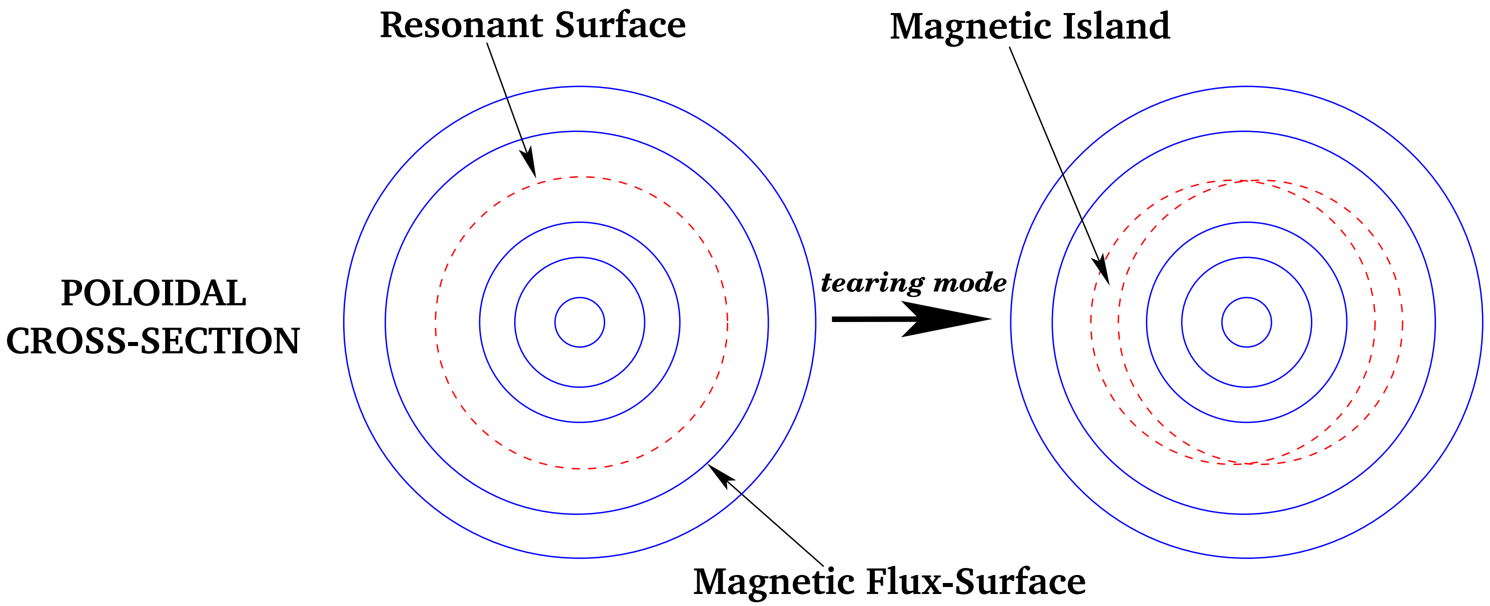
Magnetic Islands in Plasmas

RICHARD FITZPATRICK

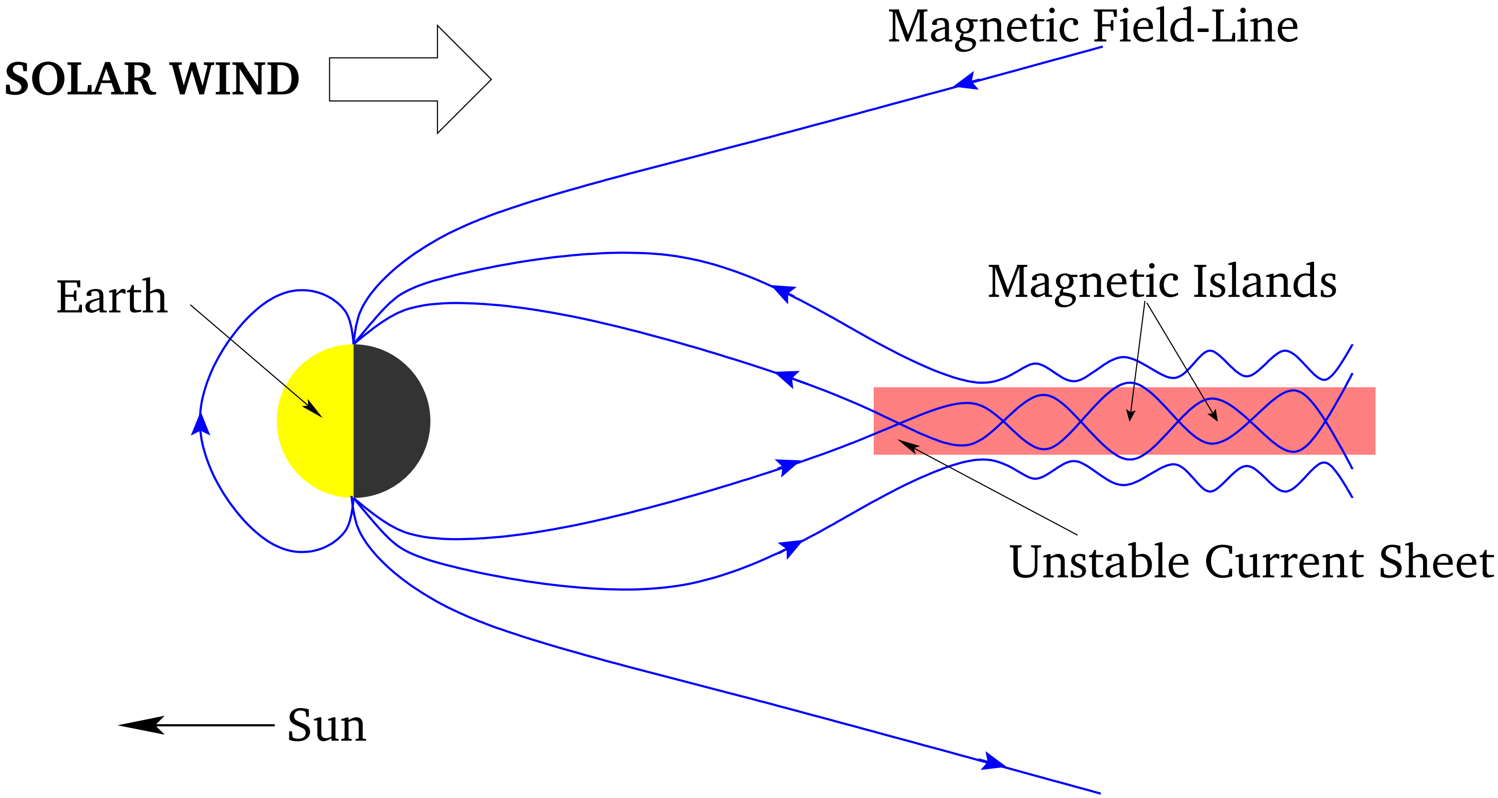
Institute for Fusion Studies
Department of Physics
The University of Texas at Austin
Austin, TX, USA

Magnetic Islands in Toroidal Plasmas



- Centered on resonant flux-surfaces which satisfy $\vec{k} \cdot \vec{B} = 0$, where $\vec{k}$ is wave-number of mode, and $\vec{B}$ is equilibrium magnetic field.
- Effectively "short-circuit" confinement by allowing heat/particles to transit island region by rapidly flowing along field-lines, rather than slowly diffusing across flux-surfaces.

Magnetic Islands in Earth's Magnetotail



- Island formation associated with energetic electron production.

Asymptotic Matching[a]

- Plasma divided into thin “inner region”, centered on resonant/neutral surface, and “outer region” which comprises remainder of plasma.
- Outer region governed by *linear ideal-MHD*. Resistive, inertial, non-linear, *etc.* effects only important in inner region.
- Solution in outer region fully specified by *tearing stability index*, Δ', which is defined as jump in logarithmic derivative of perturbed normal magnetic field across inner region.
- Must asymptotically match solutions at boundary between inner and outer regions.

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[a] H.P. Furth, J. Killeen, M.N. Rosenbluth, Phys. Fluids **6**, 459 (1963).

Constant-ψ Approximation

- Perturbed normal magnetic field approximately *constant* in inner region. Yields island chain in which individual islands separated by X-points. Also implies $W \propto \psi^{1/2}$, where W is island width, and ψ is reconnected magnetic flux.
- Approximation valid provided

  $$|\Delta'|\,W \ll 1.$$

  Only holds for weakly unstable tearing modes.
- Approximation implies that perturbed plasma current associated with island *much smaller* in magnitude than equilibrium current.

Rutherford Equation

- For constant-ψ island, asymptotic matching gives *Rutherford island width evolution equation*:[a]

$$\frac{0.823}{\eta}\frac{dW}{dt} \simeq \Delta' - 0.41\,\frac{W}{a^2},$$

where a is effective current sheet width.

- Island width grows *algebraically* on *resistive time-scale* (assuming $\Delta' > 0$), and *saturates* at

$$W_0 = 2.44\,a^2\,\Delta'.$$

- Constant-ψ approximation holds provided $W_0 \ll a$.

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[a] P.H. Rutherford, Phys. Fluids **16**, 1903 (1973). F. Militello, F. Porcelli, Phys. Plasmas **11**, L13 (2004). D.F. Escande, M. Ottaviani, Physics Lett. A **323**, 278 (2004).

Non-Constant-ψ Magnetic Islands

- As tearing mode becomes more unstable, $\Delta' W \rightarrow 1$, and constant-ψ approximation starts to break down.
- When $\Delta' W$ exceeds some critical value, which is $\mathcal{O}(1)$, island X-point splits into two Y-points separated by Sweet-Parker layer.[a]

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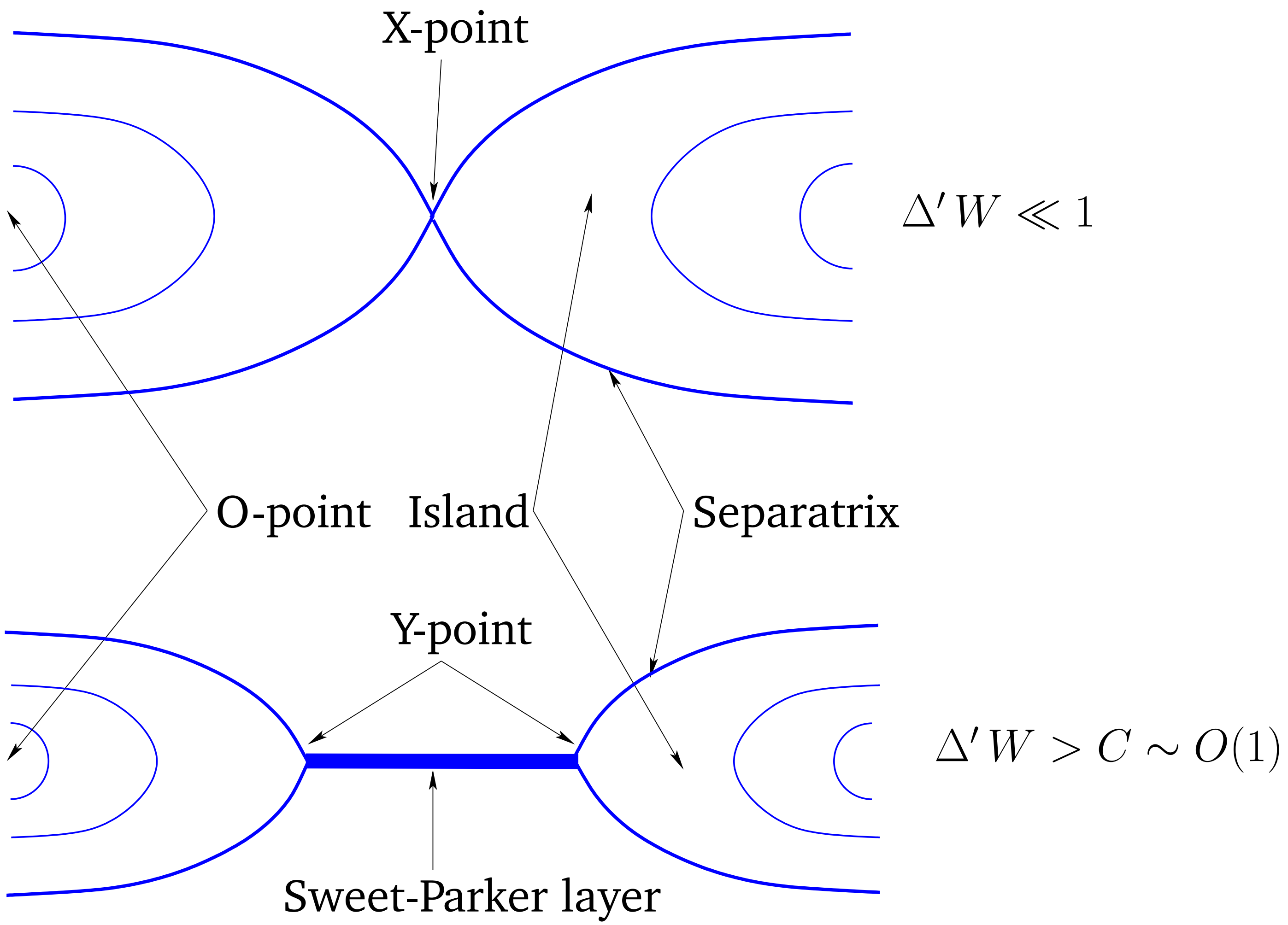
[a] F.L. Waelbroeck, Phys. Fluids B **1**, 2372 (1989). N.F. Loureiro, S.C. Cowley, W.D. Dorland, *et al.*, Phys. Rev. Lett. **95**, 235003 (2005).

Y-Point Formation



Non-Constant-ψ Magnetic Island Growth

- Island growth controlled by Sweet-Parker layer physics:[a]

$$\frac{d\psi}{dt} \propto \eta^{1/2}\, B_*^{3/2}\, L^{-1/2},$$

where, B_* is reconnecting field at edge of layer, and L is layer length.

- When L is of order size of plasma, island width grows as[b]

$$W \propto (t/\tau_{SP})^2,$$

where $\tau_{SP} = \sqrt{\tau_A\, \tau_R} \sim \eta^{-1/2}$ is Sweet-Parker time-scale.

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[a] P.A. Sweet, *IAU Symp. 6: Electromagnetic Phenomena in Cosmical Plasmas*, 123 (1958). E.N. Parker, J. Geophys. Res. **32**, 509 (1957). S.I. Syrovatskii, Sov. Phys. JETP **33**, 933 (1971).

[b] F.L. Waelbroeck, Phys. Fluids B **1**, 2372 (1989).

Secondary Magnetic Islands

- Sweet-Parker layer constitutes thinner more intense current sheet than original plasma equilibrium.
- Layer itself becomes tearing unstable when [a]

  $$L/\delta \gtrsim 50,$$

  where $\delta \sim L/(\tau_R/\tau_A)^{1/2}$ is Sweet-Parker layer width.
- Instability gives rise to formation of *secondary magnetic islands*.

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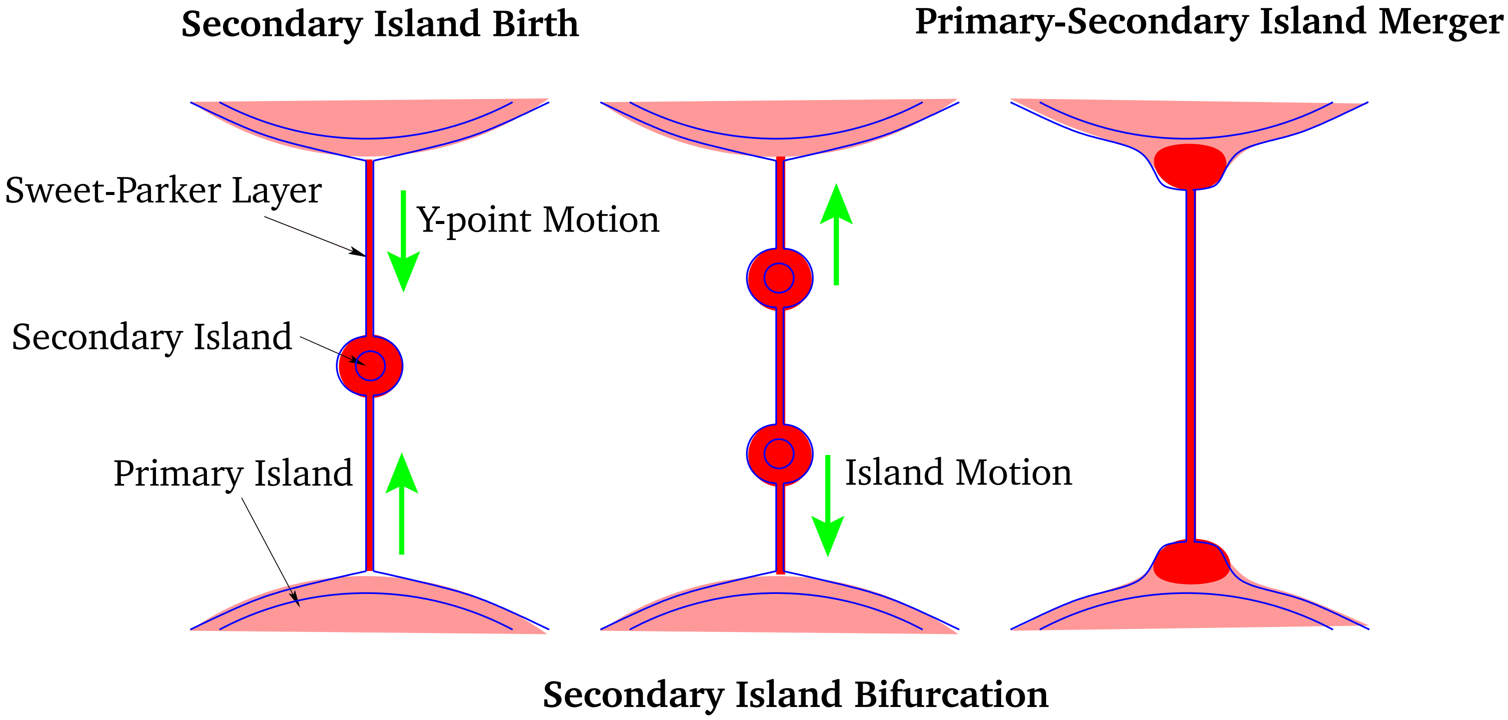
[a] D. Biskamp, *Nonlinear Magnetohydrodynamics* (CUP, 1993).

Secondary Island Evolution [a]



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[a] N.F. Loureiro, S.C. Cowley, *et al.*, Phys. Rev. Lett. **95**, 235003 (2005).

Particle Acceleration by Secondary Islands

- As secondary island forms its length rapidly contracts.
- Charged particles trapped in contracting secondary island efficiently accelerated to high energies via Fermi mechanism.[a]
- Possible origin of energetic electrons which constitute up to 50% of energy output of both solar flares and magnetic reconnection events in Earth's magnetotail.[b]

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[a] J.F. Drake, M. Swisdak, H. Che, M.A. Shay, Nature **443**, 553 (2006).

[b] L.-J. Chen, A. Bhattacharjee, *et al.*, Nature Physics **4**, 19 (2008).

Fast Magnetic Reconnection

- Sweet-Parker reconnection rate *much slower* than observed reconnection rates in most fusion and all space plasmas
- But, when Sweet-Parker layer width, δ, falls below critical value—which is $\mathcal{O}(d_i)$ for $\beta \gtrsim \mathcal{O}(1)$ plasma, and $\mathcal{O}(\rho_s)$ for $\beta \ll 1$ plasma—ion and electron fluids *decouple*, leading to greatly accelerated reconnection rate that is *independent* of resistivity.[a]
- For $\beta \gtrsim \mathcal{O}(1)$ plasma:[b]

$$\frac{d\psi}{dt} \propto d_i\, B_*^{2}\, L^{-1}.$$

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[a] D. Biskamp, *Magnetic Reconnection in Plasmas* (CUP, 2000). J. Birn, J.F. Drake, *et al.*, J. Geophys. Res. **106**, 3715 (2001).

[b] L. Chacón, A.N. Simakov, and A. Zocco, Phys. Rev. Lett. **99**, 235001 (2007). A.N. Simakov, and L. Chacón, Phys. Rev. Lett. **101**, 105003 (2008). L. Malyshkin, Phys. Rev. Lett. **101**, 225001 (2008).

Two-Fluid Effects on Magnetic Island Growth

- In two-fluid plasma, Rutherford equation generalizes to[a]

$$\frac{0.823}{\eta}\,\frac{dW}{dt} \simeq \Delta' - 0.41\,\frac{W}{a^2} + g\,\frac{(V - V_{EB})\,(V - V_i)}{W^3},$$

  where g is $\mathcal{O}(1)$ *positive* constant, V island phase velocity, V_{EB} local $\mathbf{E} \times \mathbf{B}$ velocity, and V_i local ion fluid velocity.

- Additional term comes from *ion polarization current*.[b] Term is stabilizing if island phase velocity lies between local $\mathbf{E} \times \mathbf{B}$ and ion fluid velocities, and destabilizing otherwise.

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[a] R. Fitzpatrick, F.L. Waelbroeck, Phys. Plasmas **12**, 022307 (2005).

[b] A.I. Smolyakov, Plasma Phys. Control. Fusion **35**, 657 (1993).

Determination of Island Phase-Velocity

- Solve flux-surface averaged transport equations to find electric field, density, and temperature profiles in vicinity of island. Profiles fix island current distribution.

- For *isolated* island, phase-velocity uniquely determined by constraint that *zero net EM torque* exerted on island region. Equivalent to constraint that $\mathbf{E}\times\mathbf{B}$ velocity profile unperturbed far from island.[a]

- For wide island (*i.e.*, $W \gg \rho_s\, L_s/L_n$): $V \simeq V_i$. For narrow island: $V \simeq V_{EB}$. For intermediate width island: $V_{EB} < V < V_i$.[b] Ion polarization has *stabilizing* effect on intermediate width island.

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[a] F.L. Waelbroeck, Phys. Rev. Lett. **95**, 035002 (2005).

[b] R. Fitzpatrick, F.L. Waelbroeck, F. Militello, Phys. Plasmas **13**, 122507 (2006).

Island Rotation Braking

- *Eddy currents* excited in vacuum vessel of toroidal confinement device by rotating magnetic island generate localized EM *braking torque* acting in island region.[a]

- Torque brakes local plasma rotation. Also brakes island rotation, since island phase velocity relative to local plasma fixed. Also brakes global plasma rotation, since local plasma viscously coupled to global plasma.[b]

- Reduction in global plasma rotation generally has deleterious effect on plasma confinement and stability, due to loss of stabilizing effect of *rotation shear*.[c]

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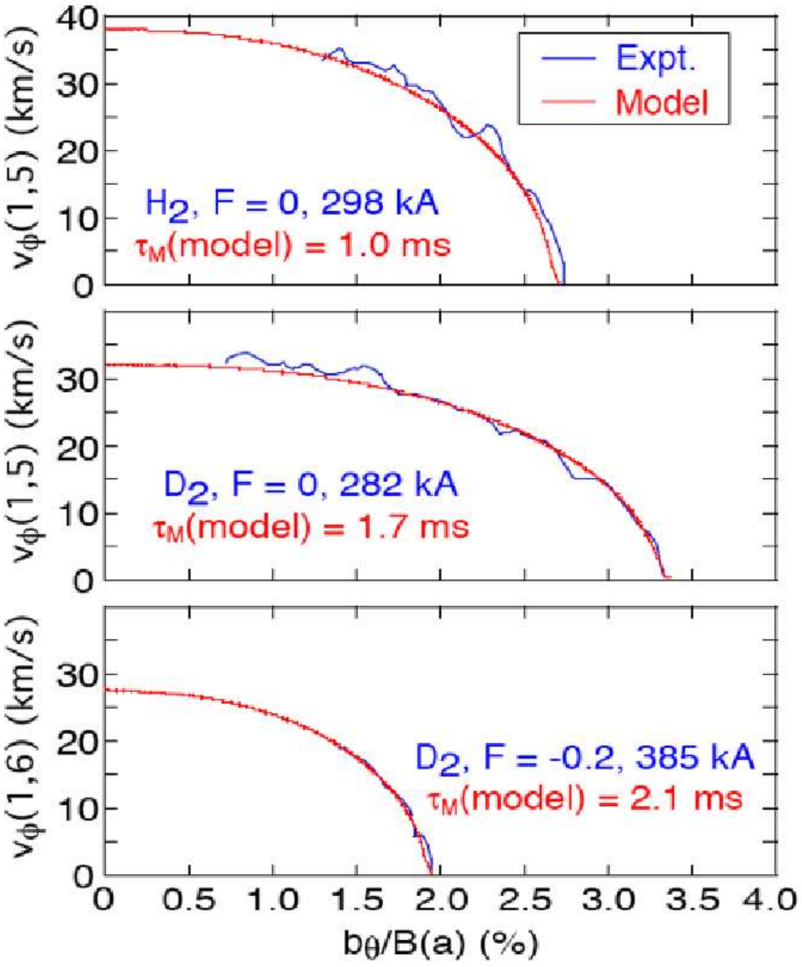
[a] M.F.F. Nave, J.A. Wesson, Nucl. Fusion **30**, 2575 (1990).

[b] R. Fitzpatrick, Nucl. Fusion **33**, 1049 (1993).

[c] R.J. Buttery, R.J. La Haye, *et al.*, Phys. Plasmas **15**, 056115 (2008).

Rotation Braking in MST Reversed-Field-Pinch [a]



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[a] B.E. Chapman, R. Fitzpatrick, *et al.*, Phys. Plasmas **11**, 2156 (2004).