_w^{-1}$ then resistive wall effectively acts as ideal wall. Rotation could stabilize resistive wall mode.
- Rotation introduced into theory by simply doppler-shifting plasma:

$$\lambda(\gamma) \rightarrow \lambda(\gamma - \mathrm{i}\, n\, \Omega),$$

where $\Omega$ is plasma toroidal angular velocity.

# Effect of Plasma Rotation on 3/1 Resistive Wall Mode



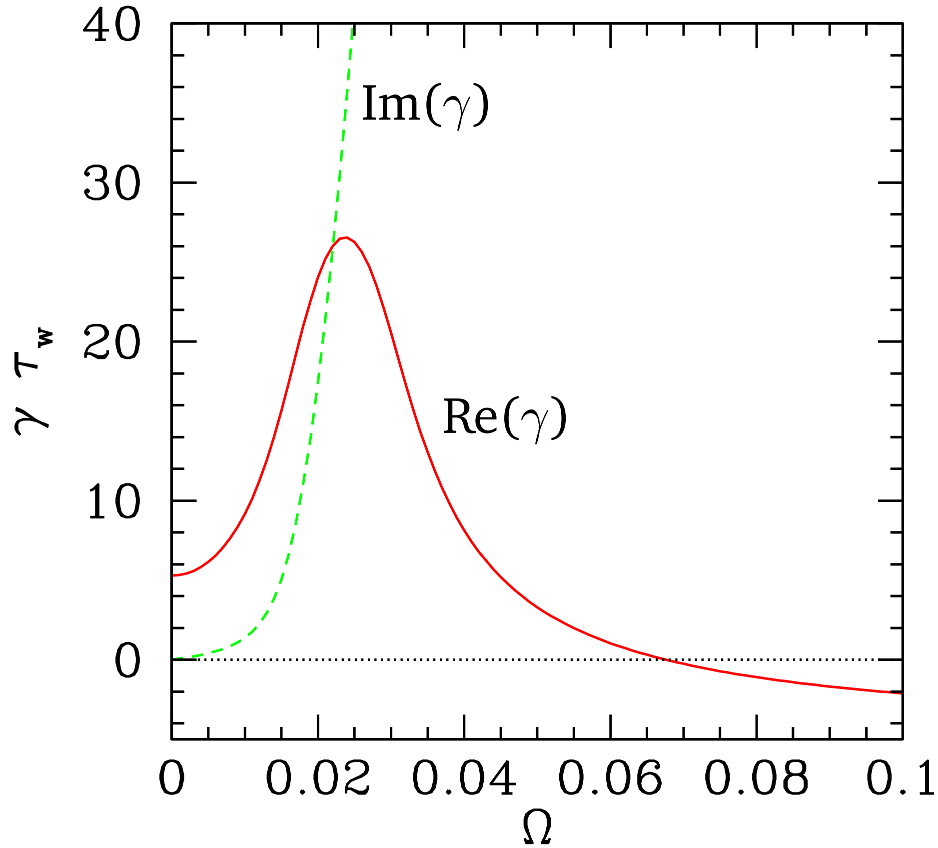

$\alpha = 0.5,\ q_0 = 1.3,\ q_a = 2.95,\ r_w = 1.1,\ \tau_w = 10^4.$

## Plasma Dissipation

- Plasma rotation alone incapable of stabilizing resistive wall mode.
- However, plasma rotation coupled with *plasma dissipation* can stabilize resistive wall mode.[a]

[a] A. Bondeson, and D.J. Ward, Phys. Rev. Lett. **72**, 2709 (1994).

## Effect of Plasma Rotation + Dissipation on 3/1 RWM




$\alpha = 0.5$, $q_0 = 1.3$, $q_a = 2.95$, $r_w = 1.1$, $\tau_w = 10^4$, $\nu = 10^{-3}$.

# Dissipation via Charge Exchange with Cold Neutrals

- All tokamak plasmas have cold neutrals close to edge.
- Hot plasma ion near edge can charge exchange with cold neutral. Neutral gains momentum and is ejected from plasma. Net effect is damping of edge plasma toroidal rotation.
- Damping rate:

$$\nu \sim n_n\, v_i\, \sigma_x,$$

where $n_n$ is number density of neutrals, $v_i$ is thermal velocity of edge ions, and $\sigma_x$ is charge exchange cross-section.

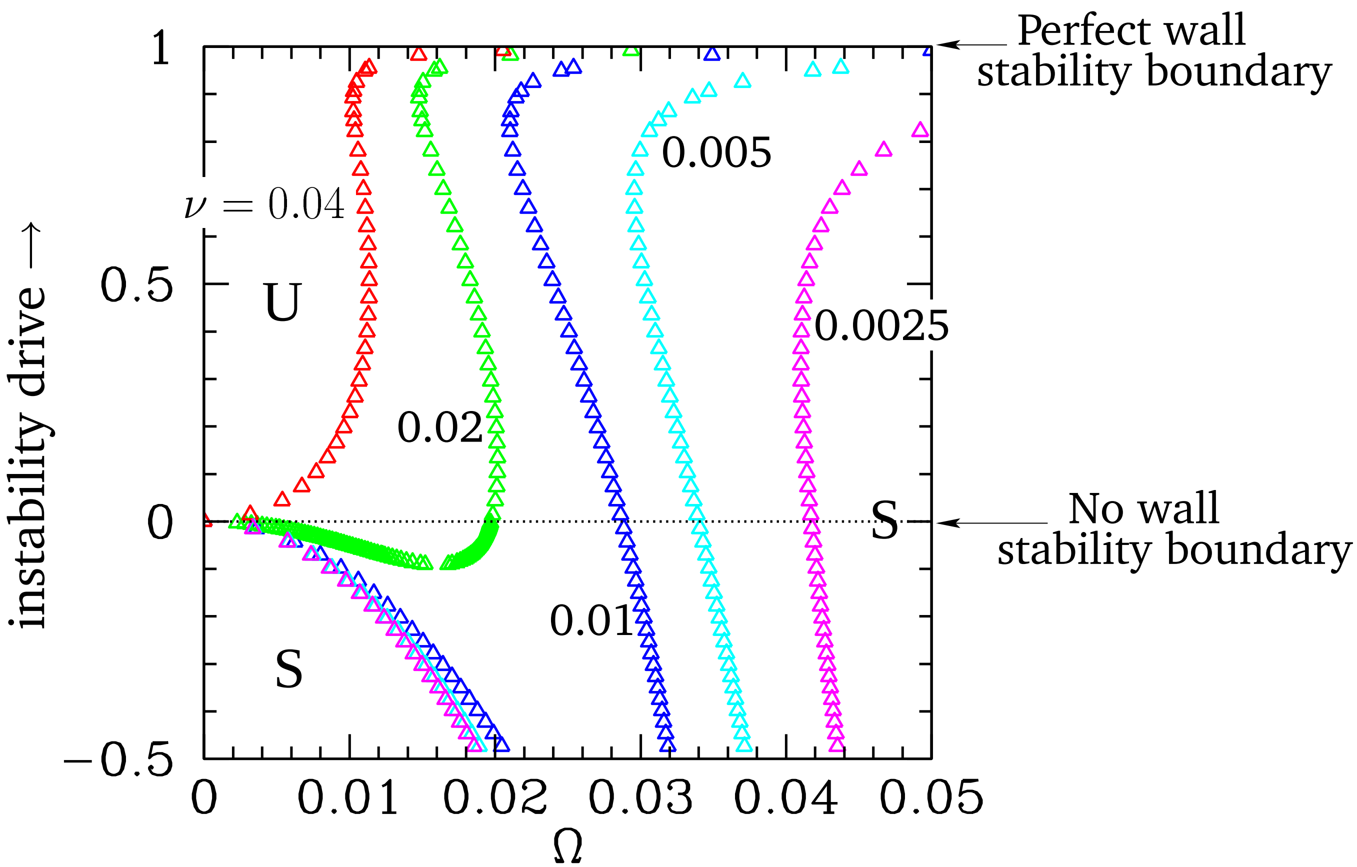
## Modeling Charge Exchange Dissipation

- Charge exchange dissipation can be included in plasma response equation by making substitution:

$$\rho\,\gamma'^{\,2} \rightarrow \rho\,(\gamma'^{\,2} + \nu\,\gamma'),$$

where $\gamma' = \gamma - \mathrm{i}\,n\,\Omega$, and $\nu$ is charge exchange damping rate normalized to $\tau_H$.

# RWM Stability with Charge Exchange Dissipation




$\alpha = 0.5$, $q_a = 2.95$, $r_w = 1.1$, $\tau_w = 10^4$.

## Charge Exchange Dissipation in HBT-EP

- HBT-EP is small tokamak operated by Columbia University. Parameters: $R_0 = 0.92$ m, $a = 0.15$ m, $B_z = 0.35$ T, $n_e(0) \simeq 8 \times 10^{18}\,\mathrm{m}^{-3}$, $n_e(\mathsf{a}) \simeq 2 \times 10^{18}\,\mathrm{m}^{-3}$, $T_i(\mathsf{a}) \simeq 10$ eV, $\tau_w \simeq 1$ ms, $r_w \simeq 1.1\,a$.

- Assuming cold neutral density at edge is $1\%$ of local electron number density, charge-exchange damping rate is $\nu \sim 400\,\mathrm{s}^{-1}$. Hydromagnetic time is $\tau_H \sim 3 \times 10^{-7}$ s.

- Normalized charge exchange damping rate is

$$\nu \sim 10^{-4}$$

  This is order of magnitude *too small* to stabilize resistive wall mode.

## Dissipation via Neoclassical Flow Damping

- Ion neoclassical parallel stress tensor damps poloidal component of perturbed flow associated with resistive wall mode. Also gives rise to perturbed radial current.
- Net effect is enhancement of plasma inertia by factor $(B_z/B_\theta)^2$, plus small dissipation.[a]

[a] K.C. Shiang, Phys. Plasmas **11**, 5525 (2004).

## Modeling Neoclassical Dissipation

- Neoclassical dissipation can be included in plasma response equation by making substitution:[a]

$$\rho\,\gamma'^{\,2} \rightarrow \rho\,\gamma'^{\,2}\left(1 + \frac{q^2}{\epsilon_0^2}\,\frac{\mu}{\gamma' + r^2\,\mu}\right),$$

  where $\gamma' = \gamma - \mathrm{i}\,n\,\Omega$, $\epsilon_0 = a/R_0$, and $\mu$ is edge neoclassical poloidal damping rate normalized to $\tau_H$.

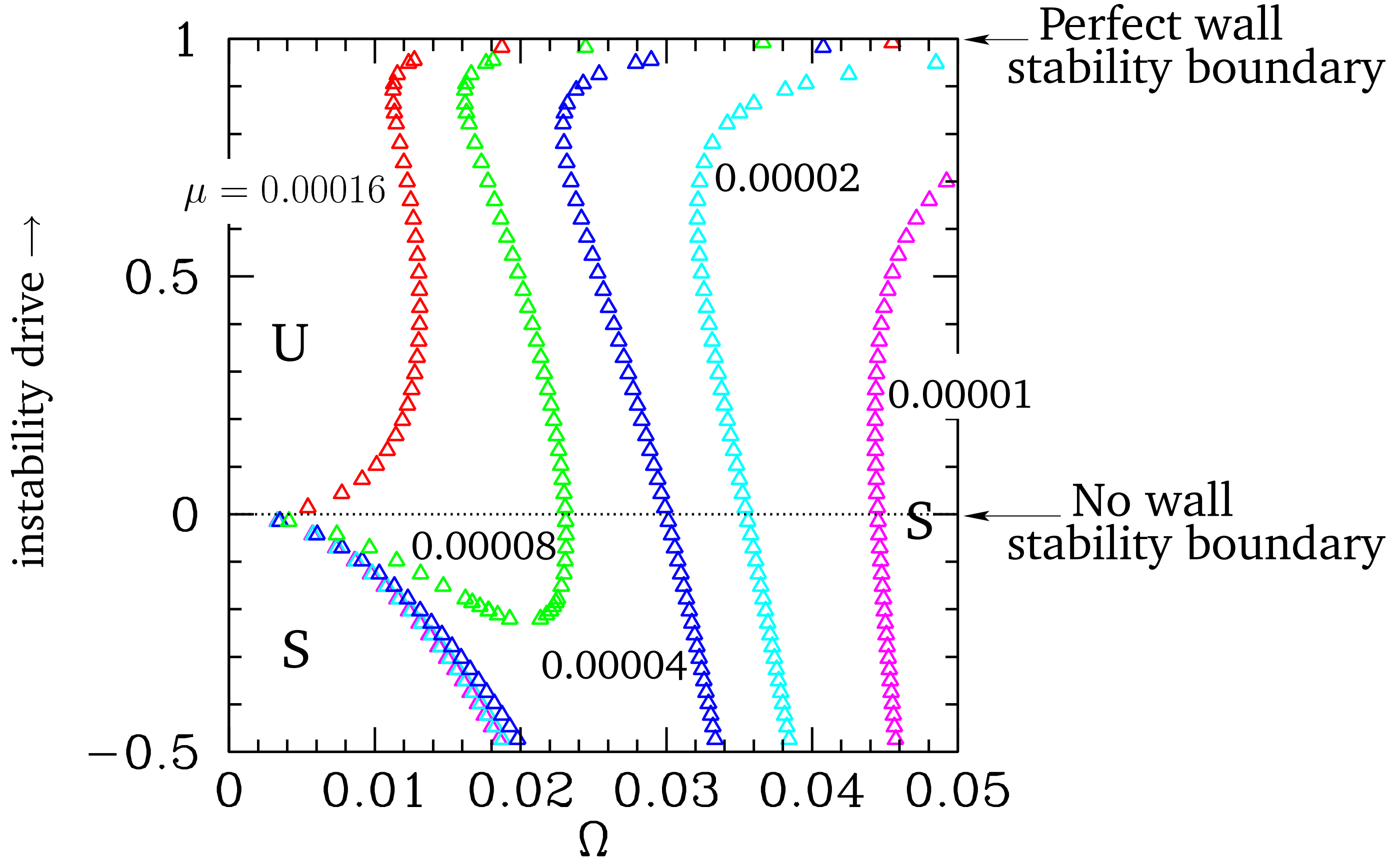
- For collisional edge,

$$\mu \simeq \frac{\epsilon_0^2\,\nu_{tr}^2}{\nu_{ii}},$$

  where $\nu_{tr}$ is ion transit frequency, and $\nu_{ii}$ is ion-ion collision frequency.

[a]K.C. Shiang, Phys. Plasmas **11**, 5525 (2004).

# RWM Stability with Neoclassical Dissipation




$\alpha = 0.5$, $q_a = 2.95$, $r_w = 1.1$, $\tau_w = 10^4$, $\epsilon_0 = 0.2$.

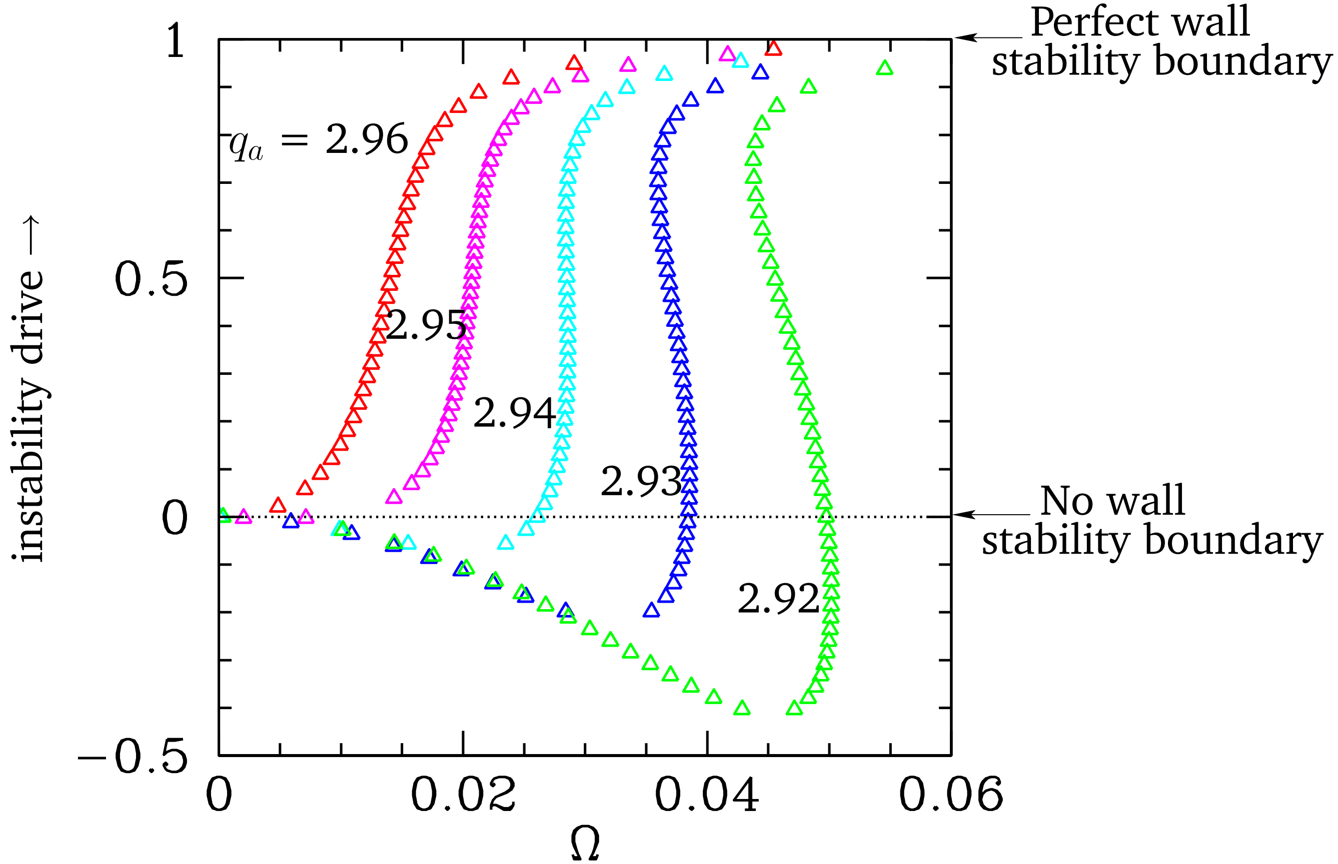
## Neoclassical Dissipation in HBT-EP

- Edge of HBT-EP is in *collisional* regime. Edge neoclassical poloidal damping rate is $\mu \sim 200\,\mathrm{s}^{-1}$.
- Normalized neoclassical damping rate is

$$\mu \sim 8 \times 10^{-5}.$$

This is sufficient to stabilize resistive wall mode in HBT-EP at rotation velocities which are *few percent* of Alfvèn velocity.

## RWM Stability in HBT-EP




$\alpha = 0.5$, $r_w = 1.1$, $\tau_w = 3000$, $\epsilon_0 = 0.16$, $\mu = 8 \times 10^{-5}$.

## Other Dissipation Mechanisms

- Alfvènic damping at toroidally coupled internal rational surface.[a]
- Sound wave damping at toroidally coupled internal rational surface.[b]
- Kinetic damping due to resonance of rwm with bounce/transit/precession frequency of thermal ions inside plasma.[c][d]

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[a] A. Bondeson, and D.J. Ward, Phys. Rev. Lett. **72**, 2709 (1994).
[b] R. Betti, J.P. Freidberg, Phys. Rev. Lett. **74**, 2949 (1995).
[c] A. Bondeson, *et al.*, Plasma Phys. Control. Fusion **45**, A253 (2003).
[d] B. Hu, *et al.*, Phys. Plasmas **12**, 157301 (2005).

## Summary

- Resistive wall mode stabilized by combination of *plasma rotation* and *plasma dissipation*.
- Typical rotation rate required to stabilize mode is

$$\Omega_c \sim k_{\|}\, v_A,$$

  where $k_{\|}$ is parallel wave-vector at *edge* of plasma, and $v_A$ is Alfvèn velocity.

- Dissipation via charge exchange with cold edge neutrals too small to stabilize rwm (in HBT-EP).
- Dissipation via neoclassical poloidal flow damping has about right magnitude to stabilize mode (in HBT-EP).