Analytic Theory of ELM Suppression by Static RMPs in DIII-D Tokamak

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Introduction

- In DIII-D H-mode discharge 158115 [a] $n = 2$ RMPs successfully used to suppress ELMs.
- TM1 is multi-harmonic, cylindrical, five-field, nonlinear, initial value code.
- Recent TM1 simulations [b] explain observed features of ELM suppression in DIII-D discharge 15811 in terms of mode penetration at 8/2 rational surface (at top of pedestal) and at 11/2 surface (at bottom of pedestal). Penetration at 11/2 surface produces density pump-out. Penetration at 8/2 surface responsible for ELM suppression.
- IFS has constructed single-harmonic analytic model to gain more exact understanding of phenomena seen in TM1 simulations.

[a] R. Nazikian, et al., PRL **114**, 105002 (2015).
[b] Q. Hu, et al., submitted to Nature Physics (2019).

IFS Analytic Model

$$\mathcal{I}\,\frac{d\hat{W}}{d\hat{t}} = -\Delta + \mathcal{A}\left(\frac{\hat{W}_v}{\hat{W}}\right)^2 \cos\varphi,$$

$$\frac{d\varphi}{d\hat{t}} = \hat{\omega},$$

$$\hat{\omega} = \hat{\omega}_0 - \hat{\omega}_\theta(\hat{r}_s,\hat{t}) - \hat{\omega}_\phi(\hat{r}_s,\hat{t}),$$

$$(1{+}2\,q_s^{\,2})\,\hat{r}^{\,3}\,\frac{\partial\hat{\omega}_\theta}{\partial\hat{t}} - \nu_\mu\,\frac{\partial}{\partial\hat{r}}\left(\hat{r}^{\,3}\,\frac{\partial\hat{\omega}_\theta}{\partial\hat{r}}\right) + \nu_\theta\,\hat{r}^{\,3}\,\hat{\omega}_\theta = \frac{\mathcal{A}\,\hat{W}_v^2\,\hat{W}^2}{\hat{W}_0^4}\,\sin\varphi\,\delta(\hat{r}-\hat{r}_s),$$

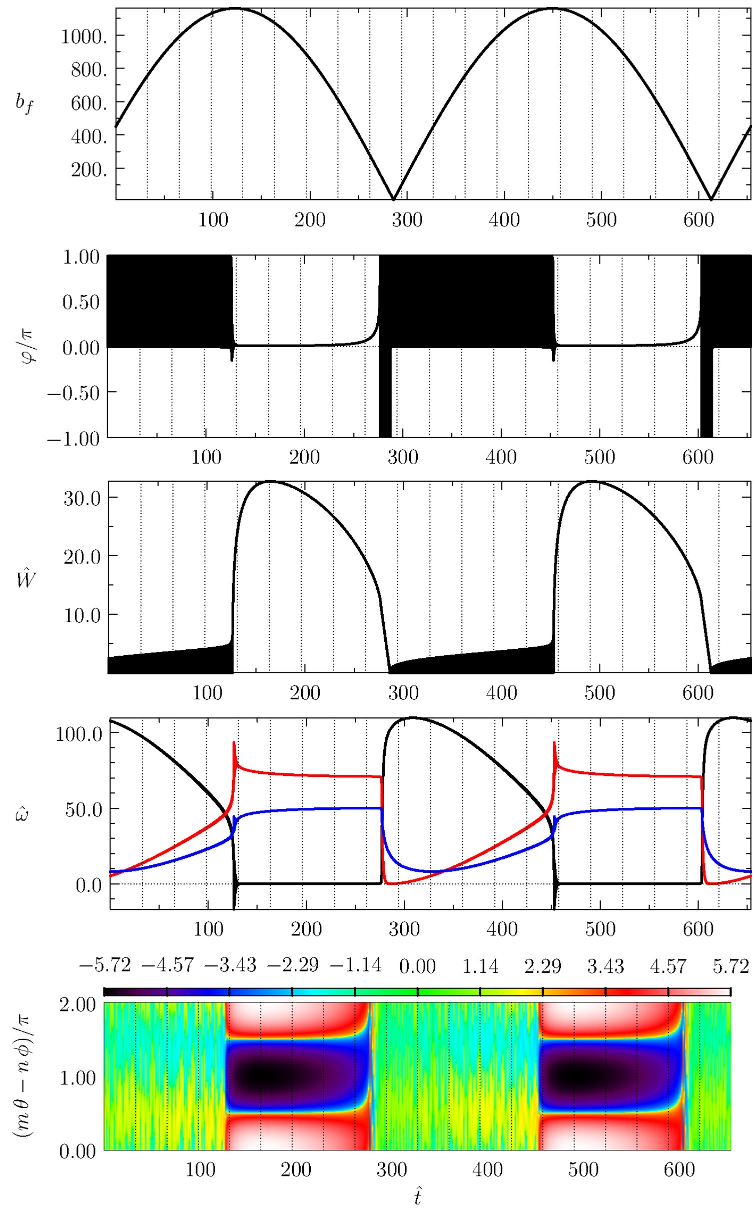
$$\hat{r}\,\frac{\partial\hat{\omega}_\phi}{\partial\hat{t}} - \nu_\mu\,\frac{\partial}{\partial\hat{r}}\left(\hat{r}\,\frac{\partial\hat{\omega}_\phi}{\partial\hat{r}}\right) + \nu_\phi\,\hat{r}\,\hat{\omega}_\phi = \left(\frac{\epsilon_a}{q_s}\right)^2\frac{\mathcal{A}\,\hat{W}_v^2\,\hat{W}^2}{\hat{W}_0^4}\,\sin\varphi\,\delta(\hat{r}-\hat{r}_s),$$

$$\frac{\partial\hat{\omega}_\theta(0,\hat{t})}{\partial\hat{r}} = \frac{\partial\hat{\omega}_\phi(0,\hat{t})}{\partial\hat{r}} = \hat{\omega}_\theta(1,\hat{t}) = \hat{\omega}_\phi(1,\hat{t}) = 0.$$

IFS Analytic Model

- Width and helical phase of RMP-driven island chain governed by Rutherford equation and no-slip constraint, respectively.
- Poloidal and toroidal angular velocities of plasma governed by respective equations of motion. Equations contain quasi-linear electromagnetic locking torque due to RMP, perpendicular viscosity, and poloidal and toroidal flow damping.
- Measured (and estimated) plasma parameters at 8/2 and 11/2 rational surfaces in DIII-D discharge 15811 used to calculate appropriate parameters in model.
- Amplitude of RMP oscillates sinusoidally at 1 Hz (as in experiment).

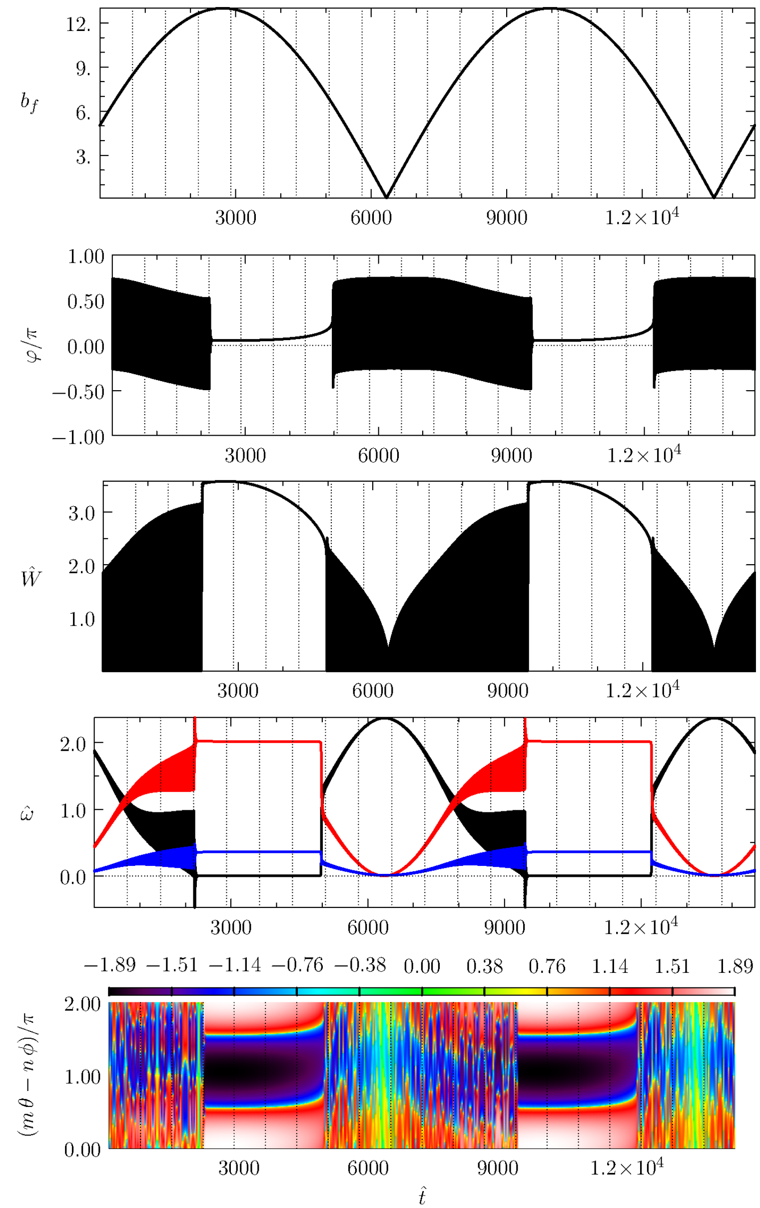
Figure 1: Response at 8/2 rational surface in DIII-D discharge 158115. Panel 1: Applied RMP. Panel 2: Helical phase of reconnected flux. Panel 3: Magnetic island width (normalized to linear layer width). Panel 4: Natural frequency (black); shift in plasma poloidal (red) and toroidal (blue) angular velocity. Panel 5: Simulated Mirnov data. Time normalized to linear reconnection time.



Response at 8/2 Rational Surface

- Model response at $8/2$ surface similar to experimental data.
- Plasma flow causes strong shielding (factor of 120) of RMP-driven reconnected flux.
- Shielding completely breaks down when RMP amplitude exceeds critical value (about 30 gauss at rational surface).
- Breakdown of shielding associated with toroidal velocity shift in co-current direction of $30\,\mathrm{km/s}$, and poloidal velocity shift (strongly localized in vicinity of rational surface) in ion diamagnetic direction of $5\,\mathrm{km/s}$.
- Even in shielded state, island width exceeds linear layer width (which is only about 2.5 mm).

Figure 2: Response at 11/2 rational surface in DIII-D discharge 158115. Panel 1: Applied RMP. Panel 2: Helical phase of reconnected flux. Panel 3: Magnetic island width (normalized to linear layer width). Panel 4: Natural frequency (black); shift in plasma poloidal (red) and toroidal (blue) angular velocity. Panel 5: Simulated Mirnov data. Time normalized to linear reconnection time.



Response at 11/2 Rational Surface

- Model response at $11/2$ surface similar to experimental data.
- Plasma flow insufficient to cause effective shielding of RMP-driven reconnected flux.
- Reconnected flux modulates sinusoidally in sympathy with RMP amplitude.
- Island width always exceeds linear layer width (which is only about $5\,\mathrm{mm}$).

Natural Frequency

- Natural frequency, ω_0, is helical phase velocity of naturally unstable tearing mode at rational surface.
- Expect mode penetration to be facilitated at given rational surface when associated natural frequency close to zero.
- After mode penetration at given rational surface, expect associated natural frequency to be pinned to zero.

Linear Natural Frequency

- According to linear theory:

$$\omega_0 = -n\,(\omega_E + \omega_{*\,e}) \equiv -n\,\omega_{\perp\,e},$$

where n is toroidal mode number, $\omega_E = E_r/(R_0\,B_\theta)$ is $\mathbf{E}\times\mathbf{B}$ frequency, and $\omega_{*\,e} = (dp_e/dr)/(e\,n_e\,R_0\,B_\theta)$ is electron diamagnetic frequency.

Nonlinear Natural Frequency

- According to nonlinear theory:

$$\omega_0 = -n\,\omega_E - n\left(1 - \frac{\eta_i\,\lambda_{\theta\,i}}{1+\eta_i}\right)\omega_{*\,i},$$

  where $\omega_{*\,i} = -(dp_i/dr)/(e\,n_e\,R_0\,B_\theta)$ is ion diamagnetic frequency, and

$$\lambda_{\theta\,i} = \frac{5}{2} - \left[\frac{Z_{\rm eff} + 1/\sqrt{2} - 1}{Z_{\rm eff} + \sqrt{2} - \ln(1+\sqrt{2}) - 1}\right].$$

- For $8/2$ rational surface in DIII-D discharge 158115, $Z_{\rm eff} = 2.5$ and $\eta_i = 1.9$, giving

$$\omega_0 = -n\,(\omega_E + 0.073\,\omega_{*\,i}) \simeq -n\,\omega_E.$$

Experimental Natural Frequency

- Experimental data from DIII-D [a] clearly indicates that natural frequency at top of pedestal similar to $\mathbf{E}\times\mathbf{B}$ frequency,

$$\omega_0 \simeq -n\,\omega_E,$$

  and significantly different from $-n\,\omega_{\perp e}$.

- Nonlinear model (which also predicts $\omega_0 \simeq -n\,\omega_E$ at top of pedestal) much more consistent with experimental data than linear models.

[a] C. Paz-Soldan, et al., NF **59**, 056012 (2019).

Summary

- Analytic model broadly consistent with experimental data.
- Strong shielding of reconnected magnetic flux at top of pedestal; almost no shielding at bottom.
- Driven island width exceeds linear layer width even in strongly shielded state. Must use nonlinear, rather than linear, theory to model plasma response.
- Nonlinear theory predicts natural frequency at top of pedestal that is consistent with experimental data. Linear theory predicts wrong natural frequency.

m	n	B_ϕ	R_0	a	n_e	T_e	T_i	η_i	$Z_{\rm eff}$
-8	2	-1.94	1.75	0.93	2.8	1.4	1.4	1.9	2.5
-11	2	-1.94	1.75	0.93	0.75	0.12	0.12	1.8	2.5
$\chi_\perp$	ω_E	$\omega_{\ast e}$	$\hat{r}_s$	s	M	Δ	$\mathcal{A}$		
1.0	-21.3	-21.5	0.853	2.3	2.0	1.0	1.0		
1.0	-9.8	-12.8	0.974	11.2	2.0	1.0	1.0		

Measured and estimated physics parameters at two rational surfaces in pedestal of DIII-D discharge 158115. m - poloidal mode number, n - toroidal mode number, B_ϕ - toroidal magnetic field (T), R_0 - major radius (m), a - minor radius (m), n_e - electron number density ($10^{19}\,{\rm m}^{-3}$), T_e - electron temperature (keV), T_i - ion temperature (keV), $\eta_i = d\ln T_i/d\ln n_e$, $Z_{\rm eff}$ - conventional measure of impurity content, $\chi_\perp$ - perpendicular momentum diffusivity (${\rm m}^2\,{\rm s}^{-1}$), ω_E - ${\bf E}\times{\bf B}$ frequency (${\rm krad\,s}^{-1}$), $\omega_{\ast e}$ - electron diamagnetic frequency (${\rm krad\,s}^{-1}$), $\hat{r}_s$ – rational surface radius normalized to the plasma minor radius, s - magnetic shear, M - majority ion mass number, $\Delta \equiv \Delta' r_s/(2\,m)$, $\mathcal{A}$ - amplification factor.