

Dynamical plasma response of resistive wall modes to changing external magnetic perturbations^{a)}

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The plasma response to external resonant magnetic perturbations is measured as a function of stability of the resistive wall mode (RWM). The magnetic perturbations are produced with a flexible, high-speed waveform generator that is preprogrammed to drive an in-vessel array of 30 independent control coils and to produce an $m/n = 3/1$ helical field. Both quasi-static and “phase-flip” magnetic perturbations are applied to time-evolving discharges in order to observe the dynamical response of the plasma as a function of RWM stability. The evolving stability of the RWM is estimated using equilibrium reconstructions and ideal stability computations, facilitating comparison with theory. The plasma resonant response depends upon the evolution of the edge safety factor, q^* , and the plasma rotation. For discharges adjusted to maintain relatively constant edge safety factor, $q^* < 3$, the amplitude of the plasma response to a quasistatic field perturbation does not vary strongly near marginal stability and is consistent with the Fitzpatrick–Aydemir equations with high viscous dissipation. Applying “phase-flip” magnetic perturbations that rapidly change toroidal phase by 180° allows observation of the time scale for the plasma response to realign with the applied perturbation. This phase realignment time increases at marginal stability, as predicted by theory. This effect is easily measured and suggests that the response to time-varying external field perturbations may be used to detect the approach to RWM instability. © 2004 American Institute of Physics. [DOI: 10.1063/1.1688793]

I. INTRODUCTION

In order to increase plasma pressure and the attractiveness of toroidal fusion reactors, a conducting wall is placed near the plasma surface.^{1–3} This prevents the growth of fast ideal external kink modes,^{4,5} which are stabilized by the opposing fields from helical eddy currents induced in the wall.⁶ Due to the finite conductivity of a real wall, eddy currents decay on the timescale of magnetic field diffusion through the wall, τ_w , allowing the growth of the resistive wall mode (RWM) at a rate inversely proportional to τ_w .^{7,8} Previous experimental studies in high beta tokamaks^{9,10} and spherical tori¹¹ have found that the resistive wall mode can be stabilized by sustained plasma rotation. Numerical^{12,13} and analytical models^{14,15,16} show that rotational stabilization of the RWM is a consequence of plasma dissipation. The physical mechanisms responsible for the dissipation remain uncertain.^{15–21} The plasma response to external magnetic perturbations near RWM marginal stability, also known as error field amplification,^{22,23} is a key phenomenon in recent

models that illustrates the effects of both plasma rotation and dissipation and that have been a focus of recent RWM experiments.²⁴

The present paper reports a recent experimental investigation of the RWM in the High-Beta-Tokamak-Extended-Pulse (HBT-EP) device^{5,6} that includes both the study of naturally growing RWM instability and measurements of plasma response to external resonant perturbations near the threshold of RWM instability.^{23,24} In order to interpret the experimental measurements and physics of RWM in HBT-EP plasmas, we solved the model equations for RWM dynamics developed by Fitzpatrick and Aydemir.^{15,16} The model provides a set of coupled ordinary differential equations for the dynamical evolution of the perturbed plasma flux, wall flux, and mode rotation rate. These equations are solved by specifying several known geometric coupling terms, the wall time constant, τ_w , and three plasma parameters: (1) the normalized stability parameter for the RWM, \bar{S} , (2) the natural plasma rotation, Ω_0 , and (3) a plasma dissipation parameter, ν_d , that Fitzpatrick and Aydemir model as viscosity near the plasma edge. Specifying the normalized stability parameter requires determination of the ideal MHD instability thresholds both with a perfectly conducting wall and without a wall, and this has been verified experimentally in HBT-EP when the position of the conducting wall is adjusted.^{5,6}

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The remainder of this article is organized into four sections. First, the HBT-EP experiment, the observed RWM instability, and the control coils used to impose external magnetic perturbations are described. Discharge programming is used to control the edge safety factor, q^* , and the rate of approach to marginal stability of the RWM. This technique allowed a systematic study of the plasma response to external resonant perturbations in HBT-EP when the RWM was either stable, marginally stable, or unstable. In particular, experiments were designed to reveal how the plasma responds to external perturbations near marginal stability and the degree of resonant error field amplification.^{23,24} Since this is a regime of focus for theoretical models, including the Fitzpatrick–Aydemir model adopted in this paper, these experiments facilitated side-by-side comparison with theory. Two types of experiments conducted in HBT-EP are reported. Section III presents observations of the external plasma response upon application of static external resonant perturbation. We find the magnitude of the amplification is consistent with the Fitzpatrick–Aydemir equations provided the viscous dissipation parameter is relatively high. In this regime, the amplitude of the response does not depend strongly upon the stability parameter, \bar{s} . In Sec. IV, measurements of the dynamic plasma response to a time-changing external perturbation are presented. The control coil polarity is reversed, corresponding to a 180° “phase flip” of the external field, at a specific instance as the plasma approaches the onset of RWM instability. In contrast with our observations of the weak variation of the magnitude of the amplification of a quasistatic perturbation, we observe the phase realignment time increases significantly as marginal stability is approached, as predicted by the Fitzpatrick–Aydemir equations. Indeed, we find the plasma’s response to a time-varying perturbation is the most sensitive indication of marginal stability in HBT-EP. Finally, Sec. V summarizes these observations and suggests that measurement of the response to a time-varying external field perturbation, such as a rapid “phase-flip,” may be a useful technique to detect the approach to RWM instability.

II. DESCRIPTION OF THE HBT-EP EXPERIMENT

The HBT-EP experiment incorporates a segmented adjustable conducting wall that has been previously used to demonstrate passive stabilization of ideal external kink modes^{5,6} and active feedback stabilization of the RWM.²⁵ Half of the wall segments are thick aluminum shells with wall constant of $\tau_w \sim 60$ ms and half are thin stainless steel with $\tau_w \sim 0.4$ ms, as illustrated in Fig. 1. Adjusting the position of the aluminum shells relative to the plasma surface changes the growth rate of external kink modes and provides a tool for investigating the RWM. Active probing of the plasma is achieved by applying nonaxisymmetric external magnetic fields generated by 30 control coils mounted on the outside of the stainless steel wall segments and used previously for active feedback control.²⁵ The current in each control coil is preprogrammed with a separate high-speed arbitrary waveform generator. The high power solid-state amplifier units that drive the control coils can supply suffi-

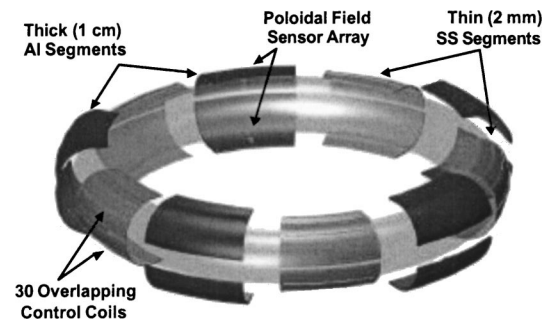


FIG. 1. Illustration of the HBT-EP experiment showing the arrangement of 20 adjustable wall segments, half of which are made of thick aluminum and half from relatively thin stainless steel. Thirty overlapping control coils are mounted on the stainless steel segments on the side not facing the plasma. A poloidal array of Mirnov sensors is decoupled from the fields of the control coils.

cient current to produce up to 2 G of resonant magnetic field on the plasma surface. The large number of control coils can provide a narrow angular Fourier mode spectrum of the applied field. For example, when the coils are energized to produce a predominantly $m/n = 3/1$ helical current structure, the relative toroidal sideband amplitudes of the applied magnetic field perturbation (i.e., $n = 2$) are more than a factor of 5 below the dominant $n = 1$ component, and the relative poloidal sidebands (i.e., $m = 2$ and 4) are reduced by a factor of 2 at the surface of the plasma.

A special discharge type was developed for systematic studies of the RWM that is similar to that used in previous experiments on RWM active feedback control in HBT-EP.²⁵ These discharges use a slightly modified fast start-up technique²⁷ and are prepared by preprogramming the equilibrium fields to achieve the desired time-evolution of the edge safety factor, the total current ramp rate, and the evolution of the plasma’s major and minor radii. A short period of rapid current ramp initializes the plasma current (~ 8 kA) and then a reduced current ramp (~ 1 MA/s) maintains the edge safety factor q^* between 2 and 3 as the major radius slowly evolves.

Figure 2 shows the growth of an $m/n = 3/1$ RWM in two example discharges. The RWM is an external kink mode. Like other external kink modes, they are distinguished from internal modes by the helical mode structure of the perturbed magnetic field measured with an array of Mirnov sensors. Internal modes, like the $m/n = 2/1$ tearing modes reported previously,⁵ have perturbations that resonant with the field-line helicity of an internal flux surface. External modes, like those discussed in this paper, have perturbations that distort the plasma surface and may resonate with a magnetic surface outside the plasma.

The location of the resonance strongly influences the time-evolution of the external kink mode. This is illustrated in the two example discharges of Fig. 2 that have different time-behaviors of the edge safety factor. In the example when q^* slowly approached 3 from below, the mode amplitude grew to a large value and caused a minor disruption (as evidenced by a rapid inward motion of the plasma major radius and a negative voltage spike on the loop voltage). As

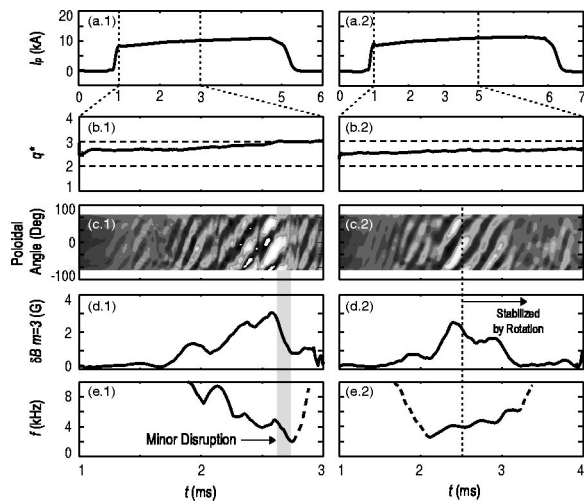


FIG. 2. Two examples of the natural growth of the RWM instability in HBT-EP. Shown parameters are: (a) total toroidal plasma current, (b) edge safety factor, (c) image of the poloidal magnetic field fluctuations at the wall measured with the poloidal Mirnov array, (d) amplitude of $m=3$ component of the poloidal magnetic field fluctuation at the wall, and (e) toroidal rotation rate of the magnetic field fluctuations. The discharge shown on the left shows the RWM as the q^* approaches 3. In this case, the electromagnetic torque applied to the plasma from eddy-currents in the wall reduced the mode rotation rate and a large-amplitude RWM causes a minor disruption. The discharge on the right shows a discharge where the RWM remained near marginal stability. In this case, the mode rotation increases and stabilizes the RWM.

the amplitude increased, the mode’s toroidal rotation slowed down to $\sim 2-3$ kHz consistent with the expected increased braking torque from the wall. In the example when q^* is kept relatively constant, the slowly-growing external kink mode had less impact on the plasma discharge, and the amplitude was observed to decrease as the mode toroidal rotation rate accelerated to $\sim 5-7$ kHz. We identify these external instabilities as resistive wall modes because the measured field perturbations have an external resonance, the perturbation’s slow growth and rotation rates are of the order of τ_w , and the instability is stabilized by sufficiently rapid toroidal rotation or when the thick aluminum wall segments are inserted closer to the plasma edge.

The phenomenon of the mode deceleration, observed in the second example (Fig. 2e.1) results from an eddy current torque applied to the plasma from the wall in so called torque balance regime.²³ In this regime the torque exerted on the mode from the wall is proportional to the product of the perturbed magnetic fluxes at the wall and at the plasma surface and is balanced by the torque from the plasma to the mode. When the rational surface is far from the plasma surface, very little torque is applied to slow mode rotation in HBT-EP. As the rational surface moves closer to the plasma edge, this product of the perturbed magnetic fluxes increases significantly causing an increase in the value of the applied torque and a deceleration of mode rotation.

In order to estimate external kink stability we reconstructed the plasma equilibrium as described previously in Ref. 6 while constraining the central current profile to be consistent with a computation of the resistive plasma current evolution described in Ref. 25. The ideal stability of the re-

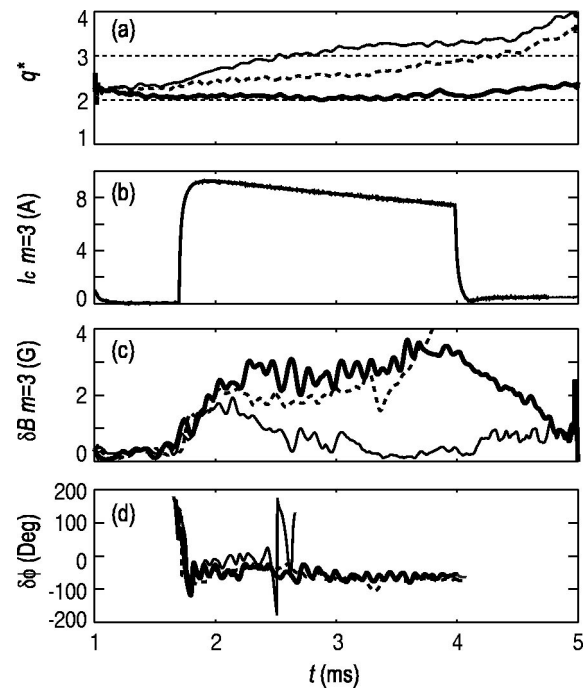


FIG. 3. Illustration of the plasma response to a quasi-static magnetic perturbation for three different discharges: (a) a discharge with relatively constant $q^* \sim 2.5$, (b) a discharge where q^* is rapidly increased above 3, and (c) a discharge when the RWM become unstable.

constructed equilibrium was then evaluated with the DCON stability code.²⁶ The results from this procedure are consistent with experimental observations and showed the plasma discharges to be near ideal MHD stability limit without a wall (i.e., the so-called “no-wall stability limit”) and can approach the stability limit for an ideal, perfectly conducting wall, as the edge safety factor approaches 3. In this paper, we define a stability parameter \bar{S} (called κ in Ref. 16) that is proportional to the normalized energy required to perturb the plasma surface with an external magnetic field fluctuation in the absence of the plasma rotation. The normalization is chosen for a given geometry of the plasma and conducting wall so that RWM is stable if $\bar{S} < 0$; the RWM is unstable if $0 < \bar{S} < 1$; and the ideal MHD kink mode is unstable for $\bar{S} > 1$, corresponding to the stability limit of the plasma with perfecting conducting wall segments. For the HBT-EP discharges discussed in this paper, the parameter \bar{S} depends most sensitively on q^* because of the edge resonance $q^* = m/n = 3/1$ and because the preprogrammed discharge parameters, like the slow plasma current ramp-rate, were kept relatively fixed. The main source of uncertainty in the calculation of \bar{S} comes from uncertainty in the determination of the edge safety factor, q^* , which is approximately $\pm 5\%$ in HBT-EP.

III. PLASMA RESPONSE TO STATIC EXTERNAL PERTURBATIONS

Observations resulting from the application of a quasi-static pulse of external magnetic field to three discharges near marginal stability are shown in Fig. 3. In all three dis-

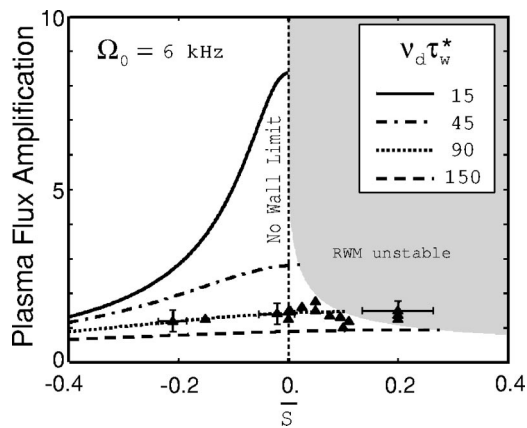


FIG. 4. The amplitude of the plasma response to a static external perturbation predicted for HBT-EP using Ref. 16 for a range of dissipation parameters and experimental measurements plotted as a function of the estimated stability parameter, \bar{S} .

charges, an external, predominantly resonant $m/n=3/1$, current structure in the control coils was applied from 1.7 to 4.0 ms. The discharges differed in their time-evolutions of the safety factor and the stability parameter, \bar{S} . The plasma response was measured independently from the applied external field using a Mirnov sensor array (as shown in Fig. 1) that was physically separated and decoupled from the control coils used to generate the external field.

When the edge safety factor was kept relatively constant, $q^* \sim 2.5$, a plasma response was observed to have a predominantly resonant $m/n=3/1$ magnetic field component that was aligned to amplify the applied field. The plasma response was also relatively constant, although the amplitude slowly evolved as a result of gradual changes in the plasma stability parameter and rotation. When the control coils were switched off, the resonant plasma response decayed, as shown in Fig. 3(c).

In discharges where q^* is ramped quickly from ~ 2.5 to more than 3.0, the resonant $m/n=3/1$ magnetic surface approaches the plasma surface from the outside and then becomes internal. We observe a strong plasma response only when the $q^* < 3$. This demonstrated that quasistatic (nonrotating) perturbations are significantly amplified by the RWM when the perturbations are externally resonant to the plasma; however, when the perturbations become internally resonant, they did not amplify or excite the $m/n=3/1$ internal tearing mode.

In the third example discharge, the edge safety factor is ramped slowly towards 3 which also gradually increases the stability parameter beyond marginal stability, $\bar{S} > 0$. The plasma response varies weakly early in the period when the external perturbation is applied; however, as $q^* \sim 3$, the external kink mode grows rapidly [Fig. 3(c)] and eventually causes a major disruption of the plasma.

The Fitzpatrick–Aydemir model equations relate the amplitude of the plasma response to external resonant static perturbations to the plasma stability parameter, the natural mode rotation, the dissipation parameter. This relationship is shown in Figs. 4 and 5 with solid and dashed lines. The model indicates a strong dependence on the dissipation pa-

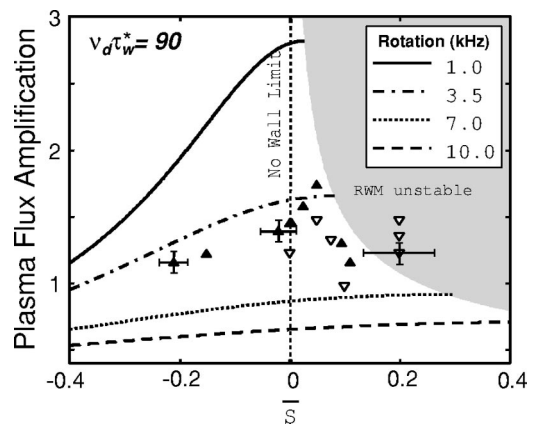


FIG. 5. The amplitude of the plasma response to a static perturbation for fixed dissipation parameter as a function of \bar{S} for a typical range of natural mode rotation rates (solid and dashed lines). Measurements of the amplitude of the plasma response taken early (solid triangles) and later (open triangles) in the discharges.

rameter ν_d . For small values of the dissipation, $\nu_d \tau_w^* \leq 15$, the model predicts a large plasma flux amplification as the RWM stability boundary is approached. But as the dissipation increases, $\nu_d \tau_w^* \geq 50$, the variation of the magnitude of the plasma amplification as \bar{S} changes becomes small for typical values of the natural mode rotation, between 3 and 8 kHz. A stronger variation in the magnitude of the amplification with \bar{S} occurs—even in the presence of strong dissipation—when the natural mode rotation decreases to approximately 1 kHz, as shown in Fig. 5.

Measurements of the plasma flux amplification as the stability parameter changed in HBT-EP are also plotted in Fig. 4. The Fitzpatrick–Aydemir model equations reproduce the measurements only when the viscous dissipation is relatively high, $\nu_d \tau_w^* \approx 90$. We use both the magnitude of the flux amplification and its weak dependence on \bar{S} as a means to define the value of ν_d that is appropriate for the RWM in the HBT-EP device.

The solid and dashed lines in Fig. 5 show the variation of the plasma flux amplification as a function of \bar{S} for plasma with high dissipation, $\nu_d \tau_w^* \approx 90$, and for various rates of natural plasma rotation. The model predicts that slowly rotating plasmas increase flux amplification and that rapidly rotating plasmas reduce flux amplification. In HBT-EP, observations of the rotation of naturally growing RWM are made when the control coils are switched off. In marginally unstable plasmas, the natural RWM toroidal rotation generally increases steadily from approximately 3 kHz early in the discharge to as high as 8 kHz later in the discharge (when the plasma is warmer and the diamagnetic flows are larger). The experimental measurements of plasma flux amplification shown in Fig. 5 are grouped into two parts: (a) those occurring early in a discharge when the natural mode rotation is relatively slow, and (b) those taken later when the rotation is higher. The measured values are bounded by the predictions of the Fitzpatrick–Aydemir equations at high dissipation when the natural rotation rates lay between 3.5 kHz and 7 kHz, and this rotation rate is consistent with observations. This general agreement between the measured, quasistatic,

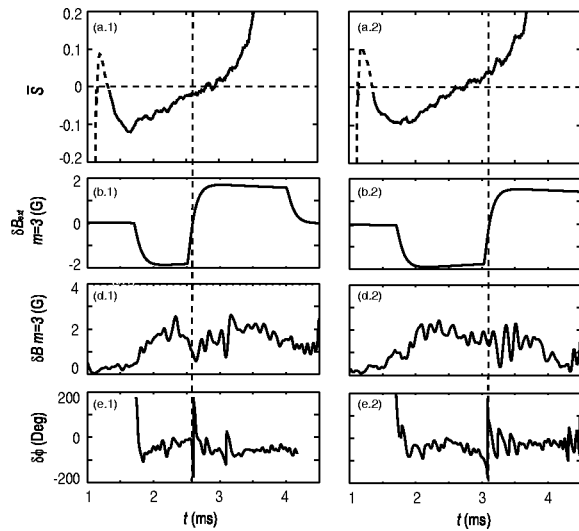


FIG. 6. Representative examples of the plasma response to a rapid 180 deg phase-flip of the external $m/n=3/1$ magnetic perturbation. The plotted values are: (a) stability parameter of $m/n=3/1$ RWM, (b) external radial magnetic field at the plasma surface, (c) amplitude of the plasma resonant response, and (d) phase difference between the applied field and the plasma response. The phase-flip occurred early in the discharge shown on the left, when the plasma was stable and relatively far from marginal stability. In the discharge shown on the right, the phase-flip occurred later in the discharge when the plasma was relatively close to marginal stability for the RWM.

flux amplification factor and the predictions of the Fitzpatrick–Aydemir equations with relatively high viscous dissipation rates serves as a starting point to the computation of the full dynamical response of the plasma to changing external perturbations described in the next section.

IV. DYNAMICAL PLASMA RESPONSE TO A “PHASE-FLIP” OF THE EXTERNAL PERTURBATION

In order to investigate the dynamical plasma response to external perturbations, experiments were conducted where a toroidal “phase-flip” of the applied external magnetic field occurred at various times in discharges near marginal stability of the RWM. Plasmas were created, like those described in the previous section, with the stability parameter of $m/n=3/1$ resistive wall mode, \bar{S} , and natural mode rotation slowly increasing in time. The nonrotating resonant current structure applied in the control coils had a rapid, preprogrammed, toroidal phase change of 180° at a predetermined instant. Although the currents in the control coils changed polarity quickly, the external magnetic field at the plasma edge changes sign more slowly, $\sim \tau_w/2m \sim 0.1$ ms, because of the finite time required for the magnetic field to penetrate the stainless steel wall segments.

The results from two typical “phase-flip” measurements are shown in Fig. 6. In the first case the phase-flip was timed to occur when the plasma was stable with $\bar{S} < 0$. In this case, shown in Fig. 6a.1, the observed plasma response field realigns itself with the applied field on a relatively short time scale (~ 0.2 ms) as seen in the rapid evolution of the relative phase between the mode and the external helical perturbation (Fig. 6e.1). In the other example, shown in Fig. 6a.2, the phase-flip of the applied field was timed to occur later in the

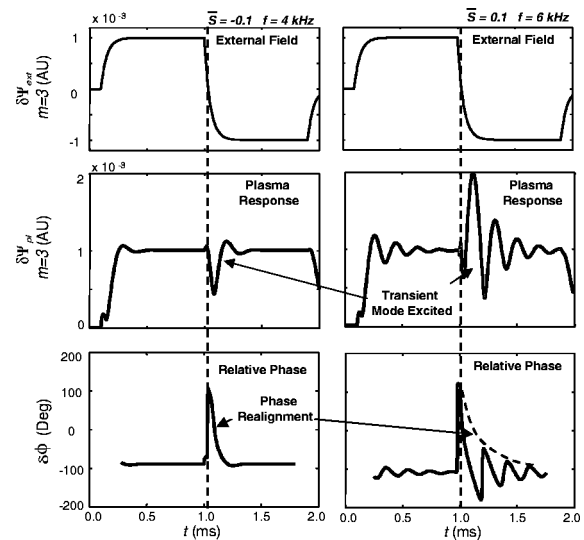


FIG. 7. Results of numerical simulations of the phase-flip experiments with Fitzpatrick–Aydemir equations for $\nu_d \tau_w^* \approx 90$ and for two values of stability parameter ($\bar{S} = -0.1, 0.1$) and plasma rotation ($\Omega_0 = 4$ kHz, 6 kHz).

discharge when the stability parameter was higher and, hence, the RWM was less stable. The plot of the plasma response, Fig. 6e.2, show oscillations of the amplitude of the $m/n=3/1$ resonant plasma response immediately after the phase-flip, oscillations of the relative phase between the perturbation and the plasma response, and an increase in the overall phase realignment time to ~ 0.5 ms. These three characteristic observations (oscillations of the amplitude and phase of the plasma response and a lengthening of the phase realignment time) were clearly and consistently evident whenever the phase-flip was programmed to occur near marginal stability.

Numerical simulations of the phase-flip experiments using the Fitzpatrick–Aydemir equations reproduced the oscillations in the amplitude of the plasma response and its relative phase caused by the phase-flip of the applied resonant field. These computations are shown in Fig. 7 and use values for the stability parameter, natural mode rotation rate, and dissipation that were selected to match the experiments shown in Fig. 6. The phase-flip excites a transient solution, responsible for the oscillations, whose $m/n=3/1$ structure oscillates back and forth like a damped toroidal pendulum and ultimately realigns with the phase of the applied field. The simulation shows the phase-realignment time is proportional to the inverse damping rate of the (stable) RWM, as calculated by the dispersion relationship.

The phase realignment times measured in several discharges are plotted in Fig. 8 as a function of the stability parameter \bar{S} , estimated at the time when the external field phase-flip occurred. The measurements are again categorized into two groups: (a) those taken early in the discharges when natural RWM rotation is low and (b) those taken later when the rotation is higher. The solid lines show the model prediction of the inverse damping rate of the RWM plotted for four different plasma rotation rates of and for the dissipation value of $\nu_d \tau_w^* = 90$. The measurements show a good agreement with model predictions of the RWM damping rate and

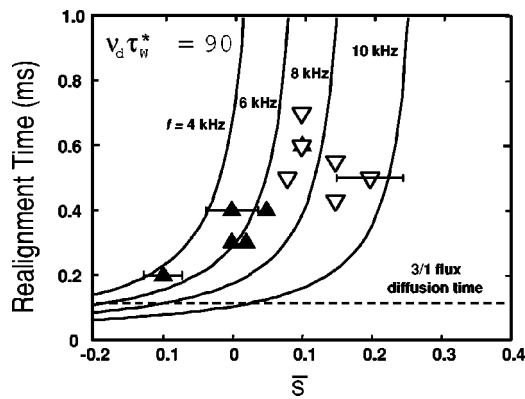


FIG. 8. Measurement of the phase realignment times in phase-flip experiments plotted versus the stability parameter, \bar{S} , early in the discharges (solid triangles) and later (open triangles). And inverse RWM damping rate predicted by Fitzpatrick–Aydemir equations (solid lines) for four different plasma rotation rates.

indicate that the model equations reproduce the dynamical behavior of the plasma response to resonant external perturbations.

V. SUMMARY

We report new measurements of the plasma response to external magnetic perturbations when the plasma is near the threshold of RWM instability. These experiments used a flexible, multichannel waveform generator to drive currents in an in-vessel array of 30 independent control coils. Additionally, the global discharge parameters were adjusted in HBT-EP to control the edge safety factor, q^* , and the rate of approach to marginal stability of the RWM. Observations of naturally growing RWM instabilities demonstrated that HBT-EP discharges are near marginal stability. High-pressure discharges created with high edge currents and with q^* programmed to approach 3 excited naturally growing RWM instabilities where the effects of strong coupling between the wall and the edge plasma slowed the natural mode rotation rate. In other discharges, when the edge safety factor was programmed to remain relatively constant, the slowly growing RWM instability had a natural mode rotation rate that was observed to gradually increase with time. When the mode rotation exceeded a critical rotation rate, the resistive wall mode stabilized. This second discharge type, with relatively constant q^* , was used for a systematic study of the plasma response to both static and time-changing external resonant magnetic perturbations. In these discharges, equilibrium reconstruction and stability calculations using the DCON ideal MHD code showed the normalized stability parameter, \bar{S} , to range from -0.5 to above $+0.5$ consistent with observations of marginal stability and previous studies of passive wall stabilization^{5,6} and active control of the RWM (Ref. 25) in HBT-EP.

Amplification of the RWM was observed upon application of static external resonant perturbation. By using observations of the natural rate of toroidal rotation of slowly-growing RWM instabilities and noting the weak dependence of the magnitude of the flux amplification on the normalized

stability parameter, \bar{S} , we determined that the strength of the dissipation parameter, used in the Fitzpatrick–Aydemir equations, must be relatively high, $\nu_d \tau_w^* \approx 90$, in HBT-EP.

The plasma response to a rapid polarity change of the external magnetic perturbation (referred to as an 180° “phase-flip”) was measured as a function of time, and this response was found to vary significantly as the phase-flip was timed to occur either early or late in the discharge as the plasma approaches the RWM stability limit. Although the magnitude of the flux amplification of a static perturbation did not change noticeably as marginal stability was approached, the phase realignment time did increase significantly at marginal stability, as predicted by Fitzpatrick–Aydemir equations. Additionally, the phase-flip excited oscillations of the amplitude and relative phase of the plasma resonant response. These effects were easily measured, and our observations suggest that detection of the resonant plasma response to time-varying external field perturbations may be used to detect the approach to RWM instability. Solutions to the Fitzpatrick–Aydemir equations show the lengthening of the phase-realignment that follows from a phase-flip is a generic feature of the RWM near marginal stability. This feature should be detectable in other toroidal devices that may have plasma parameters and wall geometries different from HBT-EP.

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