A Simple Ideal-MHD Model of VDEs in Tokamaks

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Vertical Displacement Events

- Tokamak discharges sometimes terminated by *major disruptions*: *i.e.*, sudden events in which plasma thermal energy and electric current both rapidly quenched.
- Tokamak plasma equilibria typically highly *vertically elongated*.
- Such plasmas often lose vertical stability during disruptions, leading to *vertical displacement events* (VDEs): *i.e.*, events in which plasma moves rapidly up (or down), and thereby comes into contact with vacuum vessel.

Halo Currents

- When plasma comes into contact with vacuum vessel part of plasma current—known as *halo current*—forced to flow through vessel in poloidal direction.
- Poloidal halo current crossed with toroidal magnetic field generates strong *outward* force on vaccuum vessel.
- VDE generated halo current force *very much larger* than forces typically experienced by vacuum vessel during normal operation.
- Halo current forces are critical issue for ITER.

Aim of Talk

- Aim of talk is to present simple model of VDE in which magnitude of halo current force calculated *directly* from ideal-MHD stability analysis.
- Examine worst-case scenario in which plasma thermal quench does not occur prior to onset of vertical instability: *i.e.*, so-called "hot plasma" VDE.
- Neglect moderating effect of vacuum vessel eddy currents on vertical instability in order to highlight moderating effect of halo current.

Sharp Boundary Plasma Equilibrium

- Model employs large aspect-ratio (*i.e.*, $\epsilon \ll 1$), high-β (*i.e.*, $\beta \sim \epsilon$), *sharp boundary* plasma equilibrium.[a]
- Plasma boundary is vertically elongated *ellipse* of horizontal radius a and vertical radius $\kappa\, a$, where $\kappa > 1$.
- Plasma interior characterized by uniform plasma pressure, zero plasma current, and zero poloidal magnetic field.
- Sheet current flows on plasma boundary, and generates poloidal field outside plasma.

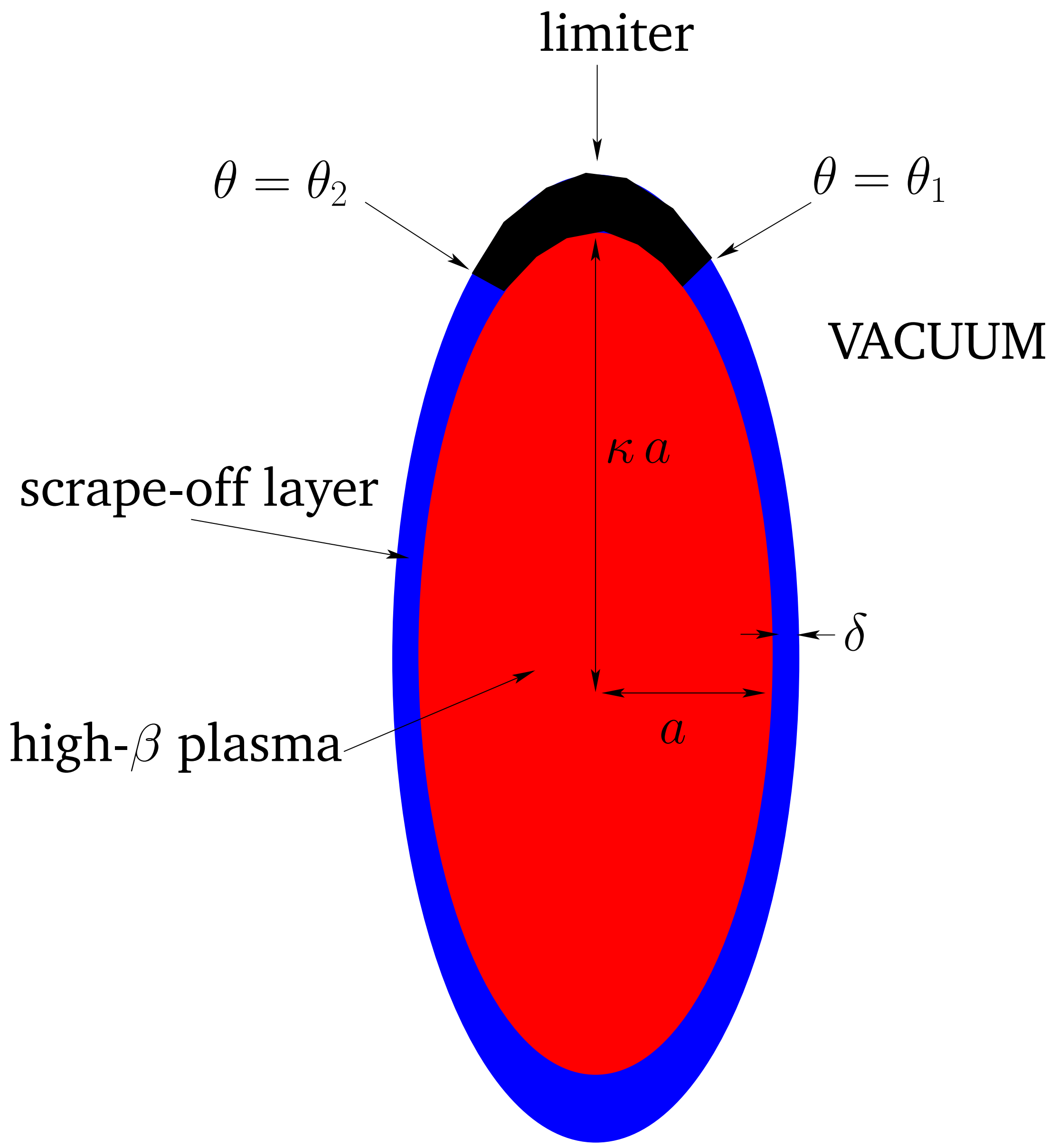
[a] J.P. Freidberg, and F. Haas, Phys. Fluids **17**, 440 (1974).

Halo

- Suppose that plasma equilibrium in contact with axisymmetric rigid conductor, termed *limiter*, which extends over poloidal angles $\theta_1 < \theta < \theta_2$.

- Scrape-off layer (SOL) is low-β plasma that immediately surrounds high-β plasma, and extends over all poloidal angles except those in range $\theta_1 < \theta < \theta_2$. All magnetic flux-surfaces in SOL intersect limiter. Let $\delta \ll a$ be SOL thickness.

- Taken together, SOL and limiter constitute *halo*.


limiter
$\theta = \theta_2$
$\theta = \theta_1$
VACUUM
$\kappa\, a$
scrape-off layer
δ
high-β plasma
a

Axisymmetric Halo Current

- Expect halo current associated with axisymmetric (*i.e.*, $n = 0$) vertical instability to be *axisymmetric*.

- Given that SOL plasma is low-β (since it is cold, due to rapid parallel heat transport to limiter), SOL halo current must be *force-free* (*i.e.*, $\mathbf{j} \propto \mathbf{B}$).

- No such constraint in limiter, since any EM force can be balanced by mechanical stresses. Expect limiter halo current to flow along path of least electrical resistance. Implies that limiter halo current flows predominately in *poloidal* direction.

Axisymmetric Limiter Force

- Axisymmetric halo current sheet density has components $i_{h\,\theta} = P_0/B_\phi$, $i_{h\,\phi} = P_0/B_\theta$ in SOL, and $i_{h\,\theta} = P_0/B_\phi$, $i_{h,\phi} = 0$ in limiter, where P_0 is a constant.
- Electromagnetic pressure acting on halo is $P_h = \mathbf{e}_n \cdot \mathbf{i}_h \times \mathbf{B}$, where $\mathbf{e}_n$ is unit normal vector at plasma boundary.
- $P_h = 0$ in SOL, and $P_h = P_0$ in limiter. Halo current generates *uniform* axisymmetric EM pressure acting on limiter.

Perturbed Matching Conditons

- Perturbed matching conditions at plasma boundary:[a]

$$
\begin{aligned}
\left[\mathbf{e}_n \cdot \delta\mathbf{B} - \mathbf{B}\cdot\nabla\xi + \xi\,\mathbf{e}_n\cdot(\mathbf{e}_n\cdot\nabla)\mathbf{B}\right]_i &= 0,\\
\left[\mathbf{e}_n \cdot \delta\mathbf{B} - \mathbf{B}\cdot\nabla\xi + \xi\,\mathbf{e}_n\cdot(\mathbf{e}_n\cdot\nabla)\mathbf{B}\right]_o &= 0,\\
\mu_0^{-1}\left[\mathbf{B}\cdot\delta\mathbf{B} + \xi\,\mathbf{e}_n\cdot\nabla(B^2/2)\right]_i^o &= 0,
\end{aligned}
$$

  where i/o refer to just inside/outside boundary, and $\xi(\theta,\phi)$ is normal plasma displacement at boundary.

- First two matching conditions ensure boundary remains flux-surface. Final condition enforces edge pressure balance.

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[a] J.P. Freidberg, and F. Haas, Phys. Fluids **17**, 440 (1974).

Modified Matching Conditions

- Halo current force modifies edge pressure balance.
- Can incorporate this effect into matching conditions by writing

$$\mu_0^{-1}\left[\mathbf{B}\cdot\delta\mathbf{B}+\xi\,\mathbf{e}_n\cdot\nabla(B^2/2)\right]_i^o=-(P_0/\delta)\,\zeta\,\xi,$$

where $\zeta=1$ in limiter, and $\zeta=0$ in SOL. Term on r.h.s. represents virtual work done on displaced plasma by reaction to limiter force.

Ideal Plasma Stability

- Perturbed magnetic field written $\delta\mathbf{B} = \nabla V$ inside and outside plasma, where $\nabla^2 V = 0$.
- Can find well-behaved solutions for V inside and outside plasma.
- Can match solutions at plasma boundary, using modified matching conditions.
- Stability problem reduces to

$$\overleftrightarrow{F}\ \vec{\xi} = \lambda\ \vec{\xi},$$

  where $\vec{\xi}$ are polodial harmonics of ξ, and $\overleftrightarrow{F}$ is self-adjoint matrix.
- According to ideal-MHD energy principle, plasma is ideally *unstable* if any eigenvalues of F-matrix are *negative*.

Effect of Limiter Force

- Suppose plasma is ideally unstable to $n = 0$ vertical mode.
- Ideal instabilities grow on Alfvén time-scale: $\tau_A \sim 10^{-7}$ s.
- But, when plasma intersects limiter, and halo current flows across it, associated force modifies edge pressure balance, and is able to moderate growth of instability.
- Expect instability to be slowed down such that it grows on resistive time-scale associated with SOL plasma. (Otherwise, absurdly large electric fields generated in SOL.) This time-scale very much longer than Alfvén time-scale: $\tau_R \sim 10^{-3}$ s.
- Crucial Insight: Moderating effect of limiter force effectively renders plasma marginally stable to $n = 0$ vertical mode.

Determination of Limiter Force

- If plasma is ideally unstable to $n = 0$ vertical mode then (in absence of limiter force) $\lambda_{0\,n=0} < 0$, where $\lambda_{0\,n=0}$ is smallest eigenvalue of $n = 0$ F-matrix.

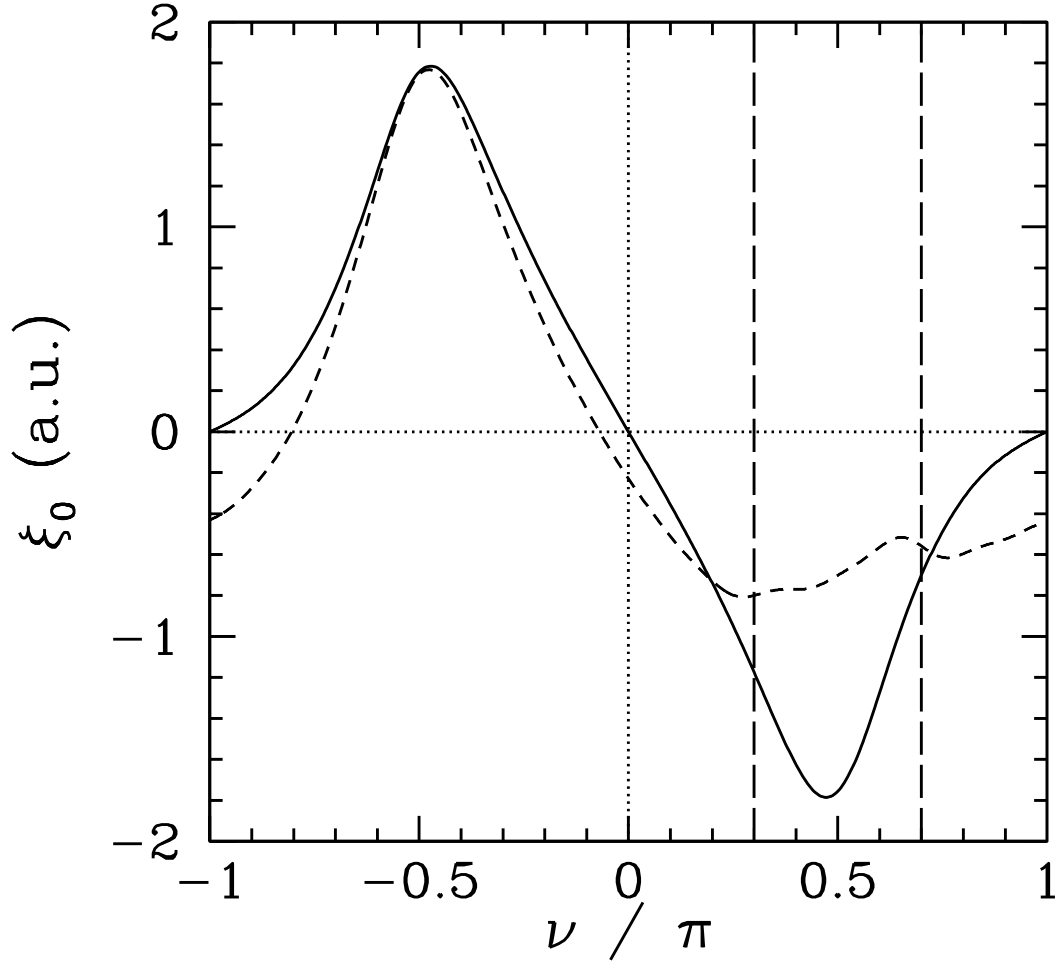
- But, $\lambda_{0\,n=0}$ is function of $n = 0$ limiter force parameter P_0.

- Limiter force must adjust itself so as to make mode marginally stable: *i.e.*, P_0 must be adjusted until $\lambda_{0\,n=0} = 0$. Required value of P_0 always *positive*: *i.e.*, limiter force always acts *outward*.

- Direct calculation of limiter force from $n = 0$ ideal-MHD stability analysis. Result independent of SOL/limiter resistivity.

Example Calculation

- Example plasma equilibrium: $\beta/\epsilon = 0.2$, $q_a = 3.0$, $\kappa = 2.0$, $\theta_1 = 0.3\,\pi$, $\theta_2 = 0.7\,\pi$.
- When $P_0 = 0.0$ find $\lambda_{0\,n=0} = -0.195$: *i.e.*, plasma ideally unstable to $n = 0$ vertical mode.
- But, when $P_0 = 0.041$ (normalized units) then $\lambda_{0\,n=0} = 0.0$.
- So, normalized outward limiter pressure $P_0 = 0.041$ renders $n = 0$ mode marginally stable. Allows mode to grow on resistive time-scale determined by SOL plasma.

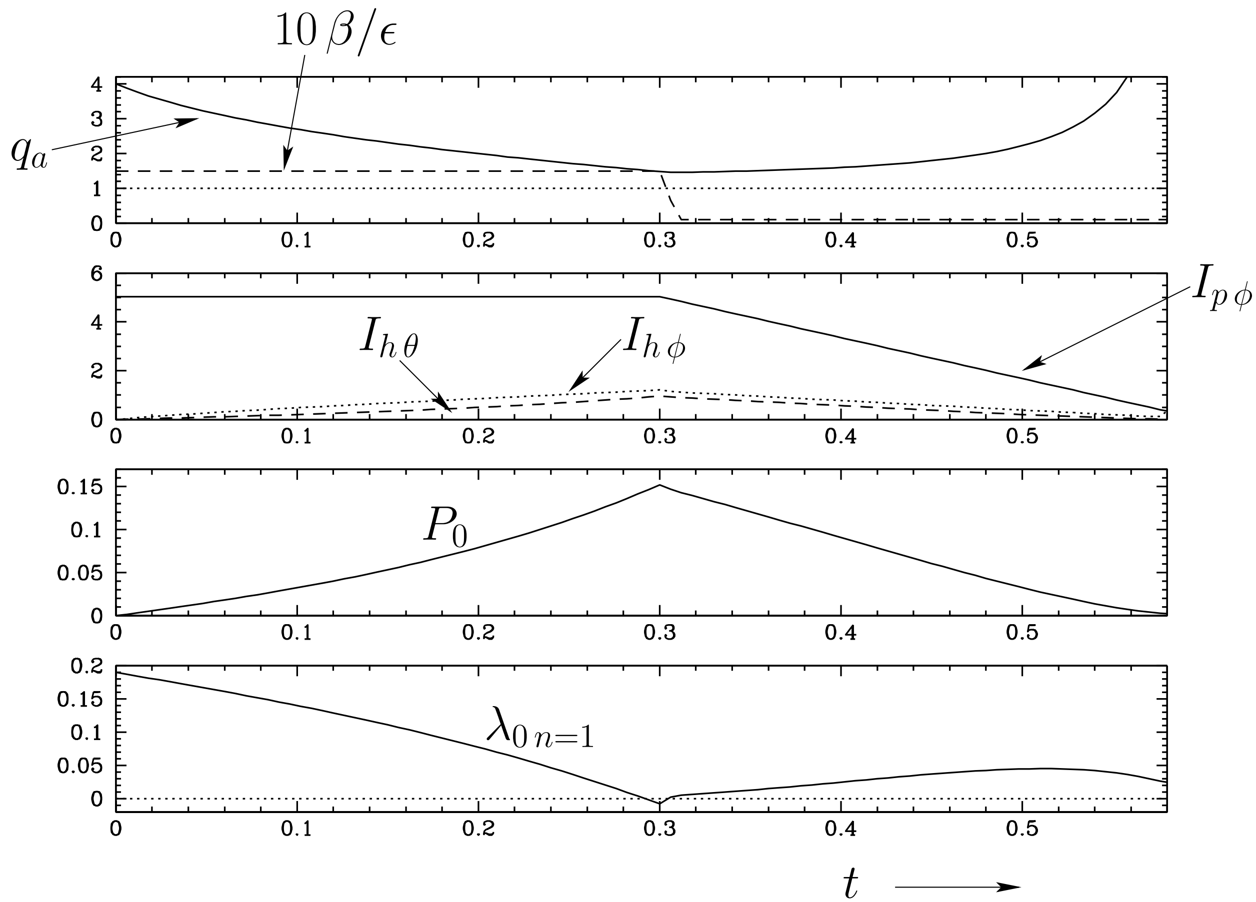


2
1
0
−1
−2
−1
−0.5
0
0.5
1
ξ_0 (a.u.)
ν / π

VDE Simulation

- Plasma moves vertically at constant velocity. As plasma flux-surfaces intersect limiter, high-β plasma core shrinks in size (at constant κ): $a = 1 - t$.
- SOL grows at expense of high-β plasma, but there is maximum possible SOL width, δ_0: $\delta = \min(t, \delta_0)$.
- Halo initially forms at constant β, and constant toroidal plasma current, $I_{p\,\phi}$.
- Thermal and current quenches simulated by linearly ramping β and $I_{p\,\phi}$ to zero. Both quenches start at t_0. End of thermal quench at t_1. End of current quench at t_2.

$$t_0 = 0.30, t_1 = 0.31, t_2 = 0.60, \delta_0 = 0.3$$



VDE Growth-Rate

- Vertical instability moderated by halo current force grows on time-scale

$$\tau_h \sim \mu_0\,\sigma_h\,a^2\,\frac{\kappa^{3/2}}{q_a^2},$$

where σ_h is halo conductivity.

- Vertical instability moderated by vacuum vessel eddy currents grows on time-scale

$$\tau_w \sim \mu_0\,\sigma_w\,a\,\delta_w,$$

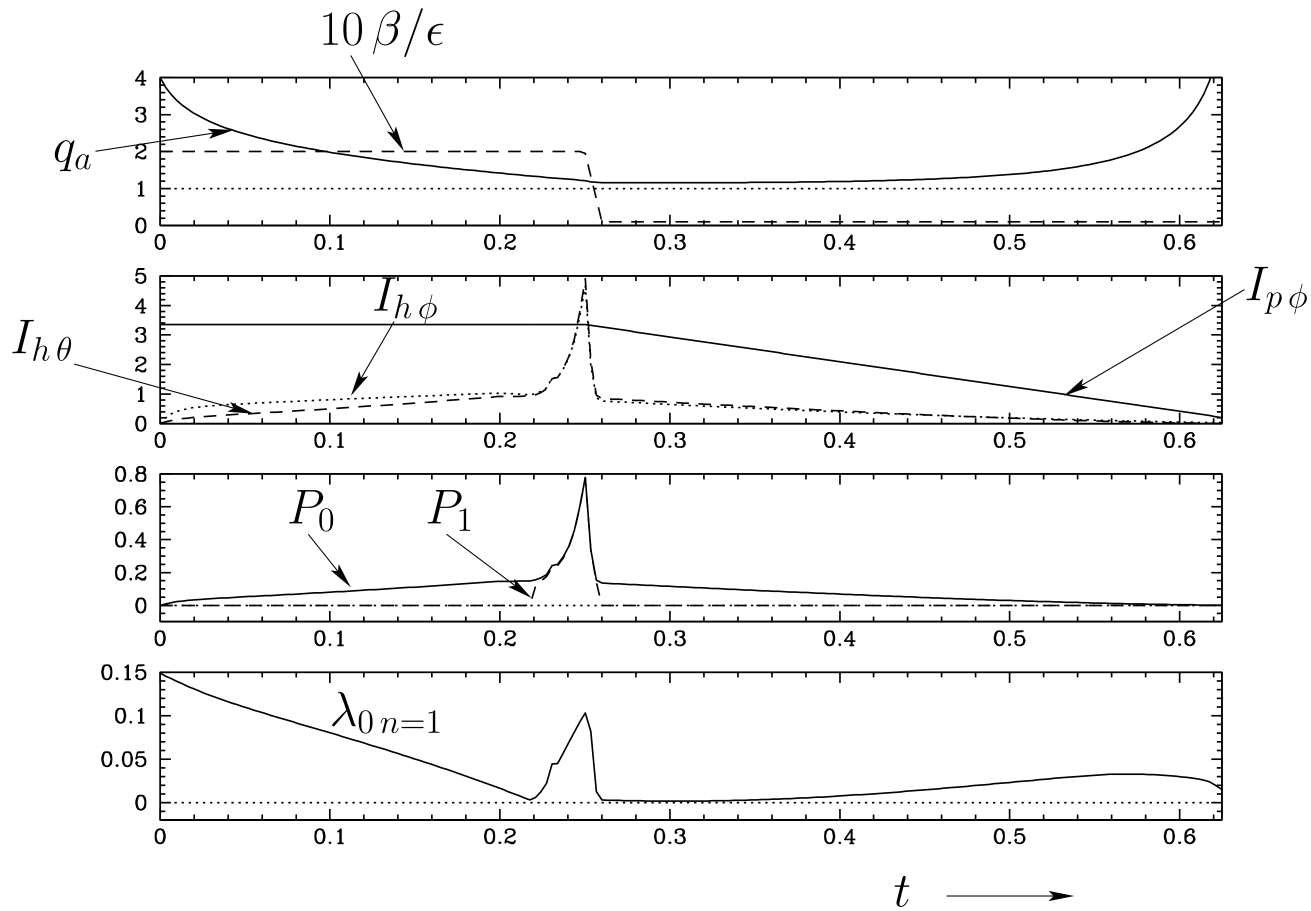
where σ_w is wall conductivity, and δ_w is wall thickness.

- For thin wall expect $\tau_h \gg \tau_w$: *i.e.*, halo current more effective at moderating growth of vertical instability than vessel eddy currents.

Non-Axisymmetric Halo Currents

- As q_a decreases during VDE, $n = 1$ kink mode stability decreases. Eventually, when $q_a \sim 1$, kink mode becomes unstable.
- Can no longer assume that halo current is axisymmetric in kink-unstable plasma.
- Halo current force has $n = 0$ component of amplitude P_0 and $n = 1$ component of amplitude P_1. $n = 1$ halo current couples $n = 0$ vertical and $n = 1$ kink modes to produce two hybrid $n = 0/n = 1$ modes.
- Must adjust P_0 and P_1 such that plasma marginally stable to hybrid $n = 0/n = 1$ modes.
- Toroidal peaking of halo current force governed by ratio P_1/P_0.

$$t_0 = 0.30, t_1 = 0.31, t_2 = 0.60, \delta_0 = 0.3$$



Summary

- Have developed simple model of VDEs in tokamak plasmas which calculates halo current force directly from $n = 0$ ideal-MHD stability.
- Model predicts plausible halo current amplitudes in crude VDE simulations.
- Model can be generalized to take $n = 1$ kink mode into account, and hence to predict toroidal peaking factor for halo current force.