

External Modes and Resistive Wall Modes

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External Kink Modes

- Principal figure of merit in tokamak plasma is

$$\beta = \frac{\text{plasma energy density}}{\text{magnetic energy density}}.$$

- Main β -limiting instability is *external-kink mode*. This is *ideal* instability driven by radial *pressure and current gradients*.
- Talk will concentrate on current driven instabilities, since these can be described by *cylindrical theory*.
- Hope is that physics of pressure (*i.e.*, β) driven instabilities is analogous to that of current driven instabilities.

Cylindrical Theory

- Treat plasma as periodic cylinder of radius a (minor radius) and periodicity length $2\pi R_0$ (major radius).
- Adopt cylindrical polar coordinates (r, θ, z) .
- Equilibrium magnetic field: $\mathbf{B} = (0, B_\theta(r), B_z)$.
- Normalize all lengths to a , all magnetic fields to B_z , all mass densities to central plasma mass density ρ_0 , all times to hydromagnetic time $\tau_H = R_0 \sqrt{\mu_0 \rho_0} / B_z$.

Plasma Equilibrium

- Density profile:

$$\rho = (1 - r^2)^\alpha.$$

- Safety-factor ($q = r B_z / R_0 B_\theta$) profile:^a

$$q(r) = \frac{q_a}{1 - (1 - r^2)^{q_a/q_0}},$$

where q_0 is central- q and q_a is edge- q .

- Toroidal plasma current:

$$J_z = J_0 (1 - r^2)^{q_a/q_0 - 1}.$$

^aJ.A. Wesson, Nucl. Fusion **18**, 87 (1978).

Ideal Magnetohydrodynamics

- Consider external-kink mode with m periods in poloidal direction and n periods in toroidal direction.
- According to ideal-MHD, radial displacement in plasma ξ satisfies

$$\frac{d}{dr} \left[r (\rho \gamma^2 + Q^2) \frac{d(r \xi)}{dr} \right] - \left[m^2 (\rho \gamma^2 + Q^2) + r \frac{dQ^2}{dr} \right] \xi = 0.$$

- Here, γ is growth-rate (in plasma frame), and

$$Q = \frac{m}{q} - n.$$

- No resonant surface (where $Q = 0$) in plasma for external-kink mode.

Edge Boundary Condition

- Let

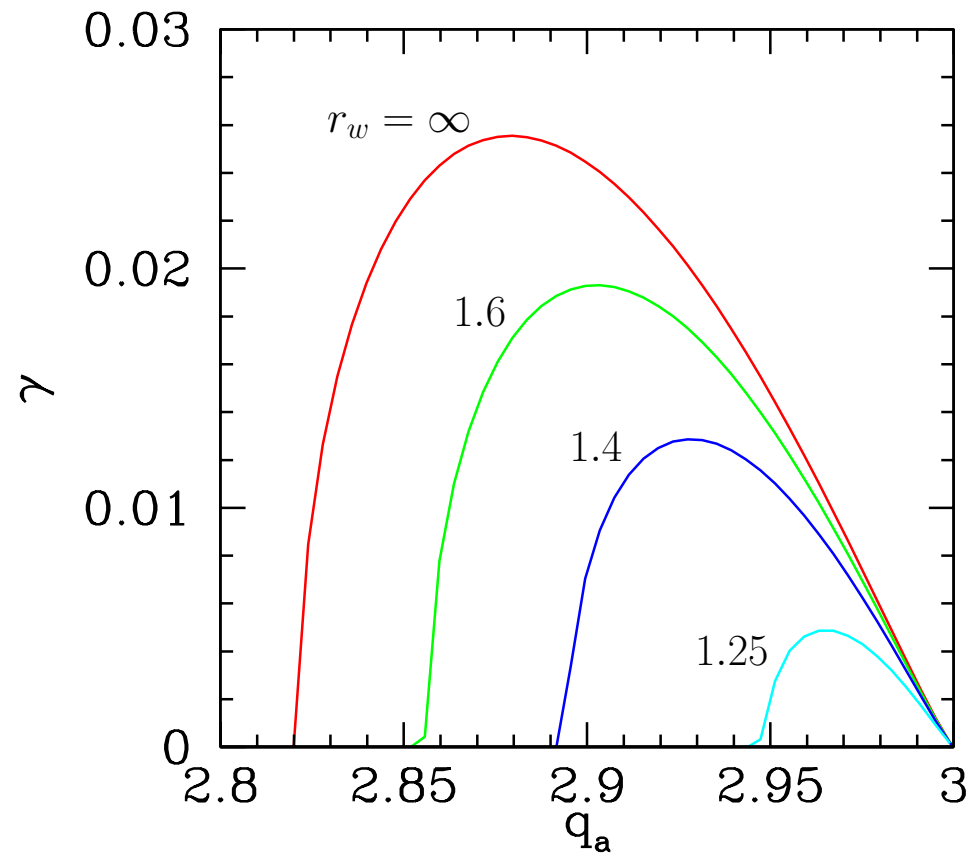
$$\lambda(\gamma) = - \left(\frac{d[\ln(r Q \xi)]}{m dr} \right)_{r=1}.$$

- If plasma surrounded by thin resistive wall of radius r_w and time-constant τ_w then

$$\lambda(\gamma) = \frac{1 + (\gamma \tau_w / 2 m) (1 + r_w^{-2m})}{1 + (\gamma \tau_w / 2 m) (1 - r_w^{-2m})}.$$

- Above dispersion relation can be solved (e.g., via Newton iteration) to give growth-rate γ of external-kink mode.

Effect of Ideal Wall on 3/1 External Kink Mode

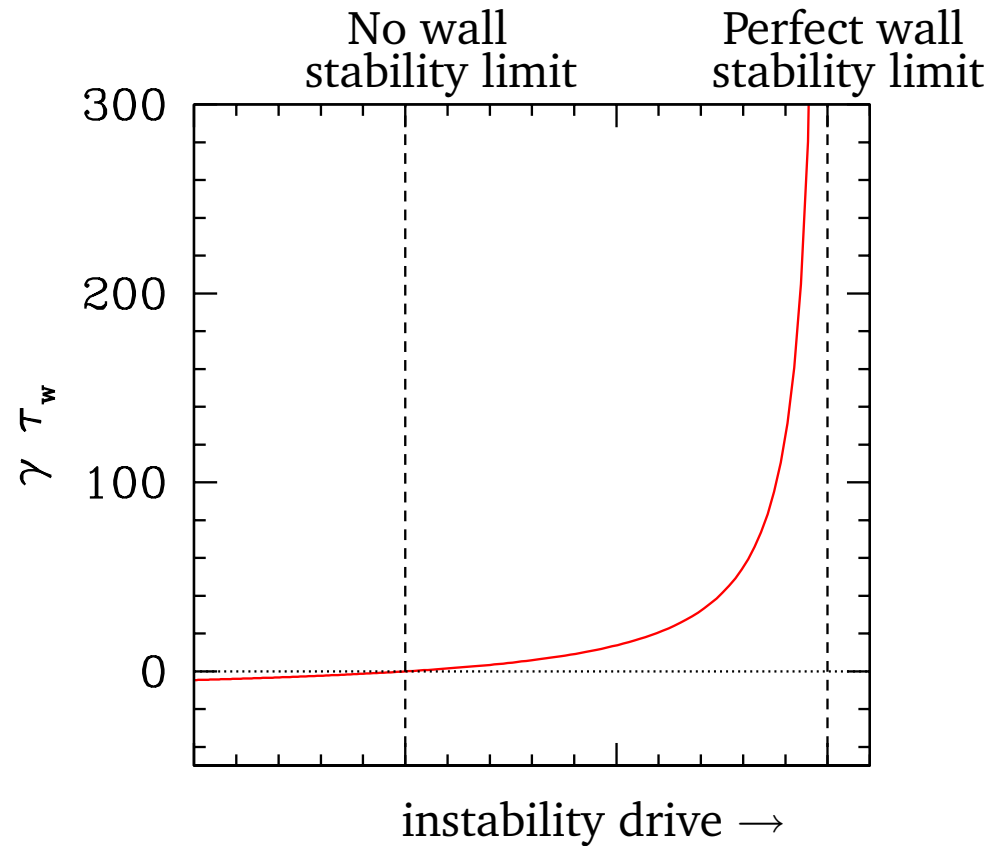


$$\alpha = 0.5, q_0 = 1.3$$

Resistive Wall Mode

- External kink mode stabilized by close-fitting perfectly conducting wall (*i.e.*, $\tau_w \rightarrow \infty$).
- Suggests that close-fitting conducting wall (*e.g.*, vacuum vessel) might increase β -limit.
- Unfortunately, conducting walls possess finite *resistivity*. Resistive wall has finite time-constant τ_w which is generally much greater than τ_H but still much less than pulse length of discharge.
- Resistive wall *does not* stabilize external kink mode. Instead, converts mode into slowly growing ($\gamma \sim \tau_w^{-1}$) *resistive wall mode*.

Effect of Resistive Wall on 3/1 External Kink Mode



$$\alpha = 0.5, q_a = 2.95, r_w = 1.1, \tau_w = 10^6.$$

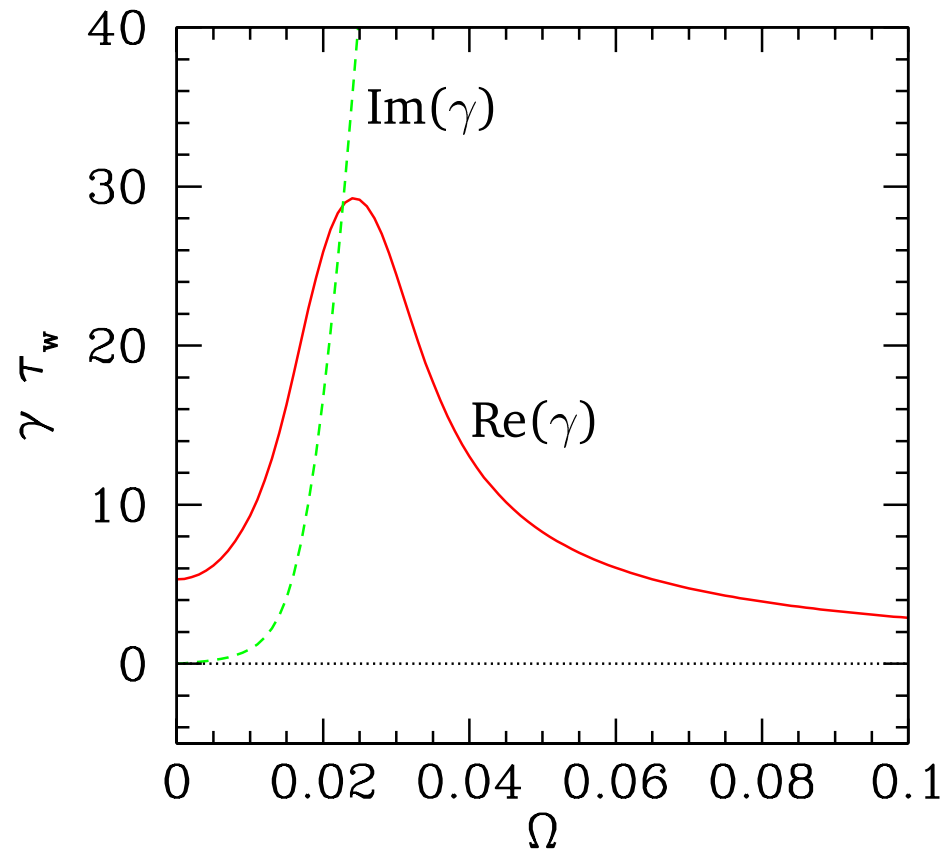
Plasma Rotation

- Tokamak plasmas rotate toroidally at few percent of Alfvén speed.
- Plasma rotation either intrinsic (not well understood) or due to unbalanced neutral beam injection.
- Since resistive wall mode is plasma instability, would expect plasma rotation to give mode *real frequency*.
- If real frequency exceeds τ_w^{-1} then resistive wall effectively acts as ideal wall. Rotation could stabilize resistive wall mode.
- Rotation introduced into theory by simply doppler-shifting plasma:

$$\lambda(\gamma) \rightarrow \lambda(\gamma - i n \Omega),$$

where Ω is plasma toroidal angular velocity.

Effect of Plasma Rotation on 3/1 Resistive Wall Mode



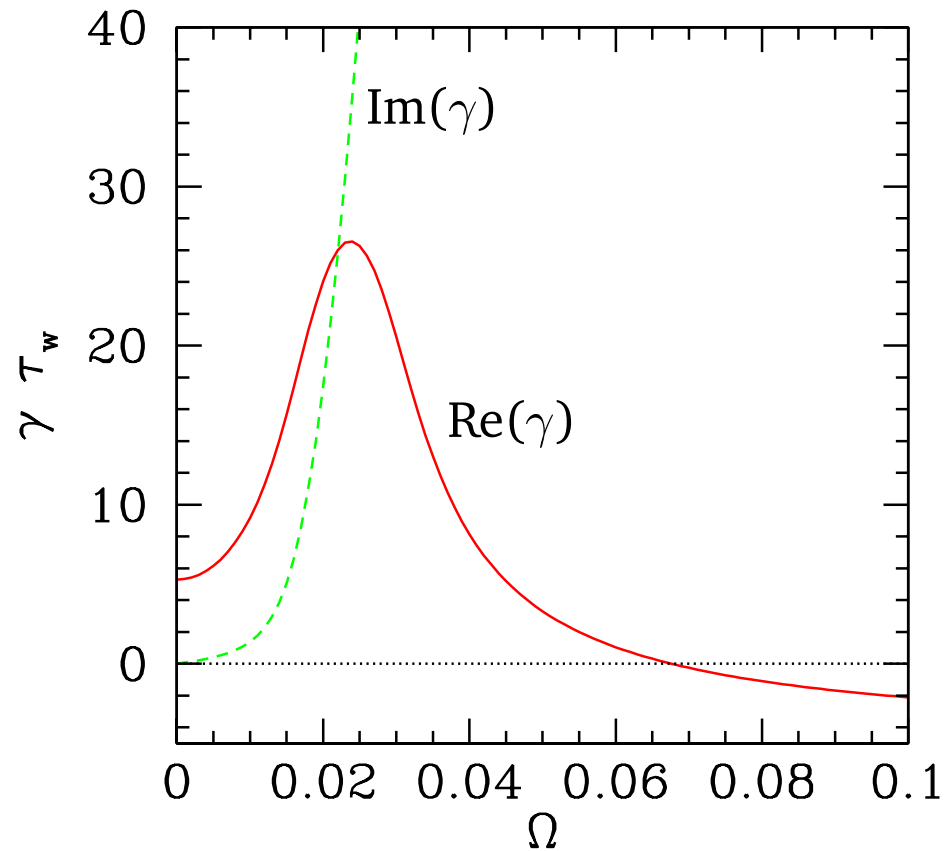
$$\alpha = 0.5, q_0 = 1.3, q_a = 2.95, r_w = 1.1, \tau_w = 10^4.$$

Plasma Dissipation

- Plasma rotation alone incapable of stabilizing resistive wall mode.
- However, plasma rotation coupled with *plasma dissipation* can stabilize resistive wall mode.^a

^aA. Bondeson, and D.J. Ward, Phys. Rev. Lett. **72**, 2709 (1994).

Effect of Plasma Rotation + Dissipation on 3/1 RWM



$$\alpha = 0.5, q_0 = 1.3, q_a = 2.95, r_w = 1.1, \tau_w = 10^4, \nu = 10^{-3}.$$

Dissipation via Charge Exchange with Cold Neutrals

- All tokamak plasmas have cold neutrals close to edge.
- Hot plasma ion near edge can charge exchange with cold neutral. Neutral gains momentum and is ejected from plasma. Net effect is damping of edge plasma toroidal rotation.
- Damping rate:

$$\nu \sim n_n v_i \sigma_x,$$

where n_n is number density of neutrals, v_i is thermal velocity of edge ions, and σ_x is charge exchange cross-section.

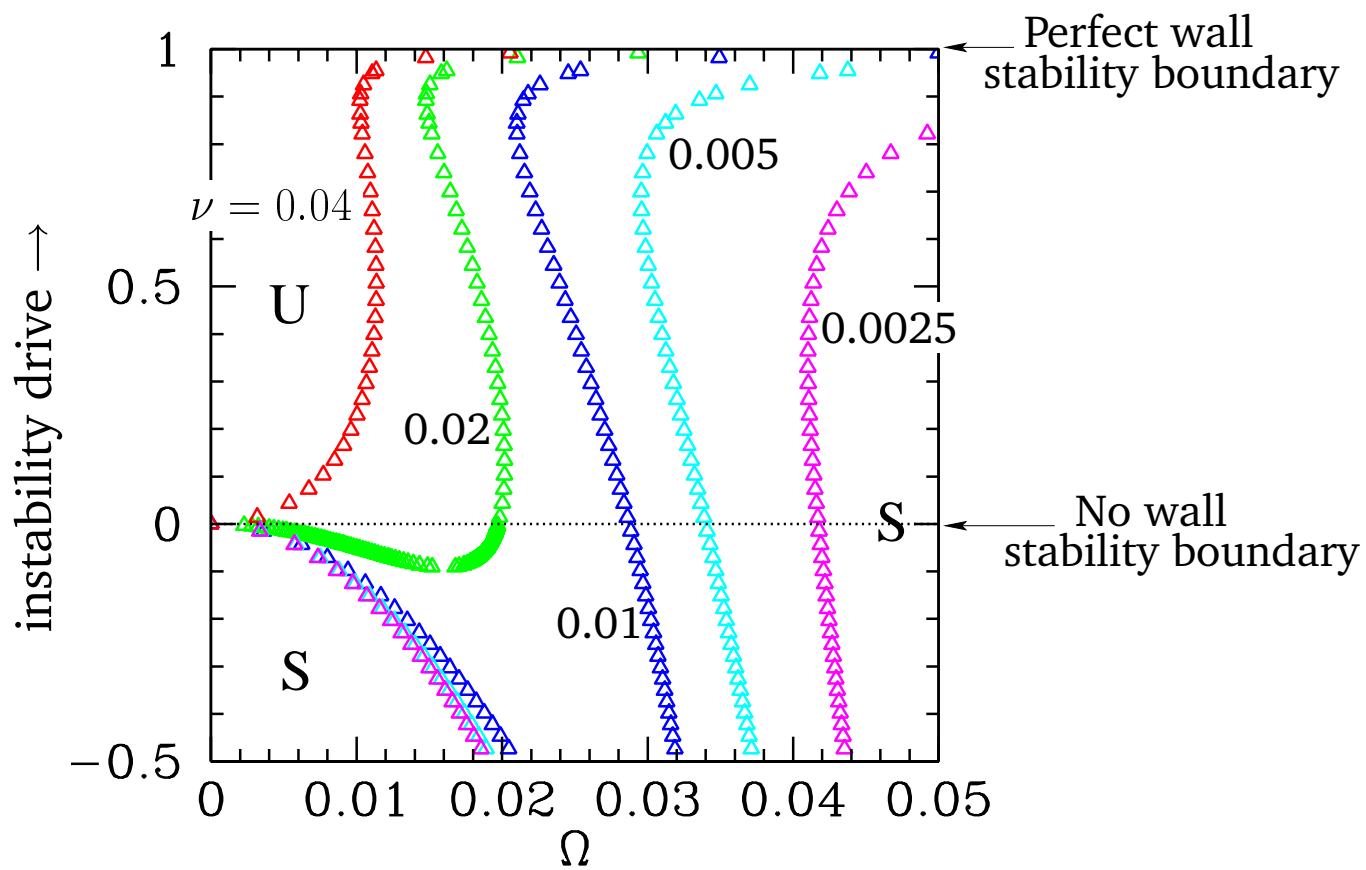
Modeling Charge Exchange Dissipation

- Charge exchange dissipation can be included in plasma response equation by making substitution:

$$\rho \gamma'^2 \rightarrow \rho (\gamma'^2 + \nu \gamma'),$$

where $\gamma' = \gamma - i n \Omega$, and ν is charge exchange damping rate normalized to τ_H .

RWM Stability with Charge Exchange Dissipation



$$\alpha = 0.5, q_a = 2.95, r_w = 1.1, \tau_w = 10^4.$$

Charge Exchange Dissipation in HBT-EP

- HBT-EP is small tokamak operated by Columbia University. Parameters: $R_0 = 0.92$ m, $a = 0.15$ m, $B_z = 0.35$ T, $n_e(0) \simeq 8 \times 10^{18} \text{ m}^{-3}$, $n_e(a) \simeq 2 \times 10^{18} \text{ m}^{-3}$, $T_i(a) \simeq 10$ eV, $\tau_w \simeq 1$ ms, $r_w \simeq 1.1 a$.
- Assuming cold neutral density at edge is 1% of local electron number density, charge-exchange damping rate is $\nu \sim 400 \text{ s}^{-1}$. Hydromagnetic time is $\tau_H \sim 3 \times 10^{-7}$ s.
- Normalized charge exchange damping rate is

$$\nu \sim 10^{-4}$$

This is order of magnitude *too small* to stabilize resistive wall mode.

Dissipation via Neoclassical Flow Damping

- Ion neoclassical parallel stress tensor damps poloidal component of perturbed flow associated with resistive wall mode. Also gives rise to perturbed radial current.
- Net effect is enhancement of plasma inertia by factor $(B_z/B_\theta)^2$, plus small dissipation.^a

^aK.C. Shiang, Phys. Plasmas **11**, 5525 (2004).

Modeling Neoclassical Dissipation

- Neoclassical dissipation can be included in plasma response equation by making substitution: ^a

$$\rho \gamma'^2 \rightarrow \rho \gamma'^2 \left(1 + \frac{q^2}{\epsilon_0^2} \frac{\mu}{\gamma' + r^2 \mu} \right),$$

where $\gamma' = \gamma - i n \Omega$, $\epsilon_0 = a/R_0$, and μ is edge neoclassical poloidal damping rate normalized to τ_H .

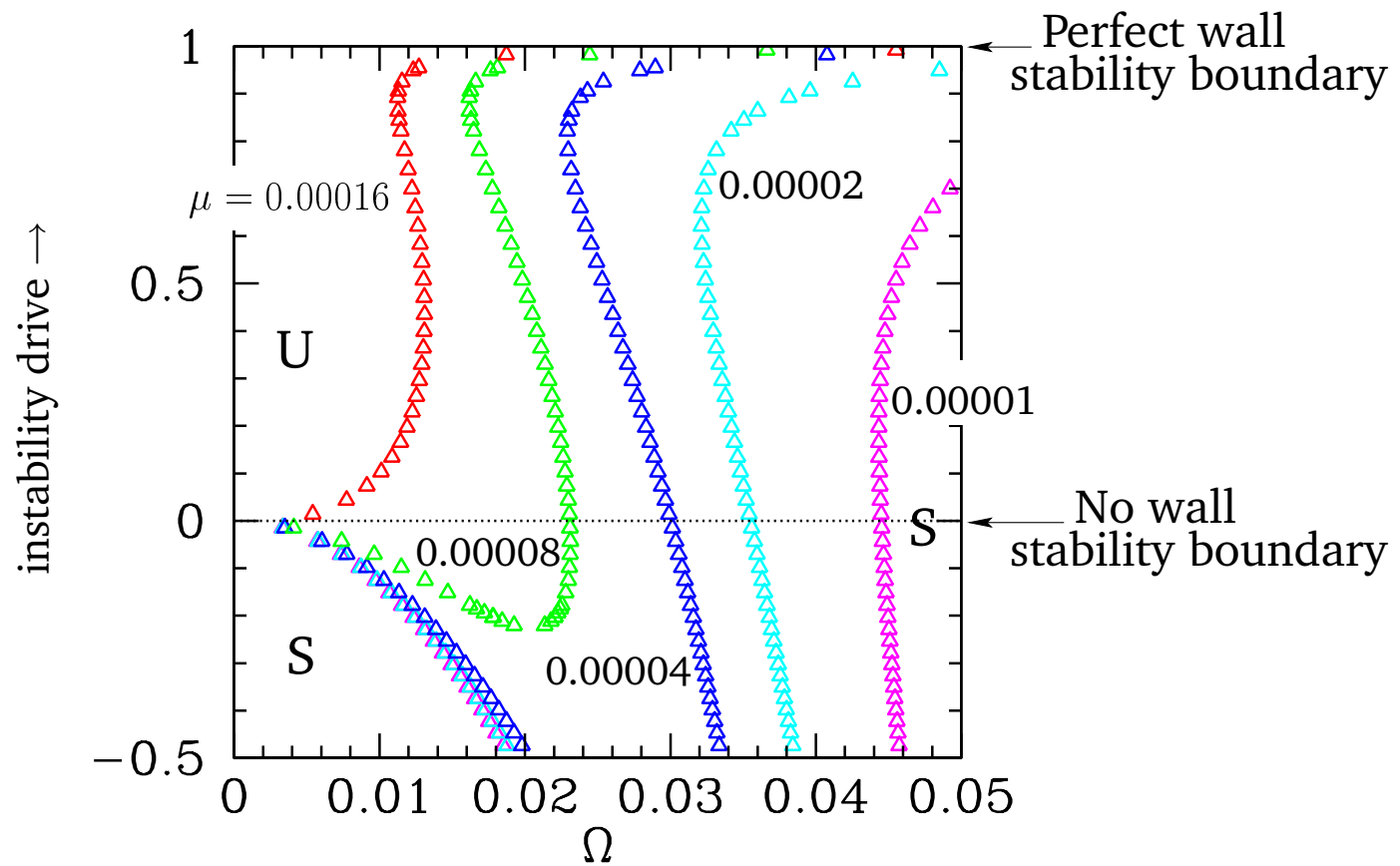
- For collisional edge,

$$\mu \simeq \frac{\epsilon_0^2 \nu_{tr}^2}{\nu_{ii}},$$

where ν_{tr} is ion transit frequency, and ν_{ii} is ion-ion collision frequency.

^aK.C. Shiang, Phys. Plasmas **11**, 5525 (2004).

RWM Stability with Neoclassical Dissipation



$$\alpha = 0.5, q_a = 2.95, r_w = 1.1, \tau_w = 10^4, \epsilon_0 = 0.2.$$

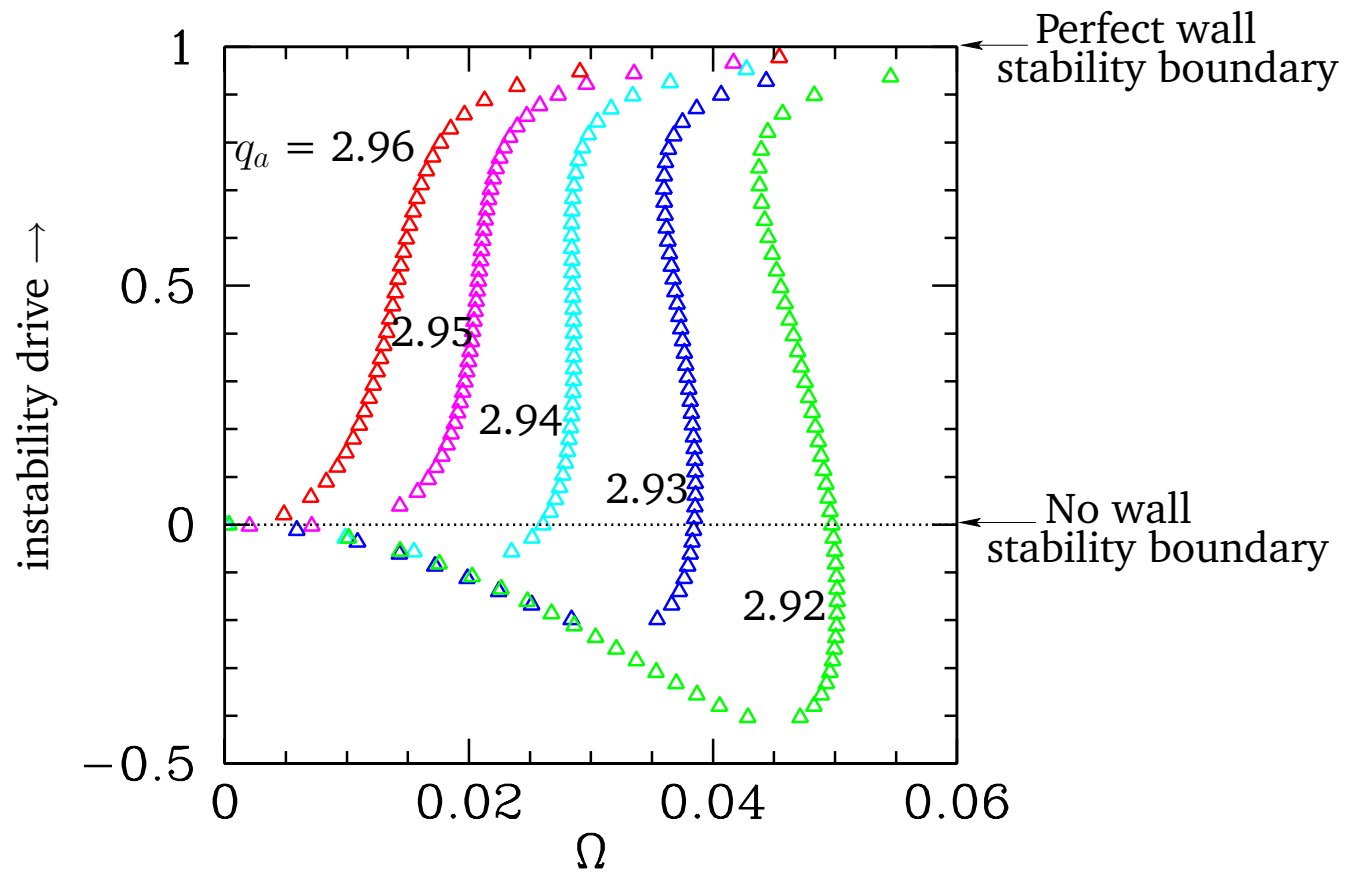
Neoclassical Dissipation in HBT-EP

- Edge of HBT-EP is in *collisional* regime. Edge neoclassical poloidal damping rate is $\mu \sim 200 \text{ s}^{-1}$.
- Normalized neoclassical damping rate is

$$\mu \sim 8 \times 10^{-5}.$$

This is sufficient to stabilize resistive wall mode in HBT-EP at rotation velocities which are *few percent* of Alfvén velocity.

RWM Stability in HBT-EP



$$\alpha = 0.5, r_w = 1.1, \tau_w = 3000, \epsilon_0 = 0.16, \mu = 8 \times 10^{-5}.$$

Other Dissipation Mechanisms

- Alfvénic damping at toroidally coupled internal rational surface.^a
- Sound wave damping at toroidally coupled internal rational surface.^b
- Kinetic damping due to resonance of rwm with bounce/transit/precession frequency of thermal ions inside plasma.^{c d}

^aA. Bondeson, and D.J. Ward, Phys. Rev. Lett. **72**, 2709 (1994).

^bR. Betti, J.P. Freidberg, Phys. Rev. Lett. **74**, 2949 (1995).

^cA. Bondeson, *et al.*, Plasma Phys. Control. Fusion **45**, A253 (2003).

^dB. Hu, *et al.*, Phys. Plasmas **12**, 157301 (2005).

Summary

- Resistive wall mode stabilized by combination of *plasma rotation* and *plasma dissipation*.
- Typical rotation rate required to stabilize mode is

$$\Omega_c \sim k_{\parallel} v_A,$$

where k_{\parallel} is parallel wave-vector at *edge* of plasma, and v_A is Alfvén velocity.

- Dissipation via charge exchange with cold edge neutrals too small to stabilize rwm (in HBT-EP).
- Dissipation via neoclassical poloidal flow damping has about right magnitude to stabilize mode (in HBT-EP).