Determination of Non-Ideal Response of a High Temperature Plasma to a Static External Magnetic Perturbation via Asymptotic Matching

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Error Fields

- Tokamak plasmas highly sensitive to externally generated, static, helical magnetic perturbations—a.k.a. "error fields".
- Error fields drive magnetic reconnection in otherwise tearing stable plasmas, giving rise to formation of non-rotating magnetic island chains on rational magnetic surfaces—a.k.a. "locked modes".
- Locked modes severely degrade plasma energy confinement, and often trigger disruptions.

Plasma Response to Error Fields

- Response very different to that predicted by naively superimposing vacuum perturbation onto equilibrium magnetic field.
- *Shielding currents* excited at rational surfaces by plasma rotation, and/or combination of pressure gradients and favorable average field-line curvature, act to suppress driven magnetic reconnection.
- *Distributed currents* generated by ideal response of realistic (i.e., highly elongated, moderate aspect-ratio, high-β) plasma equilibrium to perturbation profoundly modify perturbation structure. Consequently, cylindrical models, or models that rely on large aspect-ratio, low-β orderings, do very poor job of predicting plasma response.

Approaches to Error Field Response Calculation

1. Solve resistive-MHD equations throughout whole plasma. Highly inefficient because resistivity and inertia only important close to rational surfaces. Plasma response elsewhere governed by much simpler equations of ideal-MHD.

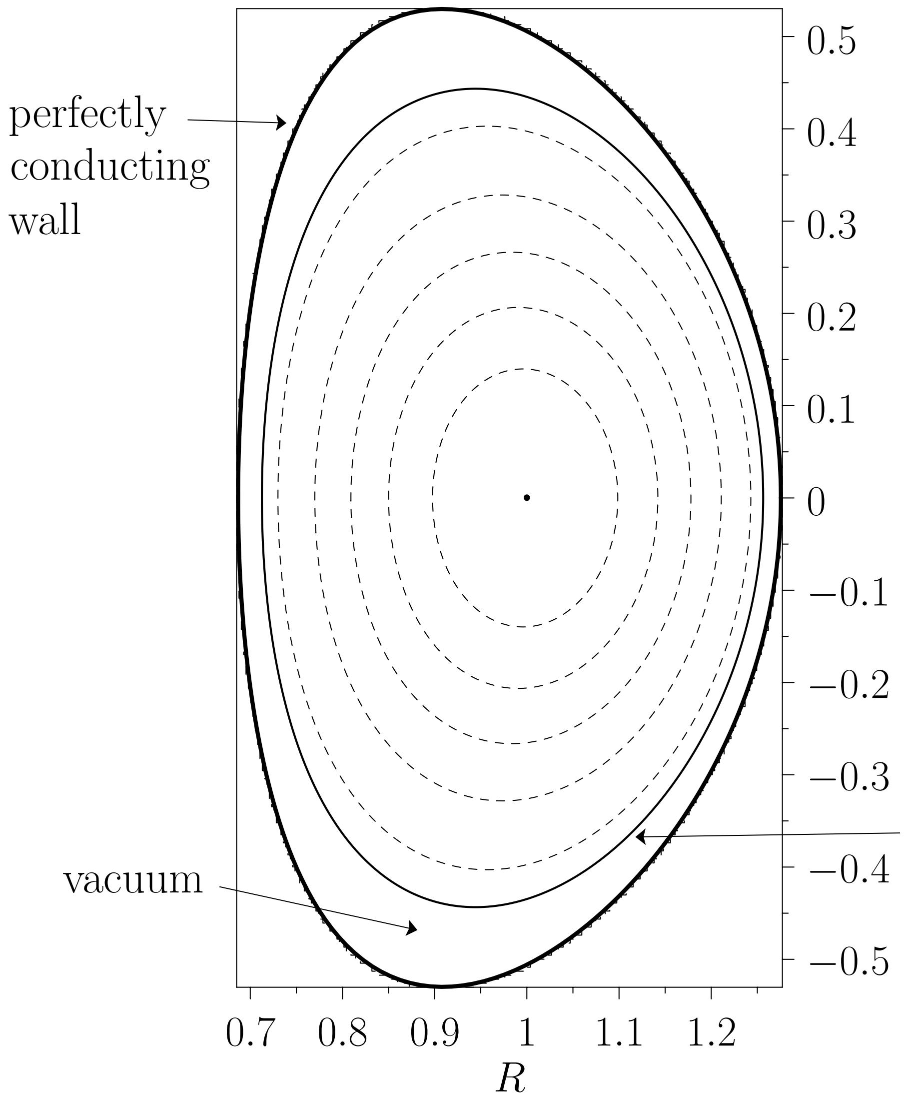
2. Solve ideal-MHD equations throughout whole plasma, placing ideal current sheets at rational surfaces. Facilitates very rapid calculations. Correctly calculates distributed currents. Neglects non-ideal response of plasma at rational surfaces.

3. Solve ideal-MHD equations throughout bulk of plasma, and asymptotically match solutions to non-ideal layer solutions at rational surfaces. More efficient than approach 1. More accurate than approach 2.

My Approach to Error Field Response Calculation

- Use recently developed TOMUHAWC code to calculate ideal-MHD response of realistic plasma equilibrium everywhere apart from immediate vicinity of rational surfaces.
- Asymptotically match ideal-MHD data obtained from TOMUHAWC to Glasser-Greene-Johnson linear layers at various rational surfaces. (GGJ linear layer response theory is simplest non-ideal response model that is consistent with realistic plasma equilibrium.)

Plasma Equilibrium



$\epsilon = 0.3$

$\kappa = 1.8$

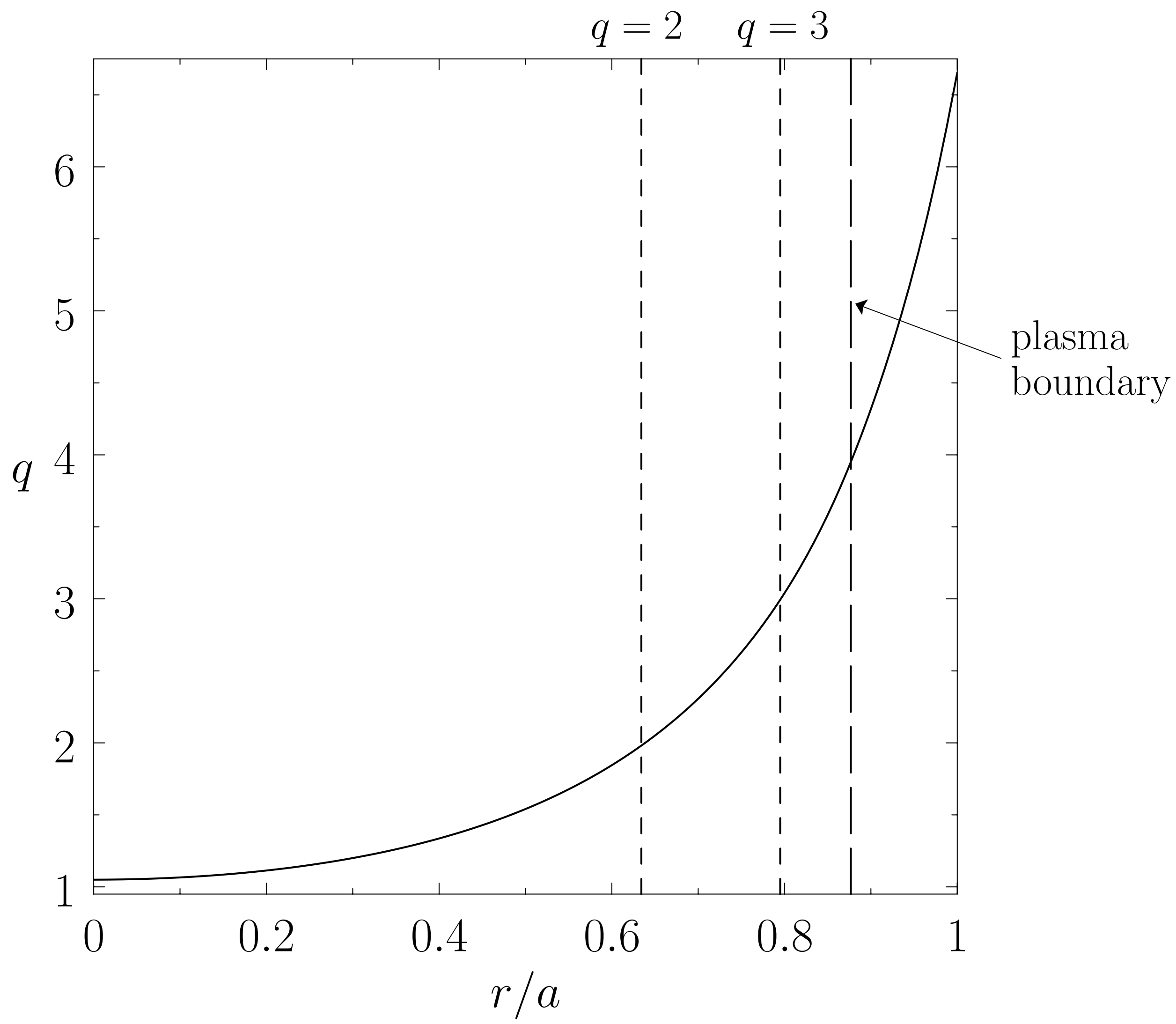
$\delta = 0.25$

$q_0 = 1.05$

$q_a = 3.95$

$\beta_N = 0.0\text{–}2.95$

Safety Factor Profile



Coordinate System

- Adopt right-handed flux coordinate system: r, θ, ϕ.
  - r - flux-surface label
  - θ - "straight" poloidal angle
  - ϕ - geometric toroidal angle
- Jacobian:

$$(\nabla r \times \nabla \theta \cdot \nabla \phi)^{-1} = r\, R^2,$$

where R is major radius.

- Flux-surface average operator:

$$\langle \cdots \rangle \equiv \frac{1}{2\pi} \oint (\cdots)\, d\theta.$$

Perturbed Magnetic Field

- Let

$$\delta\mathbf{B}\cdot\nabla r = i\sum_j \frac{\psi_j(r)}{r\,R^2}\,\exp[i\,(m_j\,\theta - n\,\phi)].$$

- In immediate vicinity of kth rational surface $[q(r_k) = m_k/n]$

$$\begin{aligned}
\psi_k(r) &= \Psi_k\,F_k\,|x_k|^{\nu_{L\,k}} + \Delta\Psi_k^{\pm}\,F_k\,\mathrm{sgn}(x_k)\,|x_k|^{\nu_{S\,k}} + A_k\,x_k,\\
x_k &= (r - r_k)/r_k,\\
F_k &= \left(\frac{m_k^2\,\langle|\nabla r|^2\rangle + n^2\,r^2}{2\sqrt{D_{I\,k}}}\right)_{r_k}^{1/2},
\end{aligned}$$

$\nu_{L\,k} = 1/2 - \sqrt{D_{I\,k}}$, $\nu_{S\,k} = 1/2 + \sqrt{D_{I\,k}}$, and $D_{I\,k}$ is GGJ ideal stability index at kth rational surface.

Asymptotic Matching Parameters

- $\Psi_{\rm k}$ - measures tearing parity reconnected magnetic flux at kth rational surface.
- $\Delta\Psi_{\rm k} \equiv \Delta\Psi_{\rm k}^{+} + \Delta\Psi_{\rm k}^{-}$ - measures tearing parity shielding currents excited at kth rational surface.
- $\Delta_{\rm k} \equiv \Delta\Psi_{\rm k}/\Psi_{\rm k}$ - determined by non-ideal tearing parity layer solution at kth rational surface.

Homogeneous Tearing Parity Dispersion Relation

- Dispersion relation:

$$\sum_{k'} (E_{kk'} - \delta_{kk'}\,\Delta_{k'})\,\Psi_{k'} = 0.$$

- TOMUHAWC code calculates elements of hermitian $\mathbf{E}$-matrix, $E_{kk'}$. Diagonal elements of matrix are toroidal generalizations of Δ' parameter of Furth-Killeen-Rosenbluth theory.

- Toroidal electromagnetic torque at kth surface:

$$\delta T_k = 2\,n\,\pi^2\,\mathrm{Im}(\Delta_k)\,|\Psi_k|^2.$$

Fact that $\mathbf{E}$-matrix is hermitian ensures that zero net torque exerted on plasma.

Error Field Generation

- Perfectly conducting wall subject to small amplitude, static, helical displacement:

$$\boldsymbol{\xi} = \delta r \, \frac{|\nabla r|}{|\nabla r|^{\,2}},$$

where

$$\delta r(\theta, \phi) = \sum_j \Xi_j \, \exp[\mathrm{i}\,(m_j\,\theta - n\,\phi)].$$

Inhomogeneous Tearing Parity Dispersion Relation

- Dispersion relation:

$$\sum_{k'} (E_{kk'} - \delta_{kk'}\,\Delta_{k'})\,\Psi_{k'} = \chi_k,$$

where

$$\chi_k = \sum_j \Xi_j\,\xi_{k,j}.$$

- TOMUHAWC code calculates $\xi_{k,j}$ parameters from homogeneous tearing eigenfunctions.

Shielding Factor

- Assume that kth rational surface locked to error field, whereas plasma at other rational surfaces rotating sufficiently rapidly to suppress driven reconnection.
- Reconnected flux driven at kth surface:

$$\Psi_k = \frac{\chi_k}{E_{kk}}\,\mathcal{S}_k,$$

where

$$\mathcal{S}_k = \frac{E_{kk}}{E_{kk} - \Delta_k}$$

is so-called *shielding factor*—i.e., factor by which driven reconnection at kth rational surface suppressed by shielding currents.

Optimum Wall Displacement

- Optimum wall displacement for driving reconnected flux at kth rational surface is

$$\Xi_j = \delta\,\hat{\xi}_{k,j},$$

where

$$\hat{\xi}_{k,j} = \frac{\xi_{k,j}}{||\xi_k||},$$

and

$$||\xi_k|| = \sum_j |\xi_{k\,j}|^2.$$

Here, δ is mean wall displacement (in r).

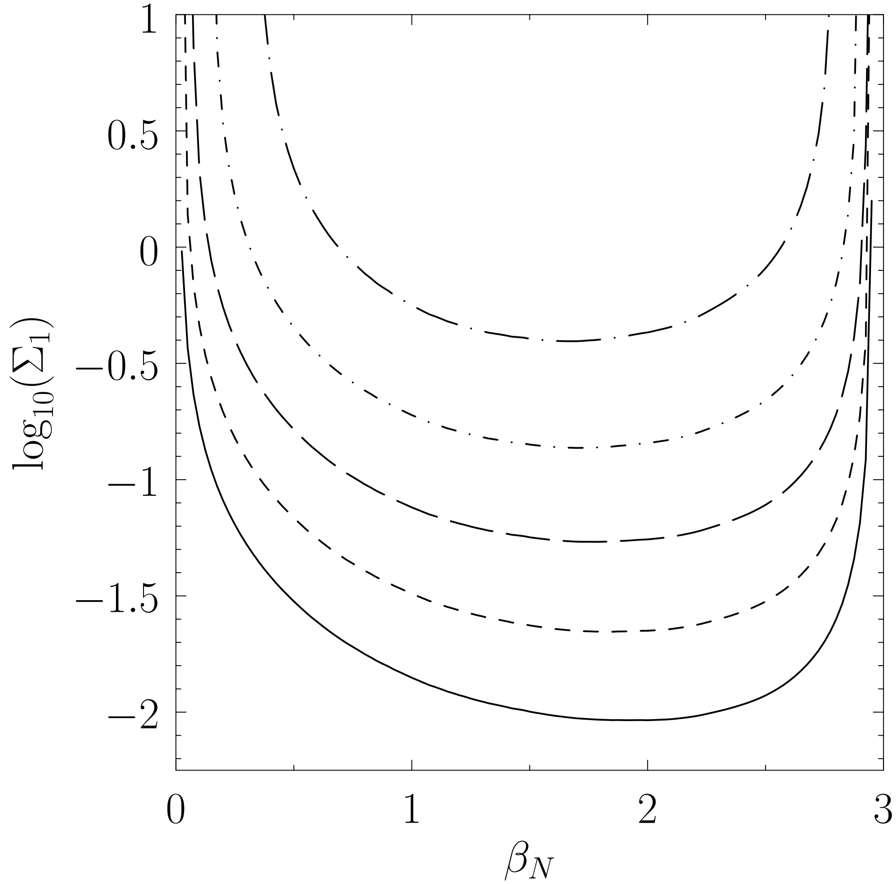
Overlap Factor

- Overlap factor

$$\alpha_{12} = \left| \sum_j \hat{\xi}_{1\,j}\, \hat{\xi}^{*}_{2\,j} \right|$$

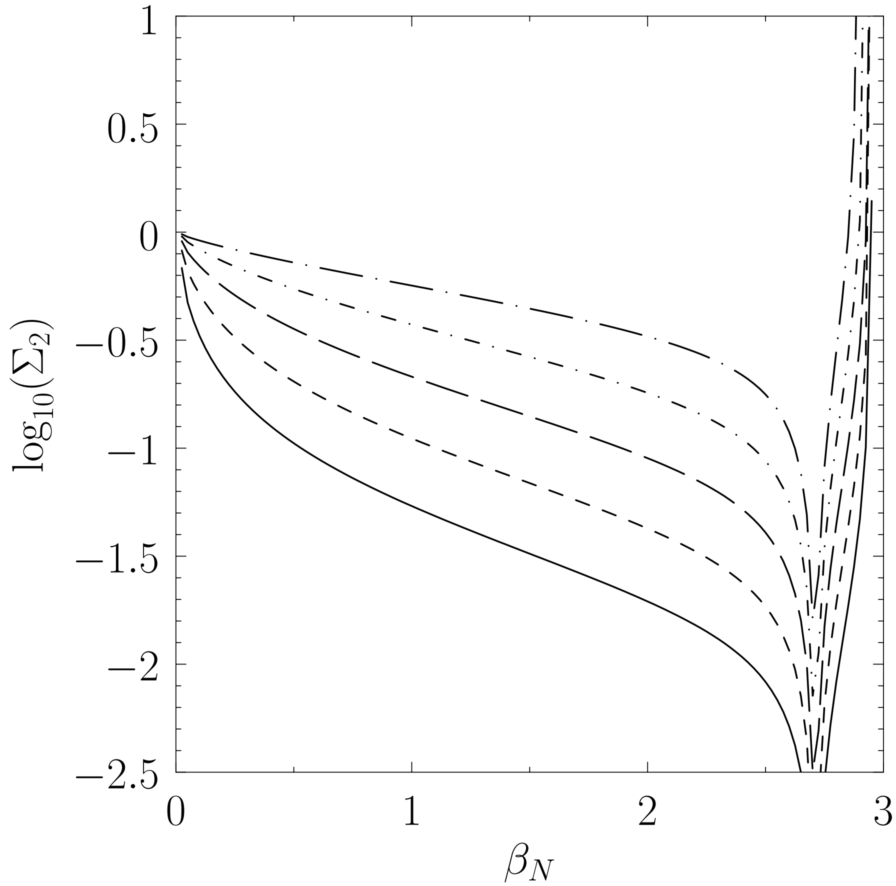
measures extent to which optimum wall displacement for driving reconnected flux at rational surface 1 is similar to optimal wall displacement for driving reconnected flux at rational surface 2. If $\alpha_{12} = 1$ then wall displacements are identical. If $\alpha_{12} = 0$ then wall displacements are independent.

Shielding Factor at $q = 2$ surface



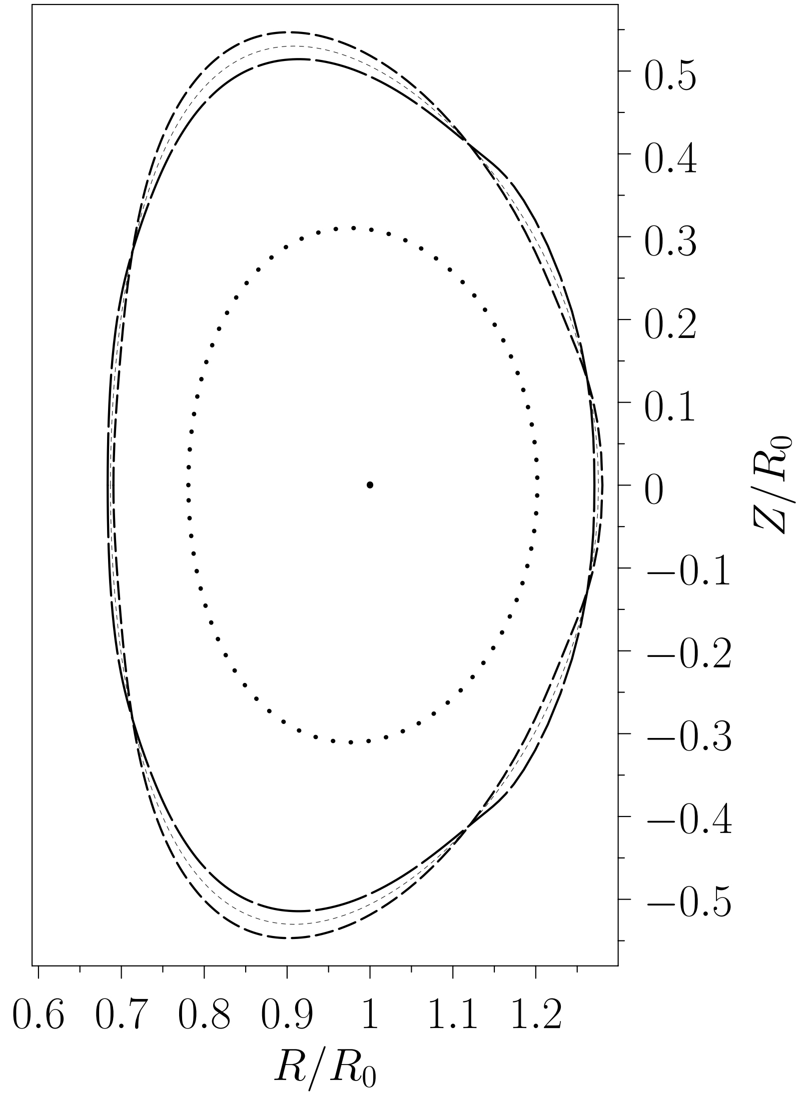
Top to bottom: $S_1 = 10^6$, $S_1 = 10^7$, $S_1 = 10^8$, $S_1 = 10^9$, $S_1 = 10^{10}$.

Shielding Factor at $q = 3$ surface

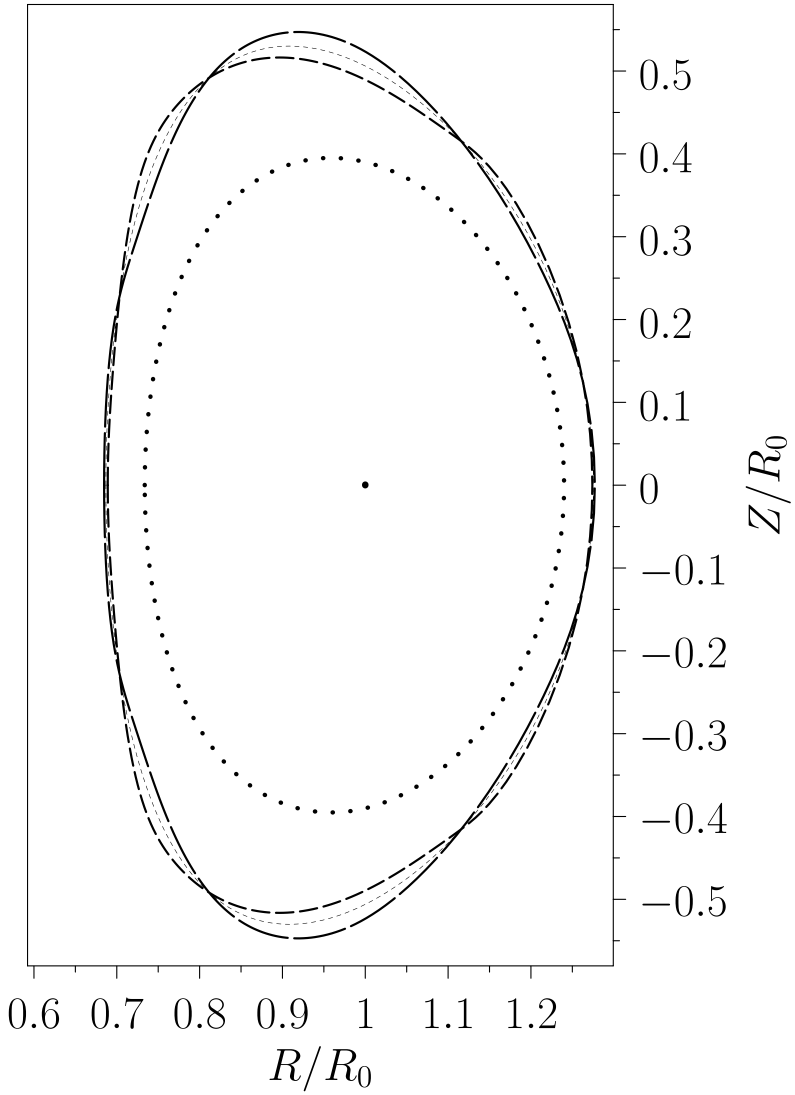


Top to bottom: $S_2 = 10^6$, $S_2 = 10^7$, $S_2 = 10^8$, $S_2 = 10^9$, $S_2 = 10^{10}$.

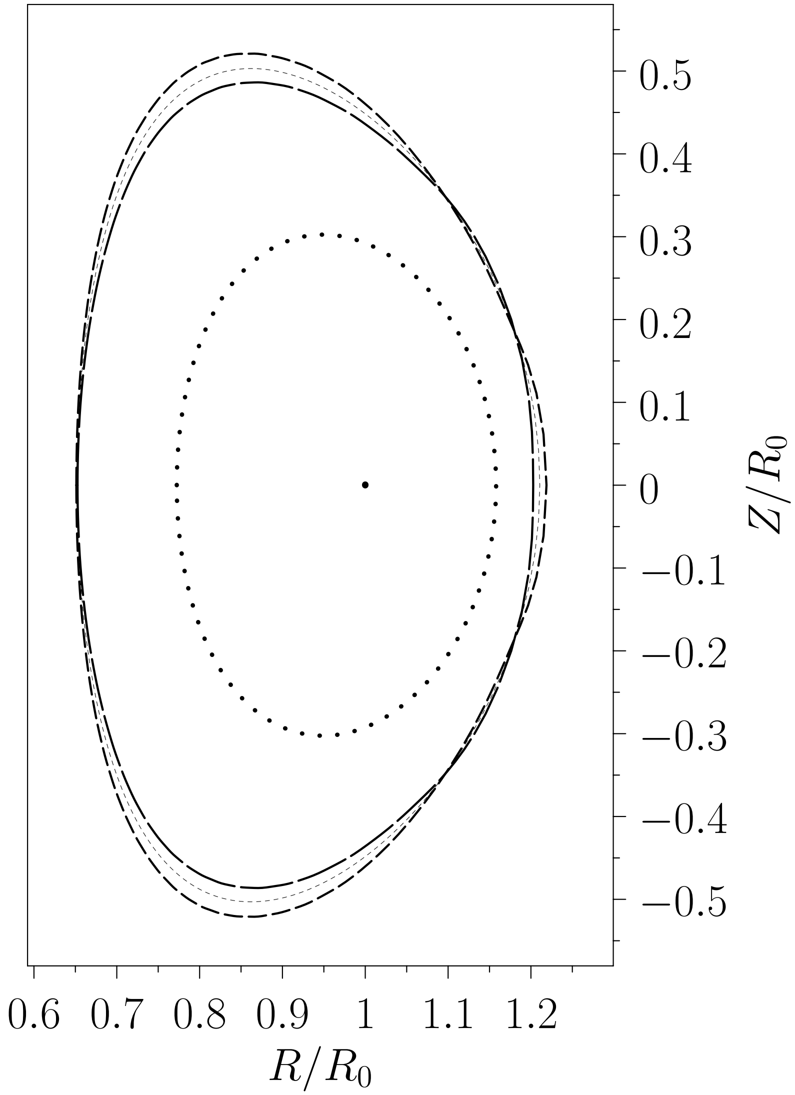
Optimum Wall Displacement: $q = 2$, Low-β



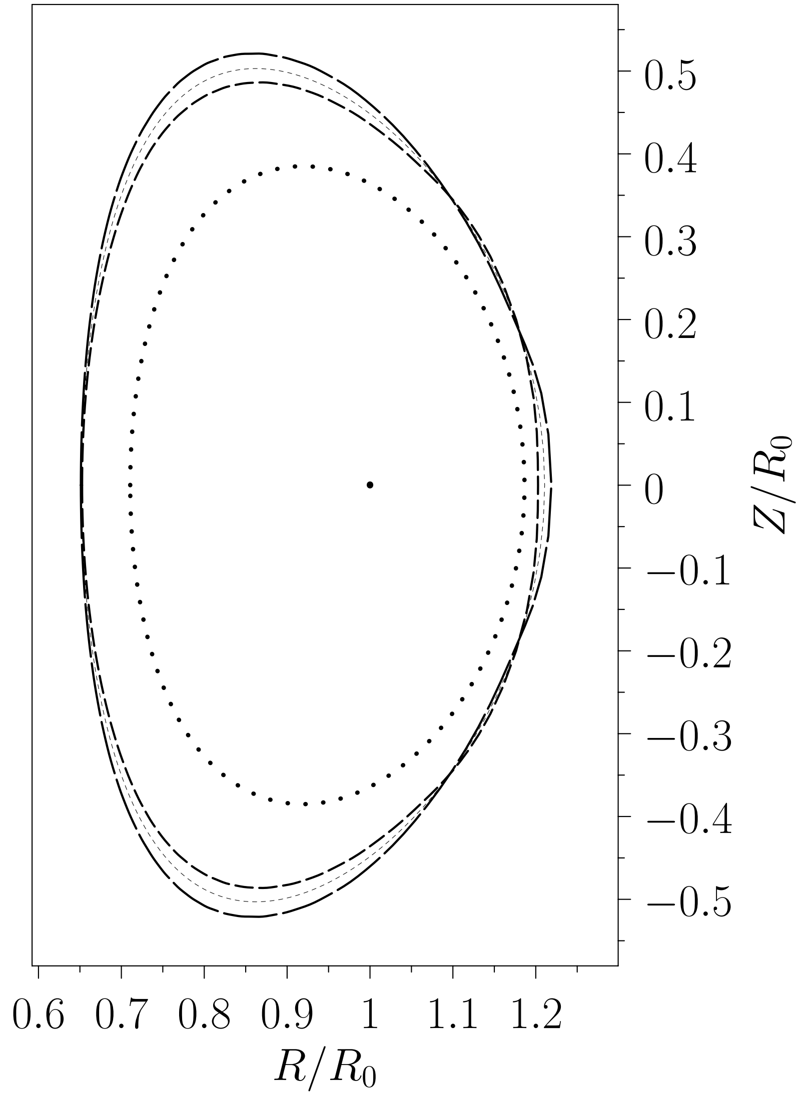
Optimum Wall Displacement: $q = 3$, Low-β



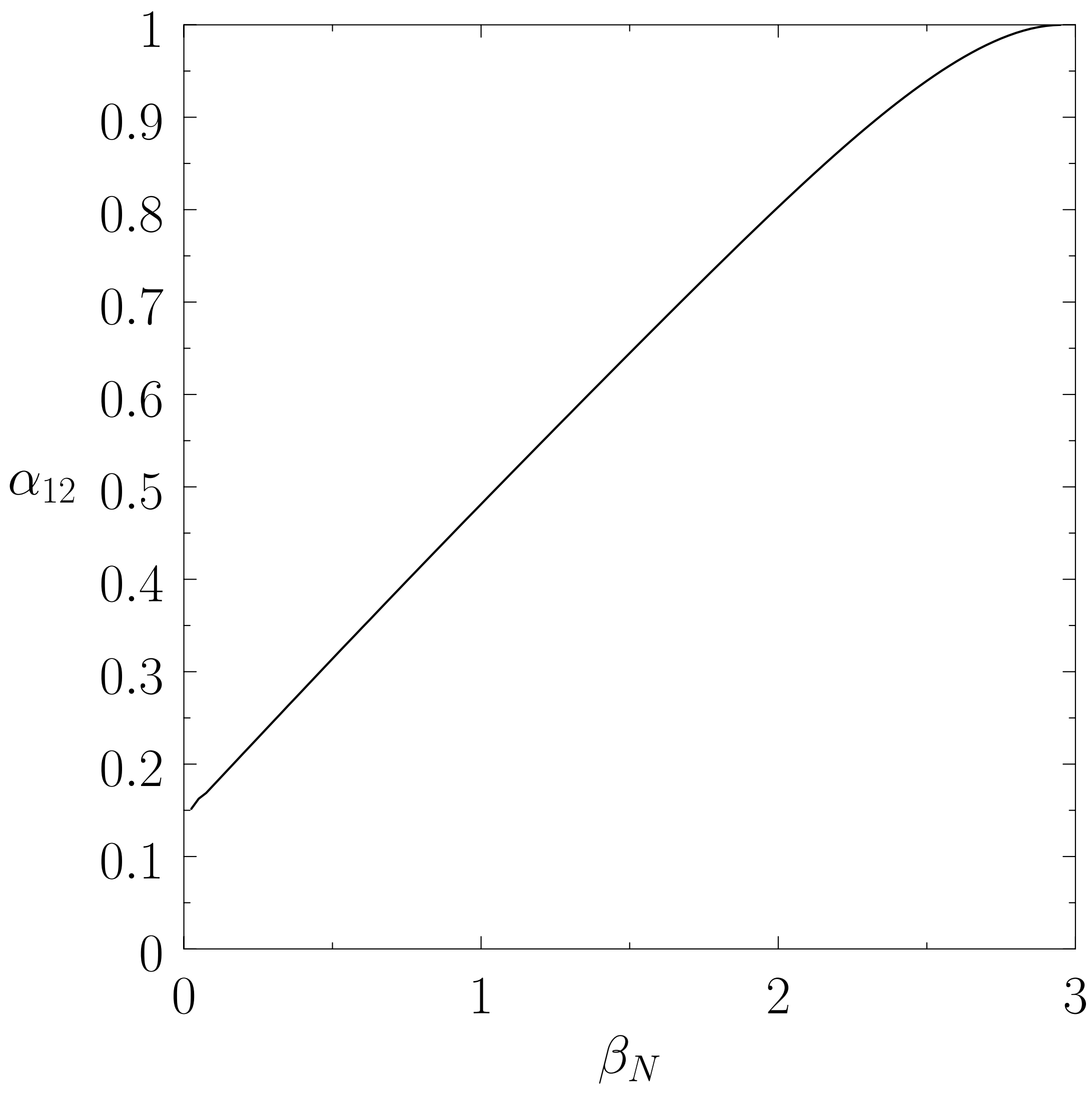
Optimum Wall Displacement: $q = 2$, High-β



Optimum Wall Displacement: $q = 3$, High-β



$q = 2/q = 3$ Overlap Parameter



Future Plans

- Include plasma rotation in linear GGJ layer response calculation.
- Generalize GGJ linear layer response model to equivalent nonlinear island response model, so as to determine threshold error-field strengths for penetration.
- Include two-fluid and low collisionality effects in linear layer response model.
- Use TOMUHAWC to calculate error-field induced drag torque due to neoclassical toroidal viscosity.