IFS Analytic Island Dynamics Model

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Introduction

- Over course of many years, IFS scientists have developed analytic, single-helicity, fully nonlinear, neoclassical, two-fluid model of magnetic island dynamics in quasi-cylindrical tokamak plasma. a

- Purpose of model to understand interaction of magnetic island with externally generated magnetic perturbation (e.g., error-field, sawtooth, ELM).

- Model used to investigate problems of great importance to DOE FES research program: e.g., penetration threshold for error-field driven locked modes; triggering of neoclassical tearing modes; ELM suppression via resonant magnetic perturbations.

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Four-Field Model

\[ \partial_t (w^2 \psi)/(w^2) = [\phi + \tau N, \psi] + \eta J, \]

\[ \partial_t (wN)/w = [\phi, N] - \rho [\alpha_n V + J, \psi] - \alpha_c \rho [\phi + \tau N, X], \]

\[ \partial_t V = [\phi, V] - \alpha_n (1 + \tau) [N, \psi], \]

\[ \epsilon \partial_t (w \partial^2_X \phi)/w = \epsilon \partial_X [\phi - N, \partial_X \phi] + [J, \psi] + \alpha_c (1 + \tau) [N, X], \]

\[ J = \beta^{-1} (\partial^2_X \psi - 1). \]

- Core of IFS model is nonlinear, reduced, two-fluid, toroidal “four-field” model of Hazeltine, Kotschenreuther, and Morrison (1985). Four fields: poloidal flux, \( \psi \); electron number density, \( N \); parallel ion velocity, \( V \); scalar potential, \( \phi \). Perturbed parallel current, \( J \), is auxiliary field.
Turbulent Perpendicular Transport

\[ \frac{\partial}{\partial t} (w N)/w = [\phi, N] - \rho [\alpha_n V + J, \psi] - \alpha_c \rho [\phi + \tau N, X] + D \partial^2_X N, \]

\[ \frac{\partial}{\partial t} V = [\phi, V] - \alpha_n (1 + \tau) [N, \psi] + \mu \partial^2_X V, \]

\[ \epsilon \frac{\partial}{\partial t} (w \partial^2_X \phi)/w = \epsilon \partial_X [\phi - N, \partial_X \phi] + [J, \psi] + \alpha_c (1 + \tau) [N, X] + \epsilon \mu \partial^4_X (\phi - N). \]

- IFS core model augmented by phenomenological terms representing turbulent perpendicular transport of density (temperature) and ion momentum.
Neoclassical Viscosity

\[
\partial_t (w^2 \psi) / (w^2) = [\phi + \tau N, \psi] + \eta J \\
+ \alpha_n^{-1} \hat{\nu}_{\theta e} [\alpha_n^{-1} J + V - \partial_X (\phi + \tau \nu_{\theta e} N) - \nu_{\theta i} - \tau \nu_{\theta e}], \\
\partial_t (w N) / w = [\phi, N] - \rho [\alpha_n V + J, \psi] - \alpha_c \rho [\phi + \tau N, X] \\
+ D \partial_X^2 N, \\
\partial_t V = [\phi, V] - \alpha_n (1 + \tau) [N, \psi] + \mu \partial_X^2 V \\
- \hat{\nu}_{\theta i} [V - \partial_X (\phi - \nu_{\theta i} N)], \\
\epsilon \partial_t (w \partial_X^2 \phi) / w = \epsilon \partial_X [\phi - N, \partial_X \phi] + [J, \psi] + \alpha_c (1 + \tau) [N, X] \\
+ \epsilon \mu \partial_X^4 (\phi - N) + \hat{\nu}_{\theta i} \partial_X [V - \partial_X (\phi - \nu_{\theta i} N)] \\
+ \hat{\nu}_{\perp i} \partial_X [-\partial_X (\phi - \nu N)].
\]

- Model completed by neoclassical electron/ion viscosity terms: represent bootstrap current, ion poloidal/toroidal flow damping.
Spatial Boundary Conditions

\[ \psi(X, \zeta, \hat{t}) \rightarrow \frac{1}{2} X^2 + \cos \zeta, \]
\[ \partial_X N(X, \zeta, \hat{t}) \rightarrow -1, \]
\[ \partial_X \phi(X, \zeta, \hat{t}) \rightarrow -\nu, \]
\[ \partial_X V(X, \zeta, \hat{t}) \rightarrow 0, \]
as \( |X| \rightarrow \infty \).

- Spatial boundary conditions: single-helicity, constant-\( \psi \), radially symmetric, magnetic island embedded in equilibrium with uniform local density gradient, and neoclassically-relaxed ion flow profile.
Asymptotic Matching

\[ 0 = \Delta' r_s + 2 m_\theta \left( \frac{w_v}{w} \right)^2 \cos \phi_p + J_c \beta \frac{r_s}{w}, \]

\[ 0 = -2 m_\theta \left( \frac{w_v}{w} \right)^2 \sin \phi_p + J_s \beta \frac{r_s}{w}, \]

\[ J_c = -2 \int_{-\infty}^{\infty} J \cos \zeta \, dX \frac{d\zeta}{2\pi}, \]

\[ J_s = -2 \int_{-\infty}^{\infty} J \sin \zeta \, dX \frac{d\zeta}{2\pi}. \]

- Terms involving \( w_v \) incorporate externally-generated resonant magnetic perturbation into problem. First matching condition yields modified Rutherford equation; second matching condition yields island phase evolution equation.
Method of Solution

- Magnetic island is helical magnetic equilibrium that evolves on comparatively slow transport timescale.
- Method of solution analogous to that employed in solving global plasma equilibrium.
- Lowest-order force balance (ignore relatively small transport and neoclassical terms) reveals that \( N = N(\psi), \phi = \phi(\psi), V = V(\psi), J = J(\psi). \)
- In order to determine form of flux-surface functions must solve transport problem across island region: i.e., include relatively small transport and neoclassical terms; annihilate dominant force balance terms via suitable flux-surface averaging. (IFS model only analytic model that incorporates second step.)
Island Propagation Frequency

- Determination of $\phi(\psi)$ profile yields island propagation frequency relative to local $E\times B$ frame.

- Linear physics: tearing mode propagates in electron diamagnetic direction relative to local $E\times B$ frame.

- Extremely narrow linear layer widths in high temperature tokamak plasmas. Detectable tearing mode already in nonlinear regime.

- Nonlinear physics: island propagates in ion diamagnetic direction relative to local $E\times B$ frame.

- Experimental confirmation of ion diamagnetic rotation of magnetic islands.\(^a\) US Fusion community do not seem to have got message.

Response of Tearing Stable Plasma to Static RMP

- Helical phase
- Island propagates in same direction as naturally unstable tearing mode
- Island width
- Toroidal ion velocity
Recent DIII-D RMP ELM Suppression Data\textsuperscript{a}

\textsuperscript{a}R. Nazikian, et al., Submitted to Nuclear Fusion.
Discussion - I

- Data can be interpreted as showing pulsating magnetic island chains driven at separate $n = 1$, $n = 2$, $n = 3$ resonant surfaces in outer regions of plasma.

- Island phase velocities and ion toroidal velocity modulate in sync with island widths in manner predicted by IFS model.

- $n = 1$ island chain rotates in electron diamagnetic direction relative to lab frame; $n = 2$, $n = 3$ chains rotate in ion diamagnetic direction.

- Due to strong ExB shear in edge region, expect driven island chains resonant just inside pedestal to rotate in electron diamagnetic direction; island chains resonant further towards edge rotate in ion diamagnetic direction.
Discussion - II

- Data can only be explained on basis of nonlinear physics. Linear physics: magnetic flux driven at rational surface by static error-field cannot propagate, even in presence of plasma rotation.

- Minimum physics requirements for analysis of RMP ELM suppression data. Model must be resistive, rather than ideal; nonlinear, rather than linear; must incorporate neoclassical viscosity (otherwise get wrong island propagation frequency); must solve transport problem to determine island frequency self-consistently (otherwise cannot determine island frequency).
Recent DIII-D RMP ELM Suppression Data

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Discussion - 1

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Discussion - II

- Data can only be explained on basis of non\textit{linear} physics. Linear physics: magnetic flux driven at rational surface by static error-field cannot propagate, even in presence of plasma rotation.

- Minimum physics requirements for analysis of RMP ELM suppression data. Model must be \textit{resistive}, rather than ideal; \textit{nonlinear}, rather than linear; must incorporate neoclassical viscosity (otherwise get wrong island propagation frequency); must solve transport problem to determine island frequency self-consistently (otherwise cannot determine island frequency).