Euclid’s Elements and Ptolemy’s Almagest

- Ancient Greece -> Two major scientific works: Euclid’s *Elements* and Ptolemy’s “*Almagest*”.

- *Elements* -> Compendium of mathematical theorems concerning geometry, proportion, number theory. Still highly regarded.

- *Almagest* -> Comprehensive treatise on ancient Greek astronomy. (Almost) universally disparaged.
Popular Modern Criticisms of Ptolemy’s Almagest

• Ptolemy’s approach shackled by Aristotelian philosophy -> Earth stationary; celestial bodies move uniformly around circular orbits.

• Mental shackles lead directly to introduction of epicycle as kludge to explain retrograde motion without having to admit that Earth moves.

• Ptolemy’s model inaccurate. Lead later astronomers to add more and more epicycles to obtain better agreement with observations.

• Final model hopelessly unwieldy. Essentially collapsed under own weight, leaving field clear for Copernicus.

Geocentric Orbit of Sun

Successive passages though VE every 365.24 days
Geocentric Orbit of Mars

Period between successive oppositions: 780 (+29/−16) days
Period between stations and opposition: 36 (+6/−6) days
Angular extent of retrograde arc: 15.5 (+3/−5) degrees

Immovability of Earth

• By time of Aristotle, ancient Greeks knew that Earth is spherical. Also, had good estimate of its radius.

• Ancient Greeks calculated that if Earth rotates once every 24 hours then person standing on equator moves west to east at about 1000 mph.

• Aristotle’s *On the Heavens* -> 1000 mph wind blowing east to west. Projectiles throw westward travel much further than those thrown eastward, et cetera.

• Aristotle -> Motion of Earth in space would generate stellar parallax. Not detectable (by naked eye).

• These arguments are not unreasonable, but we now know them to be mistaken. Atmosphere co-rotates with Earth because of friction and inertia. Projectiles also co-rotate with Earth because of inertia. Stellar parallax undetectable by naked eye because of great distances of stars from Earth.

• Moot point because Earth appears stationary to observer standing on it.
Necessity for Uniform Circular Motion of Celestial Bodies

- Aristotle’s *On the Heavens* -> heavens (i.e., region beyond lunar orbit) and heavenly bodies *eternal* and *immutable*.

- Eternal immutable bodies must be *perfect*. (Imperfect bodies would eventually change and ultimately disintegrate.)

- Circles are most perfect closed geometric figure -> celestial orbits are *circular*.

- Celestial bodies must move *uniformly* around their circular orbits. Non-uniform motion imperfect -> could not be eternal.

Hipparchus’ Model of Geocentric Solar Orbit

Sun moves uniformly about geometric center of orbit, C
If orbital radius is 1 then distance C–Earth is 0.0334
Angle A is 77.1 degrees
Origin of Epicycle-Deferent Model

- If mean radius unity then CS = e, where e is orbital eccentricity. e is small compared to unity.
- Difference between CA and CB second-order in e. Orbit circular to first-order.
- SP sweeps out equal areas in equal time intervals. Motion of P is non-uniform, to first-order in e, about either S or C, but is uniform about equant, Q

Keplerian Orbit
Ptolemy’s Epicycle Model

- Planet stands to center of epicycle as Sun, S, stands to Earth, E.
- Ptolemy’s model is poor approximation to low-eccentricity Keplerian orbit.
- Orbit is eccentric circle. Earth shifted $2e$ from geometric center, C.
- Sun rotates uniformly about geometric center: i.e., geometric center shifted onto equant.
- Model get angular location of Sun relative to Earth correct to first order in $e$.
- First-order variation of Earth-Sun distance exaggerated by factor 2.

Ptolemy’s Deferent Model

- Center of epicycle, P, stands to Earth, E, as planet stands to Sun.
- Ptolemy’s model is good approximation to low-eccentricity Keplerian orbit.
- Orbit is eccentric circle. Earth shifted $e$ from geometric center, C.
- P rotates uniformly around equant, Q, which is geometrically opposite E w.r.t. C.
- Model gets both relative angular location and relative distance of EP correct to first order in $e$. No other placement of Q or E does better job.
Summary and Conclusions

• Ptolemy’s thinking not completely shackled by Aristotelian philosophy. Fact that model is geocentric irrelevant, because purpose of model is to determine positions of celestial bodies relative to Earth. Constraint that deferents and epicycles must be circular actually excellent approximation. Ptolemy introduced equant, in direct violation of Aristotle’s maxim of uniform heavenly motion, because this was only simple way of getting agreement with observations.

• Epicycle of superior planet not a kludge -> represents Earth’s orbit around Sun, just as deferent represents planet’s orbit around Sun.

• Ptolemy’s model actually very accurate. Certainly sufficient for naked eye observations.

• Story that later astronomers had to add more and more epicycles to Ptolemy’s model to get decent agreement with observations has no basis in fact.