Locked Magnetic Islands Chains in Toroidally Flow Damped Tokamak Plasmas

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Abstract. The physics of a locked magnetic island chain maintained in the pedestal of an H-mode tokamak plasma by a static, externally generated, multi-harmonic, helical magnetic perturbation is investigated. The non-resonant harmonics of the external perturbation are assumed to give rise to significant toroidal flow damping in the pedestal, in addition to the naturally occurring poloidal flow damping. Furthermore, the flow damping is assumed to relax the pedestal ion toroidal and poloidal fluid velocities to fixed values determined by neoclassical theory. The resulting neoclassical ion flow causes a helical phase-shift to develop between the locked island chain and the resonant harmonic of the external perturbation. Furthermore, when this phase-shift exceeds a critical value, the chain unlocks from the resonant harmonic, and starts to rotate, after which it decays away, and is replaced by a helical current sheet. The neoclassical flow also generates an ion polarization current in the vicinity of the island chain which either increases or decreases the chain's radial width, depending on the direction of the flow. If the polarization effect is stabilizing, and exceeds a critical amplitude, then the helical island equilibrium becomes unstable, and the chain again decays away. The critical amplitude of the resonant harmonic of the external perturbation at which the island chain either unlocks or becomes unstable is calculated as a function of the pedestal ion pressure, the neoclassical poloidal and toroidal ion velocities, and the poloidal and toroidal flow damping rates.

1. Introduction

The ITER tokamak [1] is designed to operate primarily with so-called *H-mode* [2] plasma discharges. These are characterized by strong density and temperature gradients localized in a radially thin, annular, *pedestal* situated just inside the last closed magnetic flux-surface. Unfortunately, such gradients drive an intermittent instability known as an *edge localized mode* (ELM) [3]. Furthermore, the large impulsive heat flux across the last closed flux-surface associated with this instability leads to an unacceptable limitation on the lifetime of the ITER divertor plates [4]. Hence, it has become imperative to find a reliable method for suppressing ELMs in H-mode discharges.

In experiments recently performed on the DIII-D [5] and JET [6] tokamaks, application of a static, externally generated, magnetic perturbation, with a broad spectrum of helical harmonics, many of which were resonant in the pedestal, to an H-mode discharge was found to either completely suppress, or greatly mitigate, the ELMs. The original motivation for these so-called *resonant magnetic perturbation* (RMP) experiments was to create a series of overlapping, static, helical magnetic island chains in the pedestal, thereby causing the magnetic field there to become ergodic. However, it appears likely that this did not happen (since there was no collapse in the pedestal electron temperature). Instead, magnetic island formation was (presumably) suppressed to a large extent by equilibrium plasma flows [7, 8, 9, 10, 11, 12, 13], and only a few non-overlapping, static, helical island chains were generated. The purpose of this paper is to investigate such chains.

For a number of reasons, the physics of a helical magnetic island chain generated in the pedestal of an H-mode discharge, during an RMP experiment, is significantly different from that of a conventional island chain: *e.g.*, a chain associated with a neoclassical tearing mode resonant in the plasma core. Firstly, an RMP induced island chain is necessarily *non-rotating*, since it is locked to that helical harmonic of the externally generated, *static* magnetic perturbation which *resonates* at its associated rational surface [14]. A neoclassical tearing mode island chain, on the other hand, is convected by equilibrium ion flows in the plasma core, and therefore rotates [15]. Secondly, the *non-resonant* harmonics of the applied magnetic perturbation in RMP experiments produce a significant *toroidal flow damping* effect [16] which relaxes the toroidal ion flow in the pedestal to a fixed value determined by neoclassical theory [17, 18, 19]. Of course, the poloidal ion flow is already relaxed to a fixed value determined by neoclassical theory [20, 21], due to the strong *poloidal flow damping* which is present in all tokamak plasmas (because of the toroidicity-induced poloidal variation of the toroidal magnetic field-strength around magnetic flux-surfaces [22]). It follows that, in RMP experiments, the pedestal poloidal and toroidal ion flow velocities are both constrained to take *fixed values.* By contrast, whereas the poloidal ion flow in conventional tokamak plasmas (*i.e.*, plasmas with no significant toroidal flow damping) is fixed, the toroidal flow is free to vary.

Now, a locked magnetic island chain presents an obstacle to equilibrium ion flow unless the flow has the *same helicity* as the chain (*i.e.*, unless the flow is parallel to the equilibrium magnetic field-lines at the associated rational surface). This follows because the ion fluid is unable to cross the chain's magnetic separatrix (assuming that the radial island width is much larger than the ion gyroradius) [14]. When a locked island chain is suddenly introduced into a conventional tokamak plasma, the toroidal ion flow in the vicinity of the chain adjusts itself such that, when combined with the fixed poloidal flow, the net flow has the same helicity as the chain. The toroidal flow profile subsequently relaxes across the whole plasma under the action of perpendicular viscosity [14]. The chain then presents no further obstacle to the ion flow. However, when a locked island chain is introduced into the pedestal of an H-mode plasma, during an RMP experiment, the toroidal ion flow in the vicinity of the chain is not free to adjust itself, because of the significant toroidal flow damping generated by the applied magnetic perturbation, which pins the flow close to its neoclassical value. It follows that the island chain does, in general, present an obstacle to the ion flow. The resulting strong helical currents induced in the vicinity of the chain are able to modify its radial width and helical phase. Indeed, it is conceivable that these modifications could become large enough to completely expel the island chain from the plasma. In this case, the chain would be replaced by a thin helical current sheet, centered on the rational surface, which shields the interior of the plasma from the resonant harmonic of the external magnetic perturbation [14].

2. Island Chain Dynamics

Consider a large aspect-ratio, low-β, circular cross-section, tokamak plasma. Let us adopt the standard toroidal coordinates (r, θ, φ), where r is the magnetic flux-surface minor radius, θ the poloidal angle, and φ the toroidal angle. The equilibrium toroidal magnetic field and toroidal plasma current are both assumed to run in the $+\varphi$ direction.

Suppose that a locked, helical, constant-ψ [23], magnetic island chain, with m_θ periods in the poloidal direction, and n_φ periods in the toroidal direction, is maintained in the plasma by the m_θ, n_φ helical harmonic (henceforth, known as the "resonant" harmonic) of a static, externally generated, magnetic perturbation. Let r_s be the minor radius of the m_θ, n_φ mode rational surface (at which the equilibrium magnetic field-lines form closed helices with m_θ periods in the poloidal direction, and n_φ periods in the toroidal direction). Suppose that the island chain is sufficiently thin (radially) that it is localized in the vicinity of this surface, so that it does not overlap any other island chains present in the plasma, whilst still being much wider than the ion gyroradius. The remaining (non-axisymmetric) helical harmonics of the perturbation (henceforth, known as the "non-resonant" harmonics) are assumed to generate significant toroidal flow damping in the vicinity of the island chain. The associated damping rate can, in principle, be calculated as a complicated weighted sum over the squared amplitudes of the non-resonant harmonics [18, 24].

The equation which governs the *radial width* of the island chain takes the form [14, 15, 25, 26, 27]

$$\frac{dw}{dt} \propto \Delta' r_s + 2m_\theta \left(\frac{w_v}{w}\right)^2 \cos\phi + J_c \left(\frac{r_s}{w}\right)^3. \tag{1}$$

Here, w is one quarter of the full radial island width, w_v one quarter of the full vacuum island width (*i.e.*, the island width obtained by simply applying the vacuum external magnetic perturbation to the plasma equilibrium), ϕ the helical phase of the island chain relative to that of the vacuum chain, Δ' the linear tearing stability index [23] for the m_θ, n_φ mode, and

$$J_c \equiv \frac{\mu_0\, L_s\, w}{\pi\, B_0\, r_s^2} \int_{r_{s-}}^{r_{s+}} \oint \delta j_\parallel\, \cos\zeta\, d\zeta\, dr. \tag{2}$$

In the above, $\delta j_\parallel(r,\zeta)$ is the perturbed parallel current density associated with the island chain, $L_s \equiv R_0\, q(r_s)/(d\ln q/d\ln r)_{r_s}$ the equilibrium magnetic shear length evaluated at the rational surface, B_0 the toroidal field-strength, $\zeta \equiv m_\theta\,\theta - n_\varphi\,\varphi - \phi$ a helical angle, R_0 the major radius of the plasma, and $q(r)$ the equilibrium safety-factor profile. [Note that $q(r_s) = m_\theta/n_\varphi$.] The first term on the right-hand side of (1) represents the intrinsic magnetohydrodynamical (MHD) free energy available to drive the growth of the island chain. Since it is assumed that the chain is actually driven by the resonant harmonic of the external magnetic perturbation, it follows that this term is negative: *i.e.*, $\Delta' < 0$. The second term parameterizes the drive from the resonant harmonic. The final term represents the effect of the *ion polarization current* produced by ion flow relative to the island chain [15, 27].

The equation which controls the *helical phase* of the island chain takes the form [14, 15, 26]

$$\frac{d^2\phi}{dt^2} \propto -2m_\theta \left(\frac{w_v}{r_s}\right)^2 \left(\frac{w}{r_s}\right)^2 \sin\phi + J_s \left(\frac{w}{r_s}\right), \tag{3}$$

where

$$J_s \equiv \frac{\mu_0\, L_s\, w}{\pi\, B_0\, r_s^2} \int_{r_{s-}}^{r_{s+}} \oint \delta j_\parallel\, \sin\zeta\, d\zeta\, dr. \tag{4}$$

Here, the first term on the right-hand side of (3) represents the *electromagnetic locking torque* due to the resonant harmonic of the external magnetic perturbation, whereas the second term represents the *drag torque* produced by ion flow relative to the island chain. Note that a locked island chain is characterized by a *constant* helical phase. Of course, a rotating island chain has a varying phase.

In deriving (1) and (3), we have assumed that the equilibrium plasma current external to the rational surface is negligible, which is reasonable when the surface lies close to the edge of the plasma [14].

3. Steady-State Locked Island Regimes

The parameters J_c and J_s, appearing in (1) and (3), only have a weak dependence on w and ϕ (see Sect. 4), and can thus be treated as constants to a good approximation. Let us search for steady-state (*i.e.*, $d/dt \equiv 0$) solutions to these equations, assuming that J_c and J_s are independent of w or ϕ.

The steady-state versions of (1) and (3) can be written in the normalized form

$$0 = -\hat{w}^3 + \hat{b}\,\hat{w}\,\cos\phi + \hat{J}_c, \quad (5)$$

$$0 = -\hat{b}\,\hat{w}\,\sin\phi + \hat{J}_s, \quad (6)$$

respectively. Here,

$$\hat{b} \equiv \left(\frac{2m_\theta}{-\Delta'\,r_s}\right)\left(\frac{w_v}{r_s}\right)^2, \quad (7)$$

$$\hat{w} \equiv \frac{w}{r_s}, \quad (8)$$

$$\hat{J}_c \equiv \frac{J_c}{(-\Delta'\,r_s)}, \quad (9)$$

$$\hat{J}_s \equiv \frac{J_s}{(-\Delta'\,r_s)}. \quad (10)$$

The parameter $\hat{b}$ is the normalized *resonant* harmonic of the *radial* magnetic field at the rational surface due to the external magnetic perturbation, and incorporates the well-known MHD amplification factor $2m_\theta/(-\Delta'\,r_s)$ [14].

Equations (5) and (6) can be solved in *three* different regimes.

In *Regime 1*, the second term on the right-hand side of (5) is neglected. This implies that the island width is maintained by the ion polarization current, rather than the resonant harmonic of the external magnetic perturbation. Of course, this is only possible if the ion polarization term in (5) is *destabilizing*: *i.e.*, $J_c > 0$. As is easily demonstrated, the neglect of the second term on the right-hand side of (5) is valid provided $|\hat{J}_s| \ll \hat{J}_c$. In Regime 1, (5) yields $\hat{w} = \hat{J}_c^{1/3}$, whereas (6) reduces to

$$\sin\phi = \frac{\hat{J}_s}{\hat{b}\,\hat{J}_c^{1/3}}. \quad (11)$$

The above equation can only be satisfied provided the magnitude of its right-hand side is less than unity (since $|\sin\phi| \leq 1$). It follows that the steady-state solution is lost when $\hat{b}$ falls below the critical value

$$\hat{b}_{cr} = \frac{|\hat{J}_s|}{\hat{J}_c^{1/3}}. \quad (12)$$

In this situation, the drag torque due to the ion flow can no longer be balance by the electromagnetic locking torque, causing the island chain to *unlock* from the resonant harmonic of the external magnetic perturbation, and then start to *rotate* [15]. Now, a *static* resonant magnetic perturbation is unable to effectively drive a *rotating* island chain [28]. Moreover, a rotating chain does not generate a strong ion polarization current [15]. It follows that the chain decays away (since it is not driven either by the external perturbation or the ion polarization current, and is intrinsically stable), and is replaced by a helical current sheet [7]. The unlocking bifurcation takes place when the relative

Table 1. Steady-state locked island regimes.

Regime	Extent	$\hat{b}_{cr}$	$\hat{w}_{cr}$	ϕ_{cr}
1	$\hat{J}_c > 0,\ \lvert\hat{J}_s\rvert \ll \hat{J}_c$	$\lvert\hat{J}_s\rvert/\hat{J}_c^{1/3}$	$\hat{J}_c^{1/3}$	$\text{sgn}(\hat{J}_s)\,\pi/2$
2	$\lvert\hat{J}_s\rvert \gg \lvert\hat{J}_c\rvert$	$(27/4)^{1/6}\,\lvert\hat{J}_s\rvert^{2/3}$	$(1/2)^{1/6}\,\lvert\hat{J}_s\rvert^{1/3}$	$\text{sgn}(\hat{J}_s)\,\cos^{-1}(1/\sqrt{3})$
3	$\hat{J}_c < 0,\ \lvert\hat{J}_s\rvert \ll \lvert\hat{J}_c\rvert$	$(3/2^{2/3})\,\lvert\hat{J}_c\rvert^{2/3}$	$(1/2)^{1/3}\,\lvert\hat{J}_c\rvert^{1/3}$	0

helical phase of the chain reaches the critical value $\phi_{cr} = \text{sgn}(\hat{J}_s)\,\pi/2$. The critical island width at the bifurcation is $\hat{w}_{cr} = \hat{J}_c^{1/3}$. Note that

$$\frac{\hat{w}_{cr}}{\hat{b}_{cr}^{1/2}} = \left(\frac{\hat{J}_c}{|\hat{J}_s|}\right)^{1/2} \gg 1. \tag{13}$$

Since $\hat{w} = \hat{b}^{1/2}$ in the absence of ion flow (*i.e.*, when $\hat{J}_c = \hat{J}_s = 0$), the above expression implies that, just prior to the unlocking bifurcation, the island chain is *strongly amplified* by the flow. Of course, the chain is completely suppressed by the flow after the bifurcation.

In *Regime 2*, the third term (*i.e.*, the ion polarization term) on the right-hand side of (5) is neglected, which is valid provided $|\hat{J}_s| \gg |\hat{J}_c|$. In this regime, (5) gives

$$\hat{w} = \hat{b}^{1/2}\,(\cos\phi)^{1/2}, \tag{14}$$

whereas (6) reduces to

$$\sin\phi\,(\cos\phi)^{1/2} = \frac{\hat{J}_s}{\hat{b}^{3/2}}. \tag{15}$$

As is easily demonstrated, the above equation can only be satisfied when the magnitude of its right-hand side is less than $(4/27)^{1/4}$. It follows that the steady-state solution is lost when $\hat{b}$ falls below the critical value

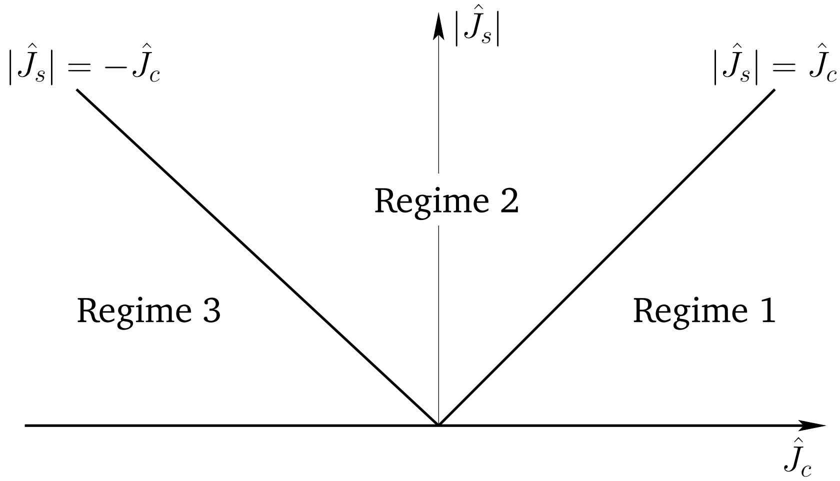
$$\hat{b}_{cr} = \left(\frac{27}{4}\right)^{1/6} |\hat{J}_s|^{2/3}. \tag{16}$$

In this situation, the drag torque due to the ion flow again forces the island chain to unlock from the resonant harmonic of the external magnetic perturbation. The chain subsequently decays, and is replaced by a helical current sheet. The unlocking bifurcation takes place when the relative helical phase of the chain reaches the critical value $\phi_{cr} = \text{sgn}(\hat{J}_s)\,\cos^{-1}(1/\sqrt{3})$. Moreover, the critical island width at the bifurcation is $\hat{w}_{cr} = (1/2)^{1/6}\,|\hat{J}_s|^{1/3}$. Finally,

$$\frac{\hat{w}_{cr}}{\hat{b}_{cr}^{1/2}} = \left(\frac{1}{3}\right)^{1/4} < 1, \tag{17}$$

which implies that, just prior to the unlocking bifurcation, the island chain is *weakly suppressed* by the ion flow. Again, the chain is completely suppressed by the flow after the bifurcation.

Figure 1. Steady-state locked island regimes.



In *Regime 3*, the second term on the right-hand side of (6) (*i.e.*, the drag term) is neglected, which is valid provided $|\hat{J}_c| \gg |\hat{J}_s|$. In this regime, (6) gives $\phi = 0$, whereas (5) reduces to

$$0 = -\hat{w}^3 + \hat{b}\,\hat{w} + J_c. \tag{18}$$

Assuming that the ion polarization term in the above equation is *stabilizing* (*i.e.*, $\hat{J}_c < 0$), it is readily demonstrated that if $\hat{b}$ exceeds the critical value

$$\hat{b}_{cr} = \left(\frac{3}{2^{2/3}}\right) |\hat{J}_c|^{2/3} \tag{19}$$

then (18) possesses *two* positive roots, the smaller of which is dynamically unstable, and the larger dynamically stable. On the other hand, if $\hat{b}$ falls below the critical value then there are no positive roots of (18), which implies that the island width evolution equation does not possesses a steady-state solution. Furthermore, it is easily shown that the non-steady-state solutions of this equation all *decay* in time. It follows that if $\hat{b}$ falls below $\hat{b}_{cr}$ then the stable solution of (18) disappears, and the island chain consequently decays away, and is eventually replaced by a helical current sheet [14, 28]. In this case, the suppression of the island chain is due to the stabilizing effect of the flow-induced ion polarization current, rather than the unlocking of the chain from the resonant harmonic of the external magnetic perturbation. The critical island width at the bifurcation is $\hat{w}_{cr} = (1/2)^{1/3}\,|\hat{J}_c|^{1/3}$. Finally,

$$\frac{\hat{w}_{cr}}{\hat{b}_{cr}^{1/2}} = \left(\frac{1}{3}\right)^{1/2} < 1, \tag{20}$$

which implies that, just prior to the bifurcation, the island chain is *moderately suppressed* by the ion flow. Of course, the chain is completely suppressed by the flow after the bifurcation.

The various different regimes for steady-state locked island chains are summarized in Table 1 and Figure 1.

4. Ion Flow in the Vicinity of a Locked Island Chain

It remains to calculate the values of the parameters $\hat{J}_c$ and $\hat{J}_s$. The first of these characterizes the effect of the flow-induced ion polarization current on the width of the locked island chain, whilst the second characterizes the flow-induced drag torque acting on the chain. In order to determine the value of these parameters, we must first calculate the *ion velocity profiles* in the vicinity of the chain.

Our calculation of the ion velocity profiles is based on the analysis of [15]. The starting point for this analysis is a *drift-MHD fluid model* of the plasma dynamics which incorporates ion and electron diamagnetic flows (including the contribution of the ion gyroviscosity tensor), but neglects electron inertia, the electron viscosity tensor, and magnetic field-line curvature. The neglect of the electron viscosity tensor, which is justified provided that the plasma in the vicinity of the island chain is sufficiently collisional, also implies the neglect of the bootstrap current. The ion fluid equation of motion incorporates a phenomenological perpendicular ion viscosity operator, as well as phenomenological poloidal and toroidal flow damping terms. The viscosity operator represents momentum transport due to small-scale plasma turbulence, whereas the toroidal flow damping term models flow damping generated by the non-resonant harmonics of the external magnetic perturbation. The drift-MHD model is first reduced to a *single helicity four-field model* [29], by means of approximations suitable to a large aspect-ratio, low β, circular cross-section, tokamak plasma. Next, the following ordering scheme (which corresponds to the so-called *intermediate poloidal flow damping regime* discussed in [15] and [30]) is adopted

$$\omega_{\ast\,i} \gg \nu_\theta \gg (\epsilon/q)^2\,\nu_\theta,\ \nu_\varphi,\ \nu_\mu. \tag{21}$$

(Alternative ordering schemes, in which the poloidal flow damping is either larger or smaller than in the above scheme, are discussed in [15] and [30].) Here,

$$\omega_{\ast\,i} \equiv k_\theta\,V_{\ast\,i}, \tag{22}$$

$$k_\theta \equiv \frac{m_\theta}{r_s}, \tag{23}$$

$$V_{\ast\,i} \equiv \frac{T_i}{e\,B_0\,L_n}, \tag{24}$$

$$\frac{\epsilon}{q} \equiv \frac{n_\varphi}{m_\theta}\,\frac{r_s}{R_0}, \tag{25}$$

$$\nu_\mu \equiv \frac{\mu}{\rho\,w^2}. \tag{26}$$

Moreover, T_i is the ion temperature, $L_n \equiv -r_s/(d\ln n/d\ln r)_{r_s}$ the density gradient scale-length, and ρ the mass density, all evaluated at the rational surface, and $n(r)$ is the equilibrium electron number density profile. It is assumed that $L_n/L_s \sim \epsilon/q \ll 1$. In (21), $\omega_{\ast\,i}$ is the *ion diamagnetic frequency*, ν_θ the phenomenological *poloidal flow damping rate*, and ν_φ the phenomenological *toroidal flow damping rate*, all evaluated at the rational surface. Furthermore, μ is the phenomenological *coefficient of perpendicular* *ion viscosity* at the rational surface. According to the ordering scheme (21), the ion diamagnetic frequency is much larger than the poloidal flow damping rate, which, in turn, is much larger than either the toroidal flow damping rate or the rate of radial momentum diffusion across the island region. This ordering scheme precludes a *neoclassical enhancement of ion inertia* [31], since this would require the poloidal flow damping force to be dominant in both the parallel ion equation of motion and the parallel ion vorticity equation, which is only the case when [30]

$$(\epsilon/q)^2\,\nu_\theta \gg \omega_{\ast\,i}. \tag{27}$$

Incidentally, the ordering scheme (21) ensures that the electric potential and the plasma density are both flux-surface functions in the island region, which leads to a great simplification in the analysis.

The magnetic flux-surfaces in the vicinity of the island chain are contours of

$$k(X,\zeta) \equiv \sqrt{X^2/4+\cos^2(\zeta/2)}, \tag{28}$$

where $X \equiv (r-r_s)/w$. The O-points are located at $X=0$ and $\zeta=\pi$, the X-points at $X=0$ and $\zeta=0$, and the magnetic separatrix at $k=1$. The regions inside and outside the separatrix correspond to $0\leq k<1$ and $1<k<\infty$, respectively.

The ion velocity profiles in the region outside the separatrix are determined by a flux-surface function, $Y(k)$, which satisfies the differential equation

$$0 = \hat{\nu}_\mu\,\frac{d}{dk}\left(A(k)\,\frac{d}{dk}[A(k)\,Y(k)]\right) - \hat{\nu}_\theta\,[A(k)\,B(k)-1]\,Y(k) - \hat{\nu}_\varphi\,B(k)\,[A(k)\,Y(k)-1], \tag{29}$$

where

$$\hat{\nu}_\theta \equiv \frac{\nu_\theta}{\omega_{\ast\,i}}, \tag{30}$$

$$\hat{\nu}_\varphi \equiv \frac{\nu_\varphi}{(\epsilon/q)^2\,\omega_{\ast\,i}}, \tag{31}$$

$$\hat{\nu}_\mu \equiv \frac{\nu_\mu}{4\,(\epsilon/q)^2\,\omega_{\ast\,i}}, \tag{32}$$

and

$$A(k) \equiv \frac{2}{\pi}\,E(1/k), \tag{33}$$

$$B(k) \equiv \frac{2}{\pi}\,K(1/k). \tag{34}$$

Here,

$$E(k) \equiv \int_0^{\pi/2}(1-k^2\,\sin^2 u)^{1/2}\,du, \tag{35}$$

$$K(k) \equiv \int_0^{\pi/2}(1-k^2\,\sin^2 u)^{-1/2}\,du. \tag{36}$$

The boundary conditions satisfied by $Y(k)$ are

$$Y(1) \;\;= \frac{\pi}{2}\left(\frac{\hat{\nu}_\varphi}{\hat{\nu}_\theta+\hat{\nu}_\varphi}\right), \tag{37}$$

$$Y(\infty) = 1. \tag{38}$$

The *ion poloidal velocity profile* in the vicinity of the island chain takes the form

$$\frac{V_{\theta\,i}-V_{\theta\,i}^{nc}}{V_p^{nc}} = \left\{\begin{array}{cc} -[\hat{\nu}_\varphi/(\hat{\nu}_\theta+\hat{\nu}_\varphi)] & 0\leq k\leq 1\\ \left([1-\cos^2(\zeta/2)/k^2]^{1/2}-A(k)\right)Y(k) & k>1\end{array}\right., \tag{39}$$

whereas the *ion toroidal velocity profile* is written

$$\frac{V_{\varphi\,i}-V_{\varphi\,i}^{nc}}{(q/\epsilon)\,V_p^{nc}} = \left\{\begin{array}{ll} [\hat{\nu}_\theta/(\hat{\nu}_\theta+\hat{\nu}_\varphi)] & 0\leq k\leq 1\\ 1-A(k)\,Y(k) & k>1\end{array}\right.. \tag{40}$$

Here, $V_{\theta\,i}^{nc}$ is *neoclassical ion poloidal velocity* (*i.e.*, the fixed velocity towards which neoclassical flow damping relaxes the poloidal flow in the vicinity of the chain) and $V_{\varphi\,i}^{nc}$ is the corresponding *neoclassical ion toroidal velocity*. Moreover,

$$V_p^{nc} \equiv V_{\theta\,i}^{nc} - \left(\frac{\epsilon}{q}\right)V_{\varphi\,i}^{nc} \tag{41}$$

is the *neoclassical island phase velocity*: *i.e.*, the phase velocity that the island chain would need to have in order not to present an obstacle to the neoclassical ion flow. Note that if $V_p^{nc}>0$ then the chain would have to rotate in the *electron* diamagnetic direction (*i.e.*, positive-θ, negative-φ), whereas if $V_p^{nc}<0$ then the chain would have to rotate in the *ion* diamagnetic direction (*i.e.*, negative-θ, positive-φ). (Here, we are assuming standard profiles such that $L_s>0$ and $L_n>0$.)

According to standard neoclassical theory [17, 20, 24, 32],

$$V_{\theta\,i}^{nc} = K_{\theta\,i}\,\eta_i\,V_{*\,i}, \tag{42}$$

$$V_{\varphi\,i}^{nc} = \;-K_{\varphi\,i}\,\eta_i\,\frac{q}{\epsilon}\,V_{*\,i}, \tag{43}$$

where $\eta_i\equiv L_n/L_{T_i}$, $L_{T_i}\equiv -r_s/(d\ln T_i/d\ln r)_{r_s}$ is the *ion temperature gradient scale-length* evaluated at the rational surface, $K_{\theta\,i}$ and $K_{\varphi\,i}$ are $\mathcal{O}(1)$ dimensionless constants which primarily depend on the ion collisionality at the rational surface, and $T_i(r)$ is the equilibrium ion temperature profile. It follows that

$$V_p^{nc} = (K_{\theta\,i}+K_{\varphi\,i})\,\eta_i\,V_{*\,i}. \tag{44}$$

Note, for example, that $K_{\theta\,i}$ and $K_{\varphi\,i}$ are both *positive* in a *collisional* plasma. Hence, in such a plasma, the neoclassical island phase velocity would be in the *electron* diamagnetic direction (*i.e.*, $V_p^{nc}>0$). (Here, we are assuming standard profiles such that $\eta_i>0$.)

Now, the equilibrium neoclassical ion velocities, $V_{\theta\,i}^{nc}$ and $V_{\varphi\,i}^{nc}$, are both proportional to the *ion temperature gradient* (via the parameter η_i). However, we would expect a sufficiently wide island chain to *flatten* the ion temperature profile in the region lying within its magnetic separatrix. Thus, it seems probable that such a chain would give rise to a local reduction in the neoclassical ion velocities. A fully self-consistent calculation of this effect would require a fluid model which incorporates temperature gradients, heat fluxes, and an energy equation (which our four-field model does not). Hence, in the following, we shall neglect any modification to the neoclassical ion velocities due to the presence of the island chain.

An examination of (39) and (40) reveals that, far from the island chain (*i.e.*, $k \gg 1$), the ion poloidal and toroidal velocities both asymptote to their neoclassical values: *i.e.*, $V_{\theta\,i} \rightarrow V_{\theta\,i}^{nc}$ and $V_{\varphi\,i} \rightarrow V_{\varphi\,i}^{nc}$. This follows because $A(k) \rightarrow 1$ and $Y(k) \rightarrow 1$ as $k \rightarrow \infty$. On the other hand, within the magnetic separatrix the flow has the same helicity as the island chain: *i.e.*,

$$V_{\theta\,i} - \frac{\epsilon}{q}\,V_{\varphi\,i} = 0, \tag{45}$$

as must be the case, given that the flow is unable to cross the separatrix. The above constraint forces the ion poloidal and toroidal velocities in the immediate vicinity of the island chain to deviate somewhat from their respective neoclassical values. However, it is assumed that the radial extent of the region in which this deviation occurs is *small* compared to the plasma minor radius.

The *ion polarization parameter*, $\hat{J}_c$, is written

$$\hat{J}_c = \hat{\beta}\,[C\,F\,(F+L)]_{k=1} + \hat{\beta}\int_1^\infty C\,\frac{d}{dk}\,[F\,(F+L)]\,dk, \tag{46}$$

where

$$\hat{\beta} \equiv \frac{\beta_i\,\rho_i^{\,2}\,L_s^{\,2}}{r_s^{\,2}\,L_n^{\,2}\,(-\Delta'\,r_s)}, \tag{47}$$

$$F(k) \equiv \hat{V}_p^{nc}\,\frac{Y(k)}{2k}, \tag{48}$$

$$L(k) \equiv \frac{\pi}{4}\,\frac{1}{k\,E(1/k)}, \tag{49}$$

$$C(k) \equiv \frac{16\,k^3}{3\pi}\left[2\left(2-\frac{1}{k^2}\right)E(1/k) - \left(1-\frac{1}{k^2}\right)K(1/k) - 3\,\frac{E^{\,2}(1/k)}{K(1/k)}\right]. \tag{50}$$

Here, $\beta_i \equiv \mu_0\,p_i/B_0^{\,2}$, p_i is the ion pressure, and $\rho_i \equiv (T_i/m_i)^{1/2}/(e\,B_0/m_i)$ the ion gyroradius, all evaluated at the rational surface, whereas

$$\hat{V}_p^{nc} \equiv \frac{V_p^{nc}}{V_{*\,i}}. \tag{51}$$

It is assumed that $\beta_i \sim (\epsilon/q)^2 \ll 1$, $\hat{\beta} \sim (\rho_i/r_s)^2 \ll 1$, and $w/r_s \sim (q/\epsilon)\,(\rho_i/r_s) \ll 1$. The first term on the right-hand side of (46) comes from a *boundary layer*, of typical thickness ρ_i, located on the chain's magnetic separatrix. This layer resolves the discontinuity in the poloidal ion velocity profile which is evident in expression (39) [15, 33]. The second term comes from the region outside the separatrix.

The *drag parameter*, $\hat{J}_s$, takes the form

$$\hat{J}_s = \hat{\beta}\,\hat{V}_p^{nc}\,\hat{\nu}_\theta\left\{\frac{\hat{\nu}_\varphi}{\hat{\nu}_\theta+\hat{\nu}_\varphi}\,8\int_0^1 D(k)\,dk + 8\int_1^\infty [A(k)\,B(k)-1]\,Y(k)\,dk\right\}$$
$$+\,\hat{\beta}\,\hat{V}_p^{nc}\,\hat{\nu}_\varphi\left(\frac{\epsilon}{q}\right)^2\left\{\frac{\hat{\nu}_\theta}{\hat{\nu}_\theta+\hat{\nu}_\varphi}\,8\int_0^1 D(k)\,dk + 8\int_1^\infty [1-A(k)\,Y(k)]\,B(k)\,dk\right\}, \tag{52}$$

where

$$D(k) \equiv \frac{2}{\pi}\,k\,K(k). \tag{53}$$

In (52), the first and second terms on the right-hand side represent the drag on the island chain due to poloidal and toroidal flow damping, respectively. The former drag is generally much larger than the latter.

5. Flow Damping Regimes

Equation (29) can be solved in *three* different flow damping regimes.

Regime I corresponds to $\hat{\nu}_\mu \gg \hat{\nu}_\varphi^{1/2}\,(\hat{\nu}_\theta+\hat{\nu}_\varphi)^{1/2}$. In this regime, the solution to (29) takes the form

$$Y(k) \simeq \frac{1}{A(k)}\left[1-\frac{\hat{\nu}_\theta}{\hat{\nu}_\theta+\hat{\nu}_\varphi}\,{\rm e}^{-(k-1)/\delta_1}\right], \tag{54}$$

where $\delta_1 \equiv (\hat{\nu}_\mu/\hat{\nu}_\varphi)^{1/2} \gg 1$. The ion poloidal velocity profile across the island O-point (*i.e.*, $\zeta=\pi$) becomes

$$\frac{V_{\theta\,i}-V_{\theta\,i}^{nc}}{V_p^{nc}} \simeq \left\{\begin{array}{lc} -[\hat{\nu}_\varphi/(\hat{\nu}_\theta+\hat{\nu}_\varphi)] & 0\leq k\leq 1\\ {[\hat{\nu}_\varphi/(\hat{\nu}_\theta+\hat{\nu}_\varphi)]\,[1/A(k)-1]} & k>1\end{array}\right., \tag{55}$$

where $k=(r-r_s)/(2\,w)$, whilst the corresponding ion toroidal velocity profile is written

$$\frac{V_{\varphi\,i}-V_{\varphi\,i}^{nc}}{(q/\epsilon)\,V_p^{nc}} \simeq \left\{\begin{array}{lc} [\hat{\nu}_\theta/(\hat{\nu}_\theta+\hat{\nu}_\varphi)] & 0\leq k\leq 1\\ {[\hat{\nu}_\theta/(\hat{\nu}_\theta+\hat{\nu}_\varphi)]\,{\rm e}^{-(k-1)/\delta_1}} & k>1\end{array}\right.. \tag{56}$$

Note that the deviations of the ion poloidal velocity from its neoclassical value are only significant within a few island widths of the rational surface (since $A(k)\rightarrow 1$ as $k\rightarrow\infty$). On the other hand, the deviations of the ion toroidal velocity from its neoclassical value remain significant within a region of radial thickness $\delta_1\,w$, centered on the rational surface, that is *much wider* than the island chain. Now, our analysis requires the thickness of this region to be *small* compared to the plasma minor radius: *i.e.*, $\delta_1\,w \ll r_s$. This leads to the constraint

$$\nu_\varphi \gg \frac{\mu}{\rho\,r_s^2}: \tag{57}$$

i.e., the toroidal flow damping rate must be much larger than the perpendicular viscous diffusion rate across the whole plasma. If the above inequality is not satisfied then this implies that the perpendicular ion viscosity is sufficiently strong to overcome the toroidal flow damping, and is thus able to relax the toroidal velocity profile across the whole plasma [14].

In Regime I, the ion polarization parameter takes the form

$$\hat{J}_c \simeq c_1\,\hat{\beta}\,\hat{U}_p^{nc}\left(\hat{U}_p^{nc}+1\right), \tag{58}$$

where

$$\hat{U}_p^{nc} \equiv \left(\frac{\hat{\nu}_\varphi}{\hat{\nu}_\theta+\hat{\nu}_\varphi}\right)\hat{V}_p^{nc}, \tag{59}$$

and

$$c_1 \equiv [C(k)\,L^2(k)]_{k=1} + \int_1^\infty C(k)\,\frac{d}{dk}[L^2(k)]\,dk = 1.38. \tag{60}$$

Note that the ion polarization effect is *stabilizing* (*i.e.*, $\hat{J}_c<0$) when

$$0 > V_p^{nc} > -\left(1+\frac{\hat{\nu}_\theta}{\hat{\nu}_\varphi}\right)V_{*\,i}, \tag{61}$$

and *destabilizing* otherwise. In other words, if the neoclassical phase velocity is in the *electron* diamagnetic direction (*i.e.*, $V_p^{nc}>0$), or strongly in the ion diamagnetic direction [*i.e.*, $V_p^{nc} < -(1+\hat{\nu}_\theta/\hat{\nu}_\varphi)\,V_{*\,i}$], then the ion polarization current induced by the neoclassical ion flow, relative to the locked island chain, is *destabilizing*. On the other hand, if the neoclassical phase velocity is only weakly in the *ion* direction then the polarization current is *stabilizing*.

The Regime I drag parameter is written

$$\hat{J}_s \simeq \hat{\beta}\,\hat{\nu}_\theta\left[s_1 + 8\left(\frac{\epsilon}{q}\right)^2\left(\frac{\hat{\nu}_\mu}{\hat{\nu}_\varphi}\right)^{1/2}\right]\hat{U}_p^{nc}, \tag{62}$$

where

$$s_1 \equiv 8\left(\int_0^1 D(k)\,dk + \int_1^\infty \frac{[A(k)\,B(k)-1]}{A(k)}\,dk\right) = 5.51. \tag{63}$$

The first and second terms in square brackets on the right-hand side of (62) come from poloidal flow damping and toroidal flow damping, respectively. Incidentally, in the limit in which the inequality (57) is not satisfied, and perpendicular viscosity is strong enough to relax the toroidal velocity profile across the whole plasma, the drag due to toroidal flow damping is converted into a conventional viscous drag [14]. Note that, in addition to its direct contribution to $\hat{J}_s$, toroidal flow damping also contributes indirectly to $\hat{J}_c$ and $\hat{J}_s$ by modifying the poloidal velocity profile.

Regime II corresponds to $\hat{\nu}_\varphi \gg \hat{\nu}_\theta,\, \hat{\nu}_\mu$. The solution to (29) in this regime is

$$Y(k) \simeq \frac{1}{A(k)}\left(1-\frac{\hat{\nu}_\theta}{\hat{\nu}_\varphi}\left[1-\frac{1}{A(k)\,B(k)}\right]\right). \tag{64}$$

Hence, the ion poloidal velocity profile across the O-point becomes

$$\frac{V_{\theta\,i}-V_{\theta\,i}^{nc}}{V_p^{nc}} \simeq \begin{cases} -1 & 0\leq k\leq 1 \\ 1/A(k)-1 & k>1 \end{cases}, \tag{65}$$

whilst the corresponding ion toroidal velocity profile is written

$$\frac{V_{\varphi\,i}-V_{\varphi\,i}^{nc}}{(q/\epsilon)\,V_p^{nc}}\simeq\left\{\begin{array}{cc}(\hat{\nu}_\theta/\hat{\nu}_\varphi) & 0\leq k\leq 1\\ (\hat{\nu}_\theta/\hat{\nu}_\varphi)\,(1-1/[A(k)\,B(k)]) & k>1\end{array}\right.. \tag{66}$$

Observe that the deviations of the ion poloidal and toroidal velocities from their respective neoclassical values are only significant within a few island widths of the rational surface (since $A(k)\rightarrow 1$ and $B(k)\rightarrow 1$ as $k\rightarrow\infty$).

In Regime II, the ion polarization parameter takes the form

$$\hat{J}_c\simeq c_1\,\hat{\beta}\,\hat{V}_p^{nc}\,(\hat{V}_p^{nc}+1). \tag{67}$$

Thus, the ion polarization effect is stabilizing when

$$0>V_p^{nc}>-V_{*\,i}, \tag{68}$$

and destabilizing otherwise.

The Regime II drag parameter is written

$$\hat{J}_s\simeq s_1\,\hat{\beta}\,\hat{\nu}_\theta\,\hat{V}_p^{nc}. \tag{69}$$

Note that the contribution to $\hat{J}_s$ from toroidal flow damping is completely negligible in this regime.

Finally, Regime III corresponds to $\hat{\nu}_\theta\gg\hat{\nu}_\varphi,\,\hat{\nu}_\mu^{\,2}/\hat{\nu}_\varphi$. The solution to (29) in this regime is

$$Y(k)\simeq\frac{\hat{\nu}_\varphi}{\hat{\nu}_\theta}\left[\frac{B(k)}{A(k)\,B(k)-1+\hat{\nu}_\varphi/\hat{\nu}_\theta}\right]. \tag{70}$$

Hence, the ion poloidal velocity profile across the O-point becomes

$$\frac{V_{\theta\,i}-V_{\theta\,i}^{nc}}{V_p^{nc}}\simeq\left\{\begin{array}{cc}\hat{\nu}_\varphi/\hat{\nu}_\theta & 0\leq k\leq 1\\ (\hat{\nu}_\varphi/\hat{\nu}_\theta)\,B(k)\,[1-A(k)]/[A(k)\,B(k)-1] & \delta_3\gg k>1,\\ (2\,\hat{\nu}_\varphi/\hat{\nu}_\theta)^{1/2}\,(k/\delta_3)^2/[1+(k/\delta_3)^4] & k\gtrsim\delta_3\end{array}\right. \tag{71}$$

whilst the corresponding ion toroidal velocity profile is written

$$\frac{V_{\varphi\,i}-V_{\varphi\,i}^{nc}}{(q/\epsilon)\,V_p^{nc}}\simeq\left\{\begin{array}{cc}1-(\hat{\nu}_\varphi/\hat{\nu}_\theta) & 0\leq k\leq 1\\ 1-(\hat{\nu}_\varphi/\hat{\nu}_\theta)\,A(k)\,B(k)/[A(k)\,B(k)-1] & \delta_3\gg k>1,\\ 1/[1+(k/\delta_3)^4] & k\gtrsim\delta_3\end{array}\right. \tag{72}$$

where $\delta_3\equiv[\hat{\nu}_\theta/(32\,\hat{\nu}_\varphi)]^{1/4}\gg 1$. Observe that the deviations of the ion poloidal and toroidal velocities from their respective neoclassical values remain significant over a region of radial thickness $\delta_3\,w$, centered on the rational surface, that is *much wider* than the island chain. We require the thickness of this region to be *small* compared to the plasma minor radius: *i.e.*, $\delta_3\,w\ll r_s$. This leads to the constraint

$$\nu_\varphi\gg\left(\frac{w}{r_s}\right)^4\left(\frac{\epsilon}{q}\right)^2\nu_\theta. \tag{73}$$

In other words, the toroidal flow damping rate cannot become too small compared to the poloidal flow damping rate.

Table 2. Flow damping regimes. Here, $\hat{U}_p^{nc} = [\hat{\nu}_\varphi/(\hat{\nu}_\theta + \hat{\nu}_\varphi)]\,\hat{V}_p^{nc}$.

Reg.	Extent	$\hat{J}_c/\hat{\beta}$	$\hat{J}_s/\hat{\beta}$
I	$\hat{\nu}_\mu \gg \hat{\nu}_\varphi^{1/2}\,(\hat{\nu}_\theta + \hat{\nu}_\varphi)^{1/2}$	$1.38\,\hat{U}_p^{nc}\,(\hat{U}_p^{nc} + 1)$	$\hat{\nu}_\theta\left[5.51 + 8\,(\epsilon/q)^2\,(\hat{\nu}_\mu/\hat{\nu}_\varphi)^{1/2}\right]\hat{U}_p^{nc}$
II	$\hat{\nu}_\varphi \gg \hat{\nu}_\theta,\, \hat{\nu}_\mu$	$1.38\,\hat{V}_p^{nc}\,(\hat{V}_p^{nc} + 1)$	$5.51\,\hat{\nu}_\theta\,\hat{V}_p^{nc}$
III	$\hat{\nu}_\theta \gg \hat{\nu}_\varphi,\, \hat{\nu}_\mu^2/\hat{\nu}_\varphi$	$0.617\,(\hat{\nu}_\varphi/\hat{\nu}_\theta)^{3/4}\,\hat{V}_p^{nc}\,(\hat{V}_p^{nc}/4 + 1)$	$0.617\,\hat{\nu}_\theta^{1/4}\,\hat{\nu}_\varphi^{3/4}\,\hat{V}_p^{nc}$

The ion polarization parameter in Regime III takes the form

$$\hat{J}_c \simeq s_3\,\hat{\beta}\left(\frac{\hat{\nu}_\varphi}{\hat{\nu}_\theta}\right)^{3/4}\hat{V}_p^{nc}\,(c_3\,\hat{V}_p^{nc} + 1), \tag{74}$$

where

$$s_3 \equiv 2^{7/4}\int_0^\infty \frac{dy}{1 + y^4}\,dy = 0.617, \tag{75}$$

$$c_3 \equiv \int_0^\infty \frac{y^4\,dy}{(1 + y^4)^2}\bigg/\int_0^\infty \frac{dy}{1 + y^4} = 1/4. \tag{76}$$

The ion polarization effect is stabilizing when

$$0 > V_p^{nc} > -4\,V_{*\,i}, \tag{77}$$

and destabilizing otherwise.

The Regime III drag parameter is written

$$\hat{J}_s \simeq s_3\,\hat{\beta}\,\hat{\nu}_\theta^{1/4}\,\hat{\nu}_\varphi^{3/4}\,\hat{V}_p^{nc}. \tag{78}$$

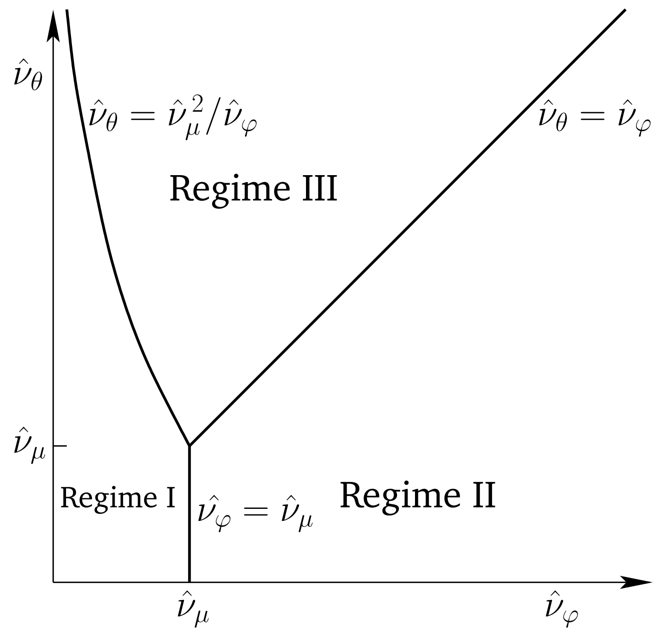
The direct contribution of toroidal flow damping to $\hat{J}_s$ is again negligible. However, as before, toroidal flow damping contributes indirectly to $\hat{J}_s$ (and $\hat{J}_c$) by modifying the poloidal velocity profile in the vicinity of the island chain.

The various different regimes for steady-state locked island chain solutions are summarized in Table 2 and Figure 2. According to these, in the limit $\hat{\nu}_\varphi \to 0$, which corresponds to Regime I, $\hat{J}_c \to 0$ and $\hat{J}_s \to 0$. In other words, in the absence of toroidal flow damping there is no drag on a locked island chain due to neoclassical ion flow. Likewise, no ion polarization current is generated in the vicinity of the chain. The reason for this is that, in the absence of toroidal flow damping, the ion toroidal velocity is free to adjust itself in such a manner that a locked island chain does not present an obstacle to the net ion flow.

6. Summary

This paper investigates the physics of a *locked magnetic island chain* maintained in the pedestal of an H-mode tokamak plasma by a static, externally generated, multi-harmonic, helical magnetic perturbation. It is assumed that the non-resonant harmonics of the external perturbation give rise to significant *toroidal flow damping* in the pedestal,

Figure 2. Flow damping regimes.



in addition to the naturally occurring *poloidal flow damping*. Furthermore, the flow damping is assumed to relax the pedestal ion toroidal and poloidal velocities to fixed values determined by neoclassical theory. We find that the resulting neoclassical ion flow produces a *drag* on the locked island chain. Moreover, this drag causes a helical phase-shift to develop between the chain and the resonant harmonic of the external perturbation. It is demonstrated that when this phase-shift exceeds a critical value, the chain unlocks from the resonant harmonic, and then starts to rotate, after which it decays away, and is replaced by a helical current sheet. We also find that the neoclassical flow generates an *ion polarization current* in the vicinity of the island chain which either increases or decreases the chain's radial width, depending on the direction of the flow. If the polarization effect is stabilizing, and exceeds a critical amplitude, then the helical island equilibrium becomes unstable, and the chain again decays away. Note that the presence of significant toroidal flow damping in the pedestal is crucial, since, in the absence of such damping, the toroidal ion flow is free to adjust itself in such a manner as to eliminate the drag and the polarization current. The analysis presented in this paper allows the critical amplitude of the resonant harmonic of the external perturbation at which the island chain either unlocks or becomes unstable to be calculated as a function of the pedestal ion pressure, the neoclassical poloidal and toroidal ion velocities, and the poloidal and toroidal flow damping rates.

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