Ptolemy's Almagest: Fact and Fiction

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Timeline:

Aristotle of Stagira: 384-322 BCE: Author of *On the Heavens,* which constituted philosophical framework of ancient Greek astronomy.

Erastothenes of Cyrene: 267-195 BCE: Credited with first reasonably accurate estimate of Earth's radius.

Hipparchus of Nicaea: 190-120 BCE: Credited with explaining the different lengths of the seasons by displacing the Earth from the center of the Sun's apparent geocentric orbit.

Geminus: 1st century BCE: Author of *Introduction to the Phenomena,* which is first work to describe the model of Hipparchus.

Claudius Ptolemy of Alexandria: 90-168 CE: Author of the *Almagest*. Credited with invention of equant.

References:

The Norton History of Astronomy and Cosmology, John North (Norton 1994).
The History and Practice of Ancient Astronomy, James Evans (Oxford 1998).
Greek Astronomy, T.S. Heath (Dover 1991).
The Copernican Revolution, T.S. Kuhn (Harvard 1957).
Ptolemy's Almagest, G.J. Toomer (tr.) (Princeton 1998).
The Exact Sciences in Antiquity, O. Neugebauer (Dover 1969).

Euclid's Elements and Ptolemy's Almagest

- Ancient Greece -> Two major scientific works: Euclid's *Elements* and Ptolemy's "*Almagest*".
- *Elements* -> Compendium of mathematical theorems concerning geometry, proportion, number theory. Still highly regarded.
- *Almagest* -> Comprehensive treatise on ancient Greek astronomy. (Almost) universally disparaged.

The modern world inherited two major scientific works from the civilization of ancient Greece. The first is *Euclid's Elements*, which is a large compendium of mathematical theorems regarding geometry, proportion, and number theory. The second is the *"Almagest"* of Claudius Ptolemy, which is a comprehensive treatise on ancient Greek astronomy. By the way, I've put Almagest in inverted commas, because the true name of this book is *Syntaxis Mathematica* (which means *"mathematical treatise"*). "Almagest" is an Arabic corruption of its Greek nickname: *"H Megeste"* (which means *"the greatest"*—presumably, the greatest treatise). This nickname gives some idea of the reputation of the Almagest in antiquity.

Euclid's Elements is still held in high regard by scientists and mathematicians. After all, this work was the standard school textbook on geometry up to about 100 years ago. Moreover, the plane geometry that we teach school kids nowadays is essentially a watered down version of that presented in the Elements.

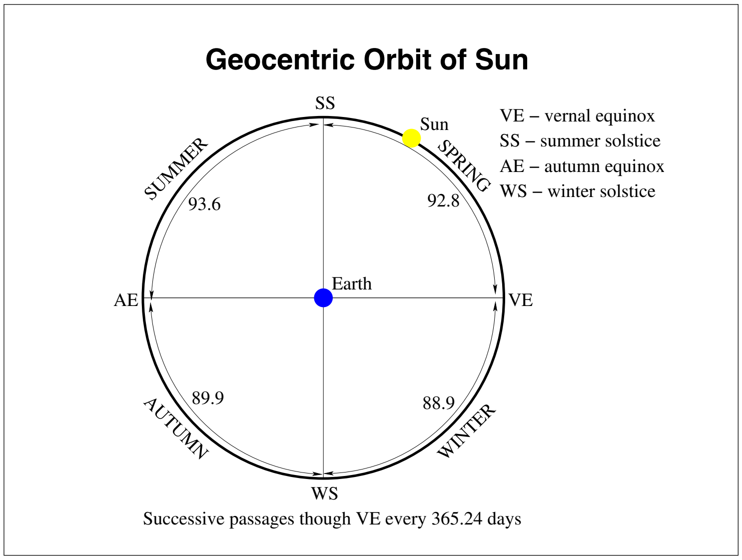
The scientific reputation of Ptolemy's Almagest has not fared as well as that of the Elements. Obviously, Ptolemy's model of the solar system has been completely superseded by that of Copernicus and Kepler, and is no longer taught. Moreover, references to the Almagest in school and college textbooks are brief, and almost uniformly disparaging in nature.

Popular Modern Criticisms of Ptolemy's Almagest

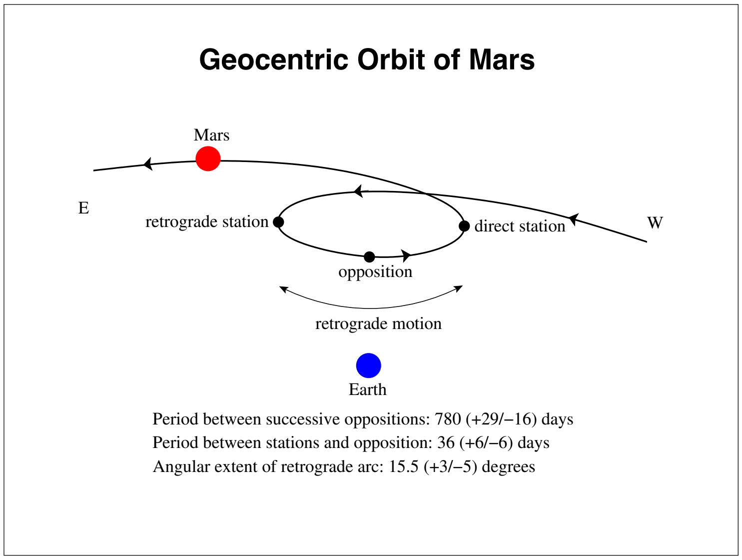
- Ptolemy's approach shackled by Aristotelian philosophy -> Earth *stationary*; celestial bodies move *uniformly* around *circular* orbits.
- Mental shackles lead directly to introduction of *epicycle* as kludge to explain retrograde motion without having to admit that Earth moves.
- Ptolemy's model *inaccurate*. Lead later astronomers to add more and more epicycles to obtain better agreement with observations.
- Final model hopelessly unwieldy. Essentially collapsed under own weight, leaving field clear for Copernicus.

The standard popular modern criticisms of Ptolemy's model of the solar system are as follows. First, it is generally thought that Ptolemy's thinking was shackled by accepted truths in ancient Greek philosophy (mostly due to Aristotle), which held, amongst other things, that the Earth was *stationary*, and that celestial bodies were constrained to move *uniformly* around *circular* orbits. Second, it is supposed that these mental shackles directly lead Ptolemy to introduce the concept of an *epicycle* as a sort of kludge to explain the observed retrograde motion of the superior planets without having to admit that this phenomenon was caused by the Earth's motion. Third, it is generally held that Ptolemy's model of the solar system was not particularly accurate, leading later Arabic and medieval European astronomers to add more and more epicycles in order to get better agreement with observations. The final version of the model is alleged to have contained an absurd number of epicycles, and to have essentially collapsed under its own weight, leaving the field clear for Copernicus and his, supposedly, much simpler, and much more accurate, heliocentric model of the solar system.

Needless to say, the popular criticisms of the Almagest that I have just outlined are almost entirely wrong. What I want to do in this talk is to describe what Ptolemy actually did in the Almagest, and to contrast this with the mistaken popular view of what he did.



Before discussing Ptolemy's model of the solar system, in detail, we need to understand the observational data that the model is intended to account for. Let us start with the apparent orbit of the Sun around the Earth. The figure shows the apparent orbit of the Sun seen from the north. The motion of the Sun is counter-clockwise. There are four cardinal points on this orbit. The *vernal equinox* is the point at which the Sun passes through the extension of the Earth's equatorial plane from south to north. The *summer solstice* is the point at which the Sun attains its most northern altitude above the Earth's equatorial plane. The *autumn equinox* is the point at which the Sun passes through the equatorial plane from north to south, and the *winter solstice* is the point at which the Sun attains its most southern altitude below the equatorial plane. Ancient Greek astronomers were able to determine the times at which the Sun passes though the cardinal points on its orbit to an accuracy of about half an hour. In fact, the Almagest explains how the times of the equinoxes were determined. They found that the period between successive passages of the Sun through the vernal equinox is always 365.24 days: this period is known as a *tropical year*. However, they also noticed that there are small irregularities in the Sun's orbit. It takes 92.8 days for the Sun to travel from the vernal equinox to the summer solstice. In other words, the season known as *Spring* is 92.8 days long. It takes 93.6 days for the Sun to travel from the summer solstice to the autumn equinox. In other words, *Summer* is 93.6 days long. It takes 89.9 days for the Sun to travel from the autumn equinox to the winter solstice, and its takes 88.9 days for the Sun to travel from the winter solstice to the vernal equinox. In other words, *Autumn* is 89.9 days long, and *Winter* is 88.9 days long. Of course, we now understand that these irregularities are a consequence of the fact that the Earth's orbit around the Sun is not a concentric circle, but rather an eccentric ellipse. Incidentally, the figures that I have quoted for the lengths of the seasons are the modern figures, which are not the same as those quoted in the Almagest. The reason for this is that the Earth's orbit about the Sun has changed somewhat since Ptolemy's time. So, the tasks facing ancient Greek astronomers when trying to explain the Sun's orbit about the Earth were twofold. First, they had to account for the regular features of the orbit: that is, the fact that the orbit repeats every tropical year. Second, and just as important, they had to to account for the slight irregularities in the orbit: that is, the fact that the seasons are not equally long



Let us now consider the geocentric orbit of a typical superior planet, such as Mars. Of course, a *superior planet* is one whose orbit (in the Copernican model) is outside that of the Earth. Generally speaking, Mars travels from west to east—that is, in the same direction as the Sun—against the backdrop of the fixed stars. This type of motion is known as *direct motion*. However, about once every two years, Mars performs a little loop the loop relative to the stars. This means that, for a brief period of time, Mars is traveling from east to west. This type of motion is known as *retrograde motion*. By the way, all the superior planets exhibit brief periods of retrograde motion. There are three cardinal points on the orbit of Mars. The *opposition* lies, more or less, at the center of the retrograde arc. At this point, Mars is directly opposite the Sun, relative to the Earth, which means that it is making its closest approach to the Earth. The two *stations* are the points at which the orbit of Mars, relative to the stars, changes direction, which means that Mars appears briefly stationary with respect to the stars at these points. The *retrograde station* is the point at which Mars switches from direct to retrograde motion, and the *direct station* is the point at which it switches back to direct motion. Ancient Greek astronomers were able to measure the positions of the martian cardinal points, as well as the times at which Mars passes though these points, to fairly good accuracy. They found that the average period between successive passages of Mars though its opposition is 780 days. However, the actual period can exceed this value by up to about 29 days, and can fall below it by about 16 days. Likewise, the mean period between passage through the retrograde station and the opposition, or the opposition and the direct station, is 36 days. Again, however, the actual period can exceed, or fall below, this value by up to 6 days. Finally, ancient Greek astronomers found that, on average, the angular size of Mars' retrograde arc is 15.5 degrees. However, the arc size can actually exceed this value by up to 3 degrees, and fall below it by up to 5 degrees. So, again, the challenges facing ancient Greek astronomers, when trying to model the geocentric orbit of Mars, were, first, to account for the regular features of the orbit, and, second, to account for the relatively small irregular features. Of course, we now understand that the irregular features of the orbit are a consequence of the fact that the heliocentric orbits of the Earth and Mars are not concentric circles, but are, instead, eccentric ellipses.

Incidentally, for the sake of brevity, I am not going to discuss the orbits of the inferior planets (that is, Mercury and Venus) in this talk.

Immovability of Earth

- By time of Aristotle, ancient Greeks knew that Earth is spherical. Also, had good estimate of its radius.
- Ancient Greeks calculated that if Earth rotates once every 24 hours then person standing on equator moves west to east at about 1000 mph.
- Aristotle's *On the Heavens* -> 1000 mph wind blowing east to west. Projectiles throw westward travel much further than those thrown eastward, et cetera.
- Aristotle -> Motion of Earth in space would generate *stellar parallax*. Not detectable (by naked eye).
- These arguments are not unreasonable, but we now know them to be mistaken. Atmosphere co-rotates with Earth because of friction and inertia. Projectiles also co-rotate with Earth because of inertia. Stellar parallax undetectable by naked eye because of great distances of stars from Earth.
- Moot point because Earth appears stationary to observer standing on it.

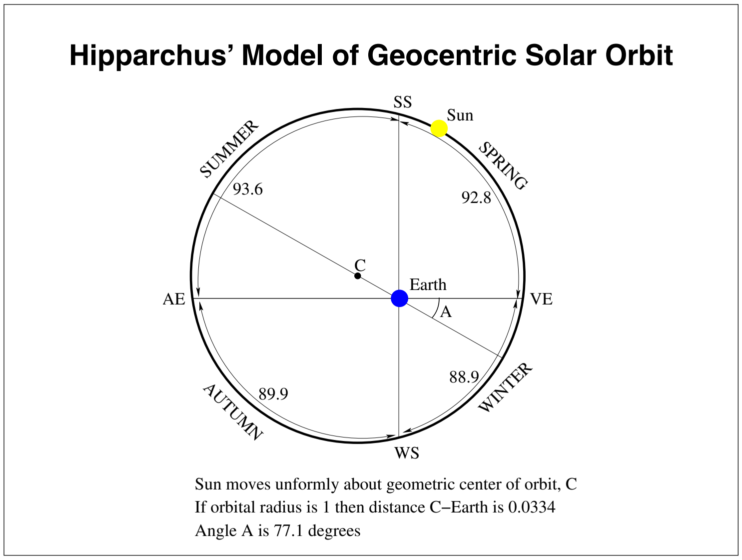
All scientists, when investigating a given phenomenon, have certain preconceptions about what they expect to find. Ancient Greek astronomers were no exception. Their preconceptions were derived from ancient Greek philosophy—in particular, the work of Aristotle. (It must be appreciated that ancient Greek astronomy did not reach its heyday until about a hundred years after the death of Aristotle.) According to Aristotelian philosophy, the Earth is *stationary*—that is, it neither rotates about its axis nor moves through space. Furthermore, celestial objects are constrained to move *uniformly* around *circular* orbits. Let us briefly examine these preconceptions, one by one.

By the time of Aristotle, the ancient Greeks knew that the Earth was spherical, and also had a fairly good estimate of its radius. Thus, they were able to calculate that if the Earth were rotating about its axis once per day then a person standing on the equator would be moving from west to east at about 1000 miles per hour. As described in Aristotle's work *On the Heavens*, the ancient Greeks reasoned that such a large velocity would have dire consequences. For instance, there would be a 1000 mile per hour wind blowing from east to west, projectiles thrown westward would travel large distances, while projectiles thrown eastward would hardly go any distance, and might even go backward. Aristotle also argued that any motion of the Earth through space would cause a shift in the apparent positions of the stars in the sky—an effect known as *stellar parallax*. However, this effect is not detectable (at least, by the naked eye.) These arguments are not unreasonable. However, we now know them to be incorrect. The Earth's atmosphere co-rotates with the Earth as a consequence of friction and inertia, so there is no 1000 mile an hour wind. Moreover, projectiles share the rotational motion of the Earth, because of inertia, so there is no great difference between their trajectories when thrown eastward and westward. Incidentally, there actually are observable consequences of the Earth's rotation, but these are much less dramatic than Aristotle supposed. Stellar parallax is a real effect, but it is far too small to be detected by the naked eye, due to the very large distances of the stars from the Earth. These distances are much greater than the ancient Greeks supposed. Finally, it should be noted that, when attempting to construct a model of the motions of the Sun and the planets, seen relative to the Earth, the stationarity, or otherwise, of the Earth is somewhat of a moot point, because the Earth appears stationary to an observer standing upon it.

Necessity for Uniform Circular Motion of Celestial Bodies

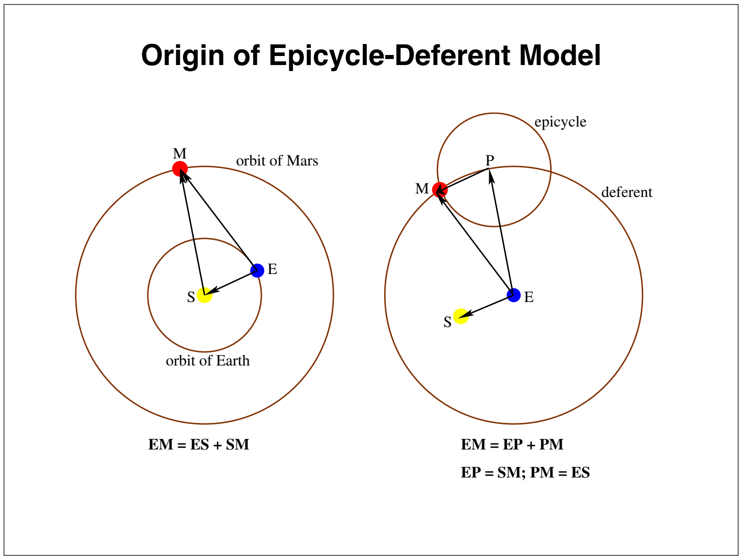
- Aristotle's *On the Heavens* -> heavens (i.e., region beyond lunar orbit) and heavenly bodies *eternal* and *immutable*.
- Eternal immutable bodies must be *perfect*. (Imperfect bodies would eventually change and ultimately disintegrate.)
- Circles are most perfect closed geometric figure -> celestial orbits are *circular*.
- Celestial bodies must move *uniformly* around their circular orbits. Non-uniform motion imperfect -> could not be eternal.

According to Aristotle's treatise *On the Heavens*, the heavens—in other words, the region situated outside the geocentric orbit of the Moon—as well as heavenly bodies, are both *eternal* and *immutable*. Aristotle presumably came to this conclusion because the Greeks, and the Babylonians and Egyptians before them, never observed any permanent or temporary changes in the heavens. (Incidentally, they regarded comets as atmospheric phenomena.) On the other hand, they observed changes—for instance, growth and decay—on the Earth all the time. Aristotle reasoned that an external and immutable body must be *perfect*, because any imperfection would eventually cause it to change and ultimately disintegrate. Thus, because a circle is the most perfect closed geometric figure, a celestial body must move in a *circle*. Everybody knows this. What is less well known is the fact that Aristotle also insisted that the circular motions of celestial bodies must be *uniform* in nature. He reasoned that if celestial bodies were continually speeding up and slowing down then these variations would eventually build up and destroy the motion. By analogy, if we saw a rotating flywheel that was wobbling then we would likely expect the wobbles to gradually get worse and worse until something disastrous happened.



The model for the geocentric orbit of the Sun described in the Almagest actually predates the Almagest by about 300 years, and is generally credited to *Hipparchus of Nicaea*. According to this model, the Sun moves uniformly about a circular orbit of center C, say. But, the Earth is shifted from the center of the orbit, as shown in the figure. This shift leads to a shift of the cardinal points on the orbit: that is, the vernal equinox, summer solstice, autumn equinox, and winter solstice. The period between successive passages through the vernal equinox is still 365.24 days. However, the lengths of the seasons are now in proportion to the lengths of the arcs between the relevant two cardinal points, and these arc lengths are no longer equal, because the Earth is no longer at the center of the orbit. The ancient Greeks calculated that, supposing the radius of the orbit to be 1 unit, the magnitude of the shift needs to be 0.0334, and the angle A, shown in the figure, which determines the direction of the shift, needs to be 77.1 degrees. This magnitude and direction of the shift gives rise to correct predictions for the lengths of all the seasons. Here, I am, again, giving the modern figures for the magnitude and direction of the shift. The figures quoted in the Almagest are different, because the Earth's orbit about the Sun has changed somewhat since Ptolemy's time. The Almagest model of the geocentric solar orbit is surprisingly accurate. It can predict the position of the Sun, relative to the stars, to an accuracy of about 1 arc minute. That is, about 1/60 th of a degree.

We can see that, in order to take account of the slightly different lengths of the seasons, the ancient Greeks had to slightly mar the prefect symmetry of their model of the heavens by making the Sun's geocentric orbit an eccentric circle. However, the orbit is, at least, still circular, and, more importantly, the Sun's motion around it is still uniform, relative to the center of the circle. Hence, there is no direct conflict with Aristotelian philosophy.

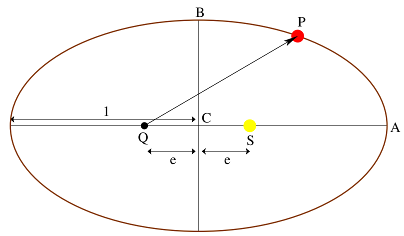


Let now consider the origin of Ptolemy's epicycle-deferent model. The left diagram shows a heliocentric model of the solar system seen from the north. All orbital motion is counter-clockwise. To determine the position of Mars. denoted M, relative to the Earth, denoted E, we move from the Earth to the Sun, denoted S, and then from the Sun to Mars. In other words, in vector notion, the Earth-Mars vector, EM, is the sum of the Earth-Sun vector, ES, and the Sun-Mars vector, SM. However, one thing we know about vector addition is that it does not matter in which order we add the component vectors. In other words, if EM equals ES plus SM then EM also equals SM plus ES. This leads to the picture shown on the right. Mars moves around a circular *epicycle* whose center P moves around a larger circular *deferent* centered on the Earth. Moreover, the vector EP is equal to the vector SM—in other words, P stands to the Earth as Mars stands to the Sun—and the vector PM is equal to the vector ES—in other words, Mars stands to P as the Sun stands to the Earth. Of course, when observed from the Earth, the apparent motion of Mars consists of two components—the first is the actual motion of Mars around the Sun, and the second is the actual motion of the Earth around the Sun. These motions are represented by the deferent and the epicycle, respectively, in Ptolemy's model. Once we understand this, we can immediately appreciate that adding additional epicycles to the model would be pointless—this would be equivalent to giving Mars a third, completely spurious, apparent motion. So, why did Arabic and Medieval European astronomers add more and more epicycles to Ptolemy's model? Actually, they didn't. This story is a complete myth.

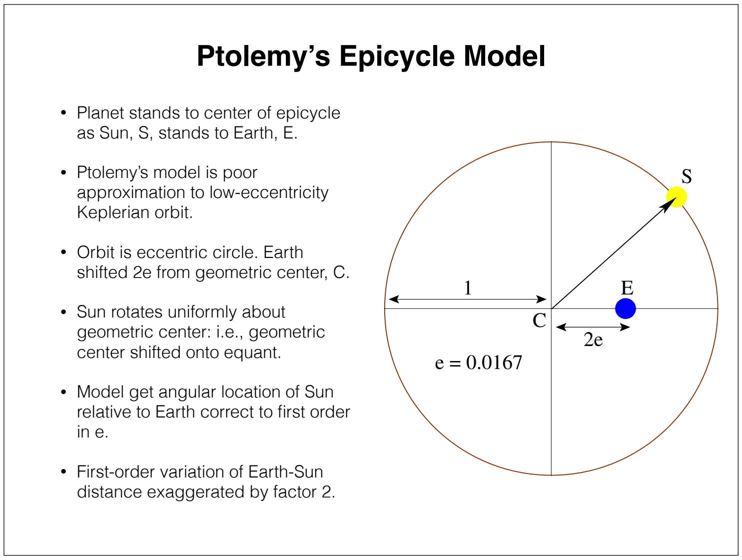
You might think that we have now come to the end of our story. By analogy with Hipparchus' model for the geocentric solar orbit, which you will recall is pretty accurate, all we need to do, in order to take the slight irregularities of the martian orbit into account, is to appropriately displace the Earth from the center of the deferent, displace the point P from the center of the epicycle, and then have P and M rotate uniformly relative to the geometric centers of the deferent and the epicycle, respectively. Unfortunately, this scheme does not work very well. In particular, it is incapable of accounting for the observed variations in the angular size of Mars' retrograde arc. In order to understand why this approach fails, we need to learn a little about the true orbits of the planets, which are, of course, Keplerian ellipses.

Keplerian Orbit

- If mean radius unity then CS = e, where e is orbital *eccentricity*. e is small compared to unity.
- Difference between CA and CB second-order in e. Orbit *circular* to first-order.
- SP sweeps out equal areas in equal time intervals. Motion of P is non-uniform, to first-order in e, about either S or C, but is uniform about *equant*, Q



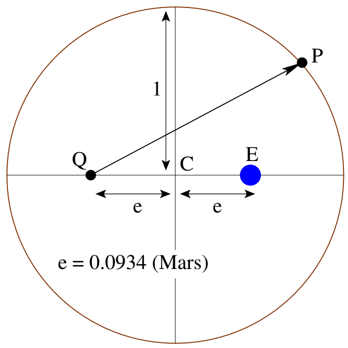
Consider a planet, denoted P, in a *Keplerian orbit* around the Sun, denoted S. As is well known, the shape of the orbit is an *ellipse*. However, the Sun is not located at the geometric center of the ellipse, denoted C. Instead, supposing that the mean radius of the ellipse is 1 unit, the Sun is displaced along the ellipse's major axis a distance e, where e is denoted the *eccentricity* of the orbit, and is generally very much less than unity. For instance, the eccentricity of the Earth's orbit is 0.0167, whereas that of Mars' orbit is 0.0934. The difference between the major radius of the ellipse, CA, and its minor radius, CB, is second order in e. In other words, it is proportional to e-squared, which implies that the ellipse is virtually indistinguishable from a circle. This leads to the important conclusion that—far from being a major weakness, as is generally supposed—Ptolemy's assumption that celestial bodies move in circular orbits is the main strength of his model. According to Kepler's second law of planetary motion, the line SP sweeps out equal areas in equal time intervals. This means that, to first order in e, the motion of P is non-uniform about both the Sun, S, and the geometric center of the orbit, C, but appears uniform about a point, known as the *equant*, Q, which is diagrammatically opposite the Sun relative to the center.



We have seen that a superior planet stands to the center of its epicycle as the Sun stands to the Earth. Thus, if we can model the motion of the Sun relative to the Earth then we can use this to determine the motion of a planet relative to the center of its epicycle. Ptolemy's model of the geocentric solar orbit can be recognized as a relatively poor approximation to a low-eccentricity Keplerian ellipse that represents the actual orbit of the Earth around the Sun. The orbit is circular because a low-eccentricity Keplerian ellipse is a circle to very high accuracy. Suppose that radius of the orbit is 1 unit. The Earth, E, is shifted a distance 2e from the geometric center of the orbit, C, where e is the eccentricity of the Earth's heliocentric orbit. Given that e equals 0.0167, this means that the appropriate shift is 0.0334 (as we saw in Slide 8). Finally, the Sun, S, rotates uniformly around the point C. In other words, the geometric center of the orbit has been shifted onto the equant. This simple scheme gets the angular position of the Sun relative to the Earth correct to first order in e. Unfortunately, it exaggerates the first-order variation in the Earth-Sun distance by a factor of 2. However, this does not really matter, because the only thing that the Earth-Sun distance affects is the apparent angular size of the Sun, which the ancient Greeks could not measure to any accuracy. For the case of an epicycle, the exaggerated variation of the radial distance of the planet from the epicycle center introduces an error into the model. However, if the radius of the epicycle is small compared to the radius of the deferent, as is the case for all of the superior planets, then this error is relatively small. (Actually, somewhat inconsistently, Ptolemy did not include the 2e shift in his epicycle model. This leads to an error of similar magnitude to that involved in using a Hipparchian orbit.)

Ptolemy's Deferent Model

- Center of epicycle, P, stands to Earth, E, as planet stands to Sun.
- Ptolemy's model is good approximation to low-eccentricity Keplerian orbit.
- Orbit is eccentric circle. Earth shifted e from geometric center, C.
- P rotates uniformly around *equant*, Q, which is geometrically opposite E w.r.t. C.
- Model gets both relative angular location and relative distance of EP correct to first order in e. No other placement of Q or E does better job.



We have seen that the center of the epicycle, P, of a superior planet stands to the Earth, E, as the planet in question stands to the Sun. However, in this case, it is vitally important to accurately represent the variation of distance EP, as well as the angular position of P relative to E. The reason for this is that the distance EP affects the apparent angular size of the epicycle seen from the Earth. Thus, if, for instance, we exaggerated the variation of this distance by a factor 2, as we did on the previous slide, then we would also exaggerate the variation of the size of the planet's retrograde arcs by the same factor, which would not agree with observations. Ptolemy's method of solving this problem is, in effect, to make the deferent a relatively good approximation to a low-eccentricity Keplerian ellipse. The deferent is circular because it represents the low-eccentricity orbit of the planet in question about the Sun, and such an orbit is circular to high accuracy. Suppose that the radius of the orbit is 1 unit. The Earth is shifted a distance e from the geometric center of the orbit, C. However, the center of the epicycle rotates uniformly about another point, Q, which is geometrically opposite the Earth relative to C. As has already been mentioned, this point is known as the *equant*. (The name derives from the Latin *punctum aequans.)* This scheme gets the angular position of P, relative to E, as well as the distance EP, correct to first order in e. Furthermore, it can be demonstrated that no other placement of the Earth, or the equant, gives rise to a better approximation (i.e., a lower second-order error) to the orbit. In other words, this is the optimum scheme—at least, to second order in the eccentricity. The invention of the equant is Ptolemy's greatest claim to fame. Of course, the fact that P is rotating uniformly about Q means that it is not moving uniformly around the deferent. In other words, this scheme is in serious conflict with the tenants of Aristotelian philosophy, which hold that heavenly motion must be uniform. In fact, Ptolemy was roundly criticized (on philosophical grounds) by later Arabic and medieval European astronomers, including Copernicus, for this feature of his model. They introduced many other schemes whose purpose was to do away with the equant, and to revert to a model in which the motion of heavenly bodies was a superposition of various uniform circular motions. These schemes, including Copernicus', are all simultaneously more complicated, and less accurate, than Ptolemy's. Incidentally, when Ptolemy's model is applied to Mars, it is capable of predicting the position of this planet, relative to the fixed stars, with a maximum error of about 14 arc minutes. In other words, about half the apparent size of the Moon's disk. However, the mean error is much smaller than this.

Summary and Conclusions

- Ptolemy’s thinking not completely shackled by Aristotelian philosophy.
  Fact that model is geocentric irrelevant, because purpose of model is to determine positions of celestial bodies relative to Earth.
  Constraint that deferents and epicycles must be circular actually excellent approximation.
  Ptolemy introduced equant, in direct violation of Aristotle’s maxim of uniform heavenly motion, because this was only simple way of getting agreement with observations.
- Epicycle of superior planet not a kludge -> represents Earth’s orbit around Sun, just as deferent represents planet’s orbit around Sun.
- Ptolemy’s model actually very accurate. Certainly sufficient for naked eye observations.
- Story that later astronomers had to add more and more epicycles to Ptolemy’s model to get decent agreement with observations has no basis in fact.

There are a number of final points that I would like to make.

First, it is not true that Ptolemy’s thinking was completely shackled by Aristotelian philosophy. As I have already mentioned, the fact that his model of the solar system is geocentric is somewhat of a moot point, because the purpose of the model is to determine the positions of heavenly bodies relative to the Earth, and to an observed standing on the Earth, the Earth appears stationary. The constraint that the epicycles and deferents in the model must be circular actually turns out to be an excellent approximation, because low-eccentricity Keplerian orbits are circular to high accuracy. Finally, Ptolemy introduced the equant into his model, in direct violation of Aristotle’s maxim that heavenly bodies must move uniformly around circular orbits, because this was the only simple way he could get his model to agree with observations. It is worth noting that no subsequent astronomer, even Copernicus, was able to come up with an more accurate approximation to a Keplerian orbit until Kepler himself.

Second, the introduction of the epicycle into Ptolemy’s model is not a kludge whose purpose is to avoid admitting that the Earth moves. The epicycle of a superior planet actually represents the Earth’s orbit around the Sun, just as the deferent represents the planet’s orbit around the Sun.

Third, Ptolemy’s model is surprisingly accurate. It is certainly sufficiently accurate to give excellent agreement with naked eye observations. It is worth noting that Kepler inherited naked eye data from Tycho Brahe of hitherto unprecedented accuracy, and this was still only just good enough to for him to detect a slight disagreement between the predictions of Ptolemy’s model and the observed orbit of Mars (which has a particularly high eccentricity).

Finally, the story that later astronomers has to add more and more epicycles to Ptolemy’s model to get decent agreement with observations has no basis in fact, and is actually completely ridiculous.