Nonlinear Dynamics of Self-Organized Plasmas

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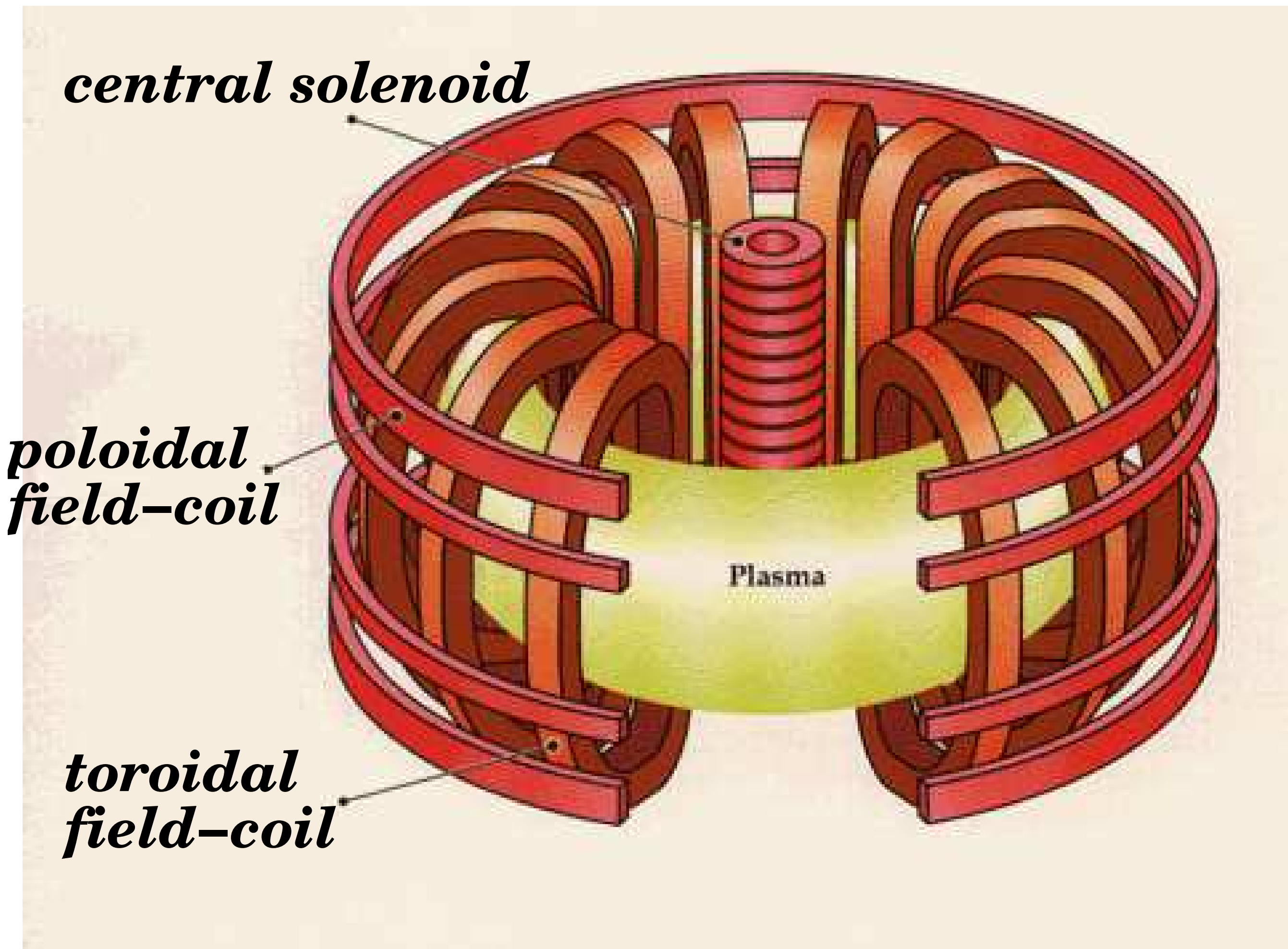
Magnetic Fusion

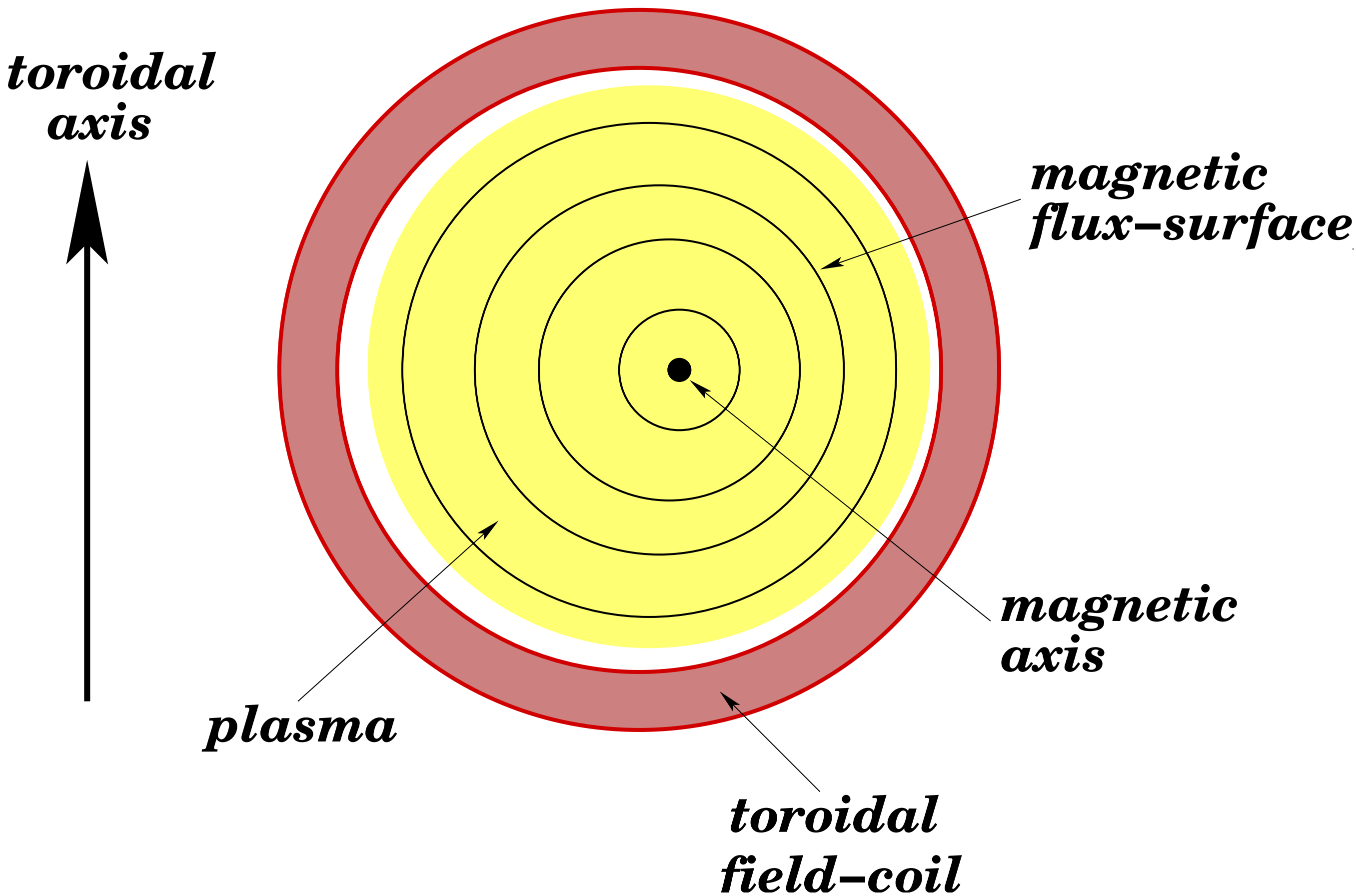
- Nuclear fusion requires plasma temperatures approximately ten times hotter than centre of sun.
- Conventional confinement via material walls out of question.
- *Magnetic fusion* aims to confine thermonuclear plasma via magnetic field. According to standard *magnetohydrodynamics* (MHD), plasma *tied* to magnetic field-lines.
- Magnetic field-lines not rigid. *Instabilities* can develop which cause field and plasma to thrash about, leading to loss of confinement.

Toroidal Pinches

- Toroidal plasma forms single-turn secondary winding of large transformer circuit.
- Flux-swing in multi-turn primary winding induces *toroidal* current in plasma, which generates *poloidal* magnetic field.
- *Toroidal* magnetic field generated by currents flowing in external coils.
- Net result is set of nested toroidal magnetic flux-surfaces on which plasma is (hopefully) confined.

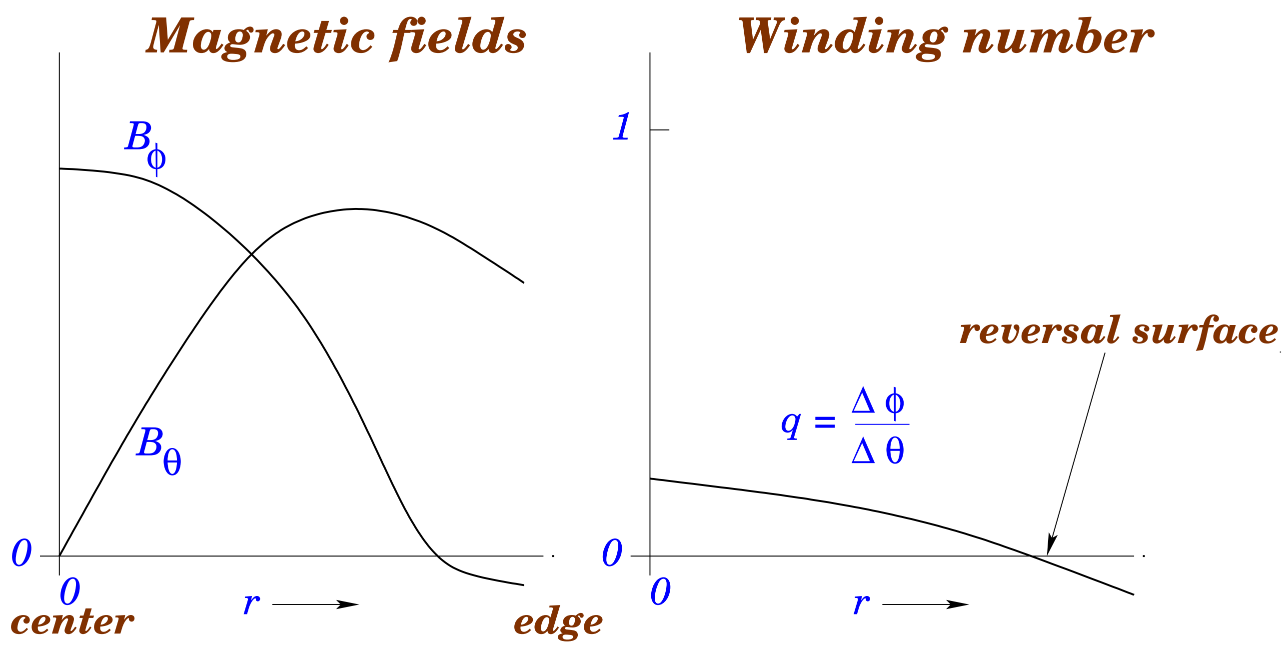

central solenoid
poloidal
field–coil
Plasma
toroidal
field–coil

Poloidal Cross–Section



Reversed Field Pinches

- In *reversed field pinch* (RFP) concept, developed at UKAEA Harwell, plasma surrounded by close-fitting conducting shell which "clamps" plasma edge in place.
- Intense internal MHD activity causes *relaxation* to stable configuration. In essence, plasma determines its own final state. Example of *self-organized plasma*.
- Relaxed state of RFP plasma characterized by *reversal* of toroidal magnetic field close to plasma edge.
- J.B. Taylor first to demonstrate that reversed configuration is *minimum energy state* of plasma subject to constraints imposed by close-fitting shell.



ϕ – *toroidal angle*

θ – *poloidal angle*

MHD Stability of RFPs

- Taylor relaxed state is *stable* to all MHD instabilities.
- Unfortunately, real-life RFP plasmas not (quite) in Taylor state:
  - Taylor state implies large edge currents. These are prevented by high resistivity of cold edge plasma.
  - Resistive evolution of plasma current profile drives equilibrium away from Taylor state.
- RFP plasmas subject to multiple, relatively benign, MHD instabilities, called *tearing modes*, which:
  - Prevent equilibrium from straying too far from Taylor state.
  - Degrade plasma confinement.

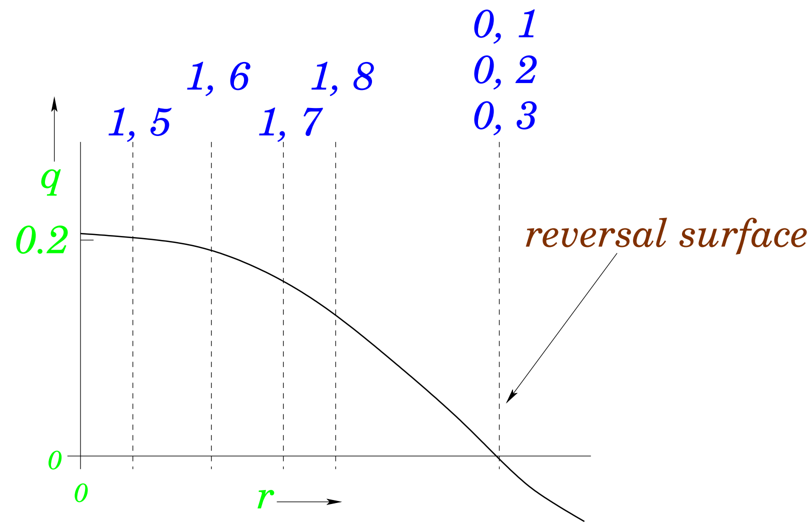
Tearing Modes in RFPs

- Tearing mode in RFP plasma identified via its *poloidal mode number*, m (number of periods short way around torus), and *toroidal mode number*, n (number of periods long way around torus).
- Tearing mode *resonates* with magnetic field when its helical pitch matches that of field. Resonance condition:

  $$m = n\,q,$$

  where $q(r)$ is *magnetic winding number*.
- Tearing modes *tear and reconnect* magnetic field-lines in vicinity of their resonant surfaces. Convert nested magnetic flux-surfaces (good confinement) into chaotic mess (poor confinement).

Dominant Tearing Modes in RFPs



- Dominant tearing modes in RFPs are $m = 1$ modes, resonant in plasma core, and $m = 0$ modes, resonant at *reversal surface*.

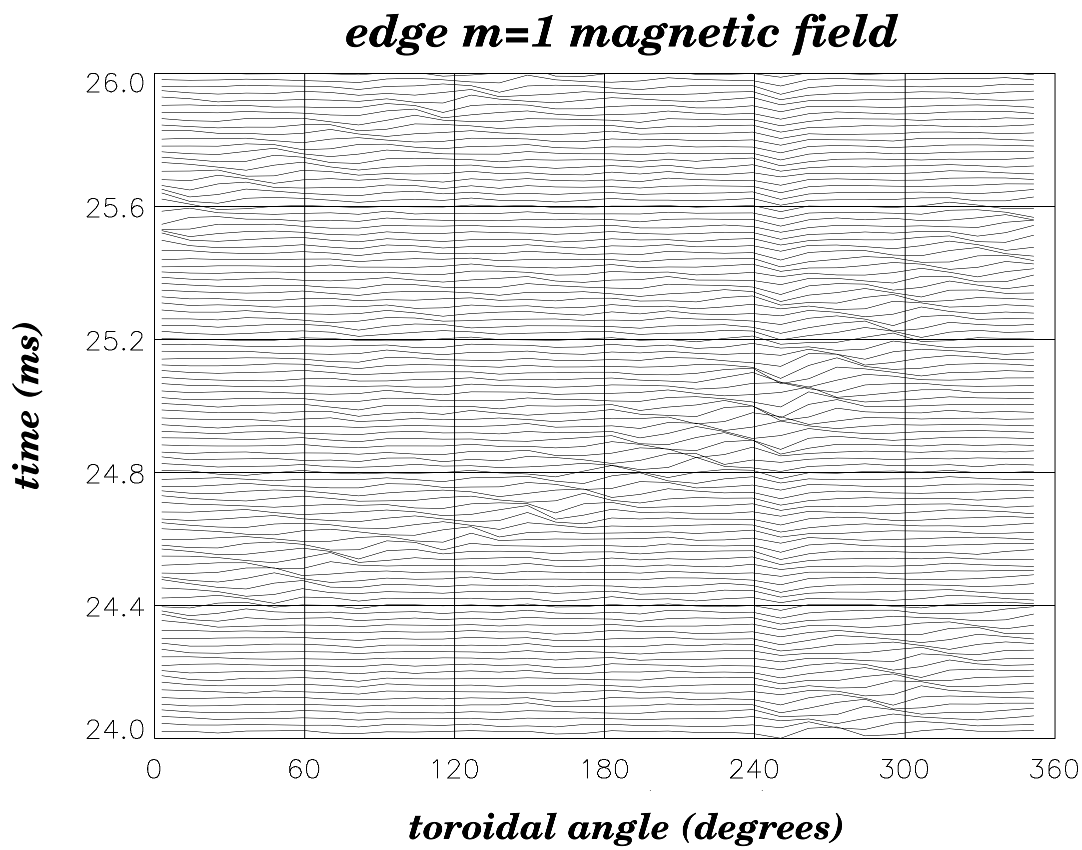
- Modes interact via *nonlinear coupling*:

$$(0, 1) + (0, 2) \rightarrow (0, 1) + (0, 3)$$

$$(1, 5) + (1, 6) \rightarrow (0, 1) + (2, 11)$$

Slinky Pattern

- Tearing mode amplitudes vary on relatively slow time-scale determined by plasma resistivity. Phases vary on much shorter time-scale.
- Tearing modes in RFP spontaneously *phase-lock* to form *toroidally localized* pattern in perturbed magnetic field known as *slinky*.
- Slinky pattern rotates toroidally in some experiments, but is stationary in others.
- Stationary slinky pattern causes severe edge loading problem which limits maximum achievable plasma current.



- Data from MST experiment (Madison, WI).

Critical Questions

- How does nonlinear coupling of tearing modes in RFP plasma lead to formation of slinky pattern?
- Can we predict properties of slinky pattern without having to perform many expensive 3D MHD simulations?

Force Minimization

- Plasma perturbed equation of motion:

$$\rho \frac{d\tilde{\mathbf{v}}}{dt} = \tilde{\mathbf{j}} \times \tilde{\mathbf{B}} + \mu \nabla^2 \tilde{\mathbf{v}}.$$

- At large mode amplitudes, nonlinear $\tilde{\mathbf{j}} \times \tilde{\mathbf{B}}$ force dominates both inertia and viscosity.

- Maybe phases of tearing modes in plasma arrange themselves so as to *minimize* nonlinear $\tilde{\mathbf{j}} \times \tilde{\mathbf{B}}$ force? Can we use this simple idea to account for formation and properties of slinky pattern?

$m = 0$ Modes

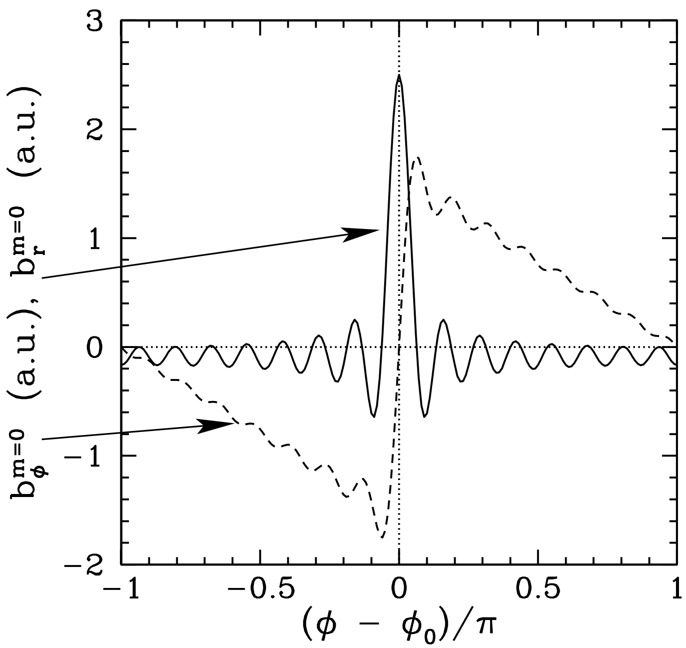
- $m = 0$ modes, resonant at reversal surface, couple nonlinearly and generate nonlinear $\tilde{\mathbf{j}} \times \tilde{\mathbf{B}}$ force at reversal surface.
- What arrangement of $m = 0$ phases minimizes net amplitude of $\tilde{\mathbf{j}} \times \tilde{\mathbf{B}}$ force at reversal surface?
- Answer:

$$\varphi^{0,n} = n\,\phi_0 \mp \pi/2,$$

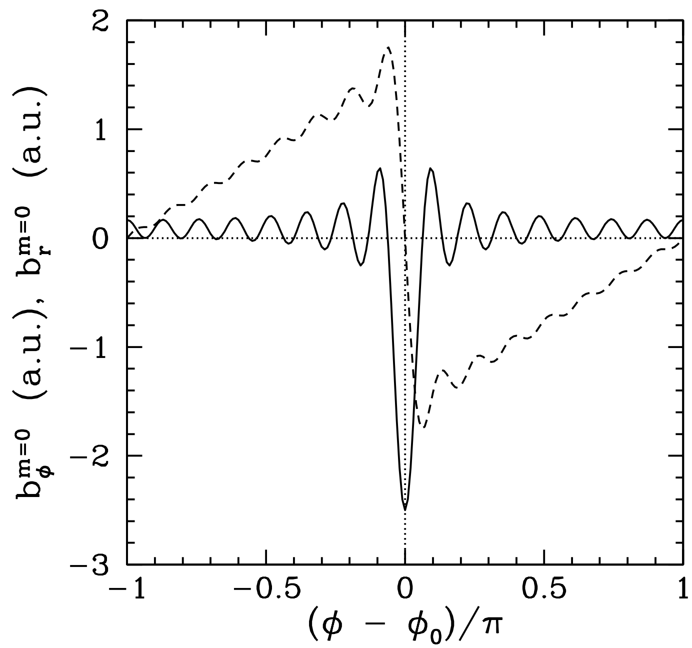
  where $\varphi^{0,n}$ is phase of $0, n$ mode, and ϕ_0 arbitrary.
- This particular phase arrangement generates two characteristic mirror-image patterns in $m = 0$ magnetic field (depending on sign chosen).

Predicted $m = 0$ Magnetic Fields

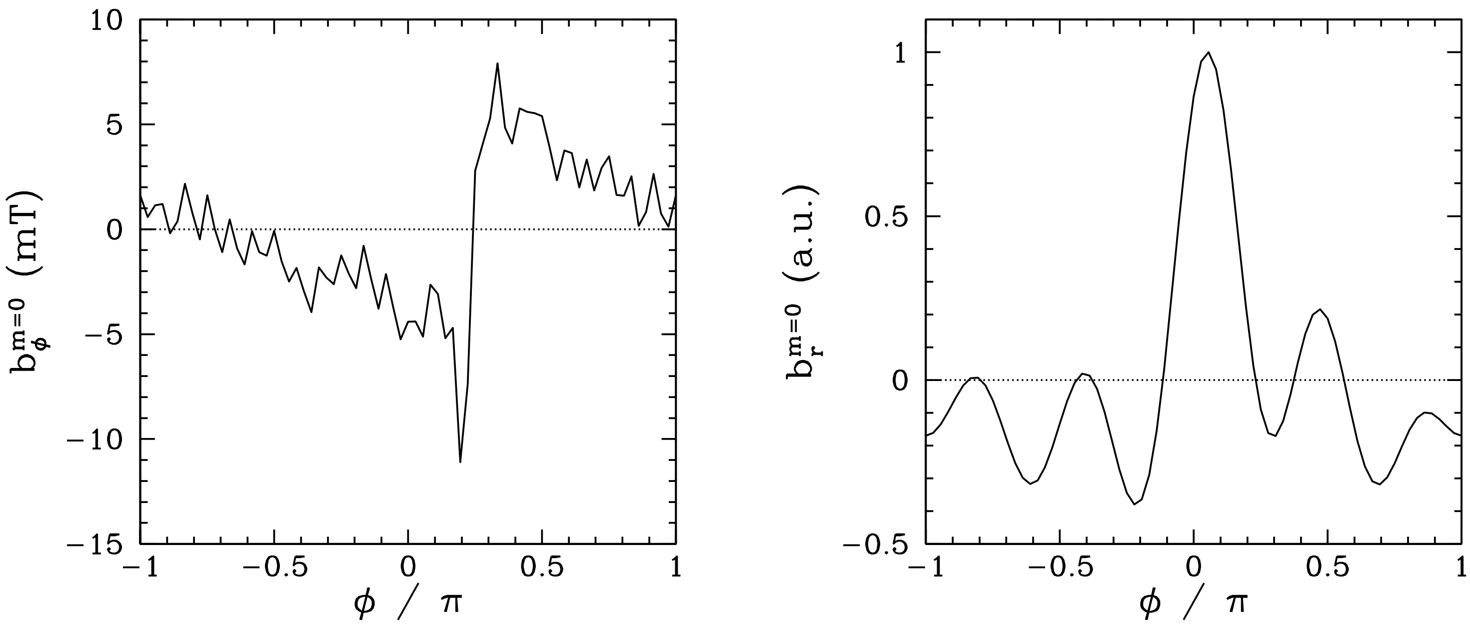
Upper sign: 15 modes



Lower sign: 15 modes

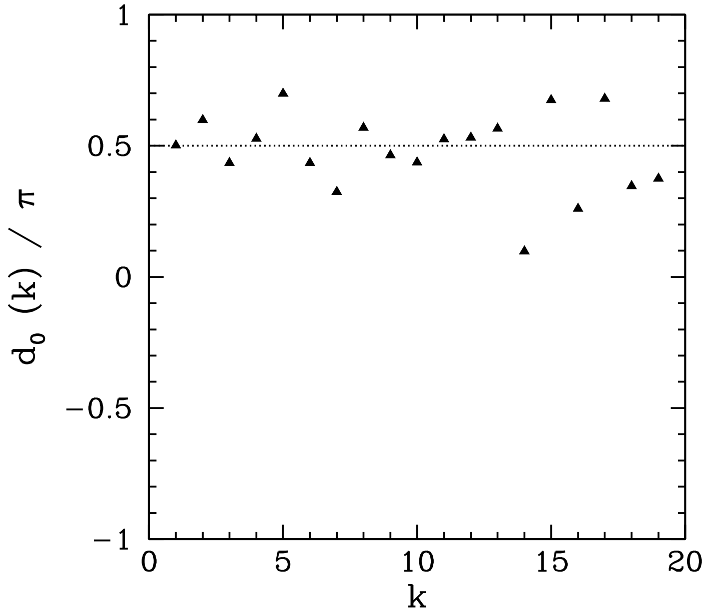


Observed $m = 0$ Magnetic Fields



- Data from RFX experiment (Padua). Consistent with upper sign.

Observed $m = 0$ Phase Correlations



- Here

$$d_0(k) = \varphi^{0,k+1} - \varphi^{0,k} - \varphi^{0,1}.$$

- Theoretical prediction is $d_0(k) = +\pi/2$.

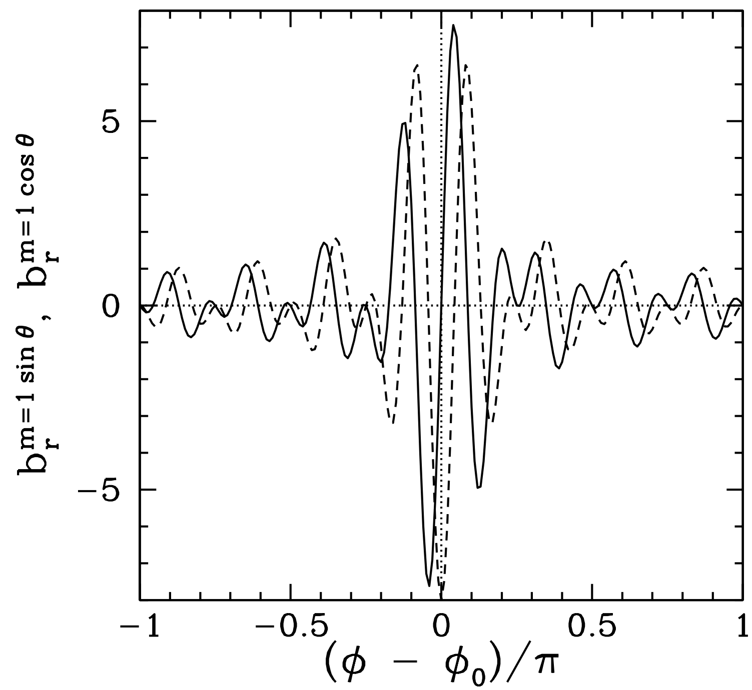
$m = 1$ **Modes**

- $m = 1$ modes couple nonlinearly through $m = 0$ modes and generate nonlinear $\tilde{\mathbf{j}} \times \tilde{\mathbf{B}}$ forces at resonant surfaces of interacting modes.
- What arrangement of $m = 1$ phases minimizes net amplitude of $\tilde{\mathbf{j}} \times \tilde{\mathbf{B}}$ forces at resonant surfaces?
- Answer:

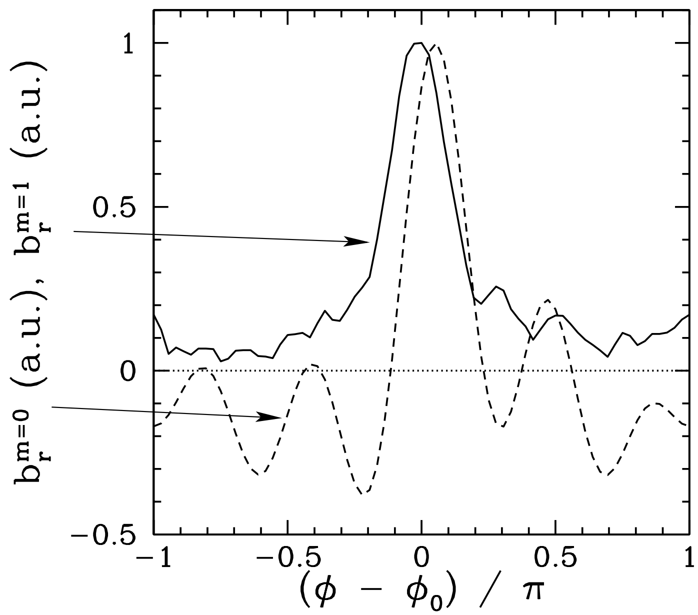
$$\varphi^{1,n} = n\,\phi_0 - \Delta_0,$$

  where $\varphi^{1,n}$ is phase of $1, n$ mode, and Δ_0 arbitrary.
- This particular phase arrangement generates toroidally localized pattern in $m = 1$ magnetic field.

Predicted $m = 1$ Magnetic Fields

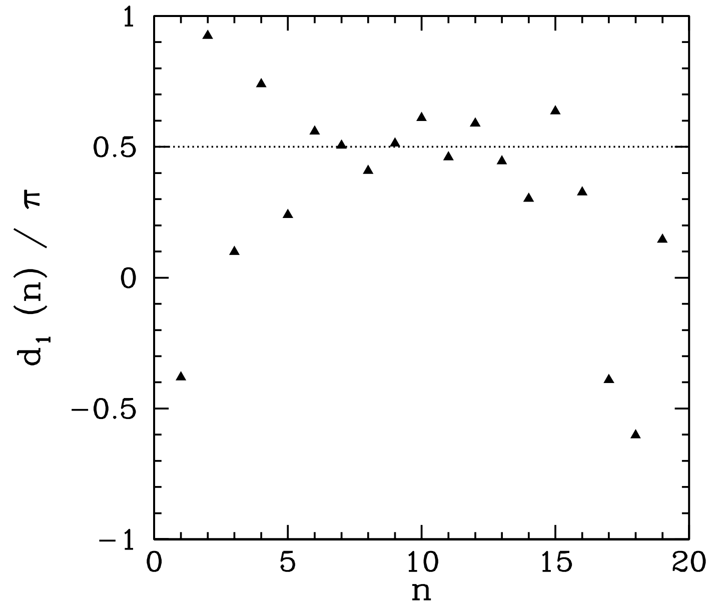


Observed $m = 1$ Magnetic Fields



- Data from RFX experiment (Padua).

Observed $m = 1$ Phase Correlations



- Here

$$d_1(n) = \varphi^{1,n+1} - \varphi^{1,n} - \varphi^{0,1}.$$

- Theoretical prediction is $d_1(n) = +\pi/2$.

Phase Locking Rules

- RFP plasma is complicated dynamical system consisting of many tearing modes interacting nonlinearly via three-wave coupling.
- Nevertheless, force minimization principle allows us to accurately predict properties of final phase-locked state of system.
- Phase locking rules are:

$$\begin{aligned} \varphi^{0,n} &= n\,\phi_0 \mp \pi/2, \\ \varphi^{1,n} &= n\,\phi_0 - \Delta_0. \end{aligned}$$

- Phase locking causes toroidally localized pattern in perturbed magnetic field, centered on $\phi = \phi_0$.

Uses of Phase Locking Rules

- Phase locking rules have been successfully used to explain interaction of slinky pattern with resonant static external magnetic perturbations.
- This understanding allows us to explain why slinky pattern rotates in some experiments, but not in others.
- Also allows us to design specific combinations of external perturbations which can be used to force pattern to rotate, or even break up pattern altogether.

Summary

- RFP plasma is complicated, intrinsically 3D, highly nonlinear, dynamical system.
- Taylor's hypothesis (minimization of energy at fixed magnetic helicity) allows us to predict proprieties of relaxed plasma equilibrium.
- New force minimization hypothesis (minimization of nonlinear electromagnetic forces via variation of mode phases at fixed mode amplitudes) allows us to predict properties of final phase locked state of tearing modes in plasma.