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Resistive wall feedback stabilization

T. H. Jensen[a] and R. Fitzpatrick[b]
General Atomics, San Diego, California 92186-5608

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A feedback system, which essentially makes a resistive wall appear ideally conducting, is discussed. Such a system applied to a resistive wall surrounding a plasma will stabilize certain modes which would be unstable in the absence of the feedback system. The system discussed is similar to the ''intelligent shell'' by Bishop [Plasma Phys. Controlled Fusion **31**, 1179 (1989)]; it utilizes a number of autonomous subsystems, each covering only a fraction of the resistive wall. A model example discussed suggests that only relatively few autonomous subsystems are needed and that the requirements of the electronics appear modest.
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I. INTRODUCTION

There exist interesting cases for which a plasma surrounded by a wall (without touching it) is stable when the wall is conducting but unstable if it is resistive. For the common case, for which the Alfvén time of the plasma is much shorter than the resistive time of the wall, the growth time of the instability is determined by the resistive wall time. Resistive wall times are typically large compared to the response time of relatively inexpensive power electronics components. Therefore, it is of practical interest to consider feedback systems which essentially make a resistive wall appear conducting to the plasma and thereby stabilize it.

For axisymmetric devices such as tokamaks it is particularly easy to institute feedback stabilization for axisymmetric modes of instability. Such systems are now well developed.[1]

It is more complicated to make such a feedback system for nonaxisymmetric modes. For this case one can follow two different strategies. Following one strategy, the system is designed to stabilize a specific large scale, known, unstable mode.[2–4] The sensing part of the system measures the amplitude and phase of the unstable mode. The sensing information is then utilized for driving coil currents, which result in fields tending to suppress the mode. Following the second strategy, many autonomous feedback systems are used instead. Each will cover only a fraction of the surface area of the resistive wall. Each of the systems may utilize a loop surrounding its area; the sensing part senses a magnetic field perpendicular to the wall; this information is used to drive a current in the loop so that the resulting magnetic field tends to oppose the field sensed. Thus such a feedback system will make the wall appear conducting in that it opposes normal magnetic field perturbations. It is therefore not just operative for a specific mode; it is, however, required that the mode wavelength is large compared to the loop size. Systems of this kind have previously been considered by Bishop[5] and Fitzpatrick and Jensen.[6] In Ref. 6, emphasis was placed on a feedback system which imitates a resistive wall moving relative to the real resistive wall. In the present paper, emphasis is placed on a feedback system which makes the resistive wall appear conductive.

In this paper we only consider systems employing the strategy of utilizing a number of autonomous feedback loops. A formulation of the problem is given, which allows determination of the stabilizing properties of realistic feedback arrangements. It is implied in this paper that the electronics of the feedback system is ideal in that there are no phase shifts or delays between the sensed perturbation (the perpendicular magnetic field) and the current driven in the loop, although the formalism given allows incorporating such effects. An estimate of the power needed for the feedback amplifiers is also given; it suggests that only modest powers are needed.

II. CONCEPTUAL FRAMEWORK

We consider perturbations of a plasma in an initial magnetohydrodynamic (MHD) equilibrium. The plasma is assumed to be surrounded by a thin, resistive wall. The perturbation of the field is $\bar{b}$. For the problem considered, we only need consider cases for which the component of $\bar{b}$, perpendicular to the wall, at the wall, is nonvanishing. This perpendicular component is termed $b_\perp(y,z)$. Here, y and z are coordinates which describe the location on the wall.

Outside the wall we assume that vacuum conditions prevail, i.e., $\bar{\nabla}\times\bar{b} = 0$. Then the magnetic field can be represented by

$$\bar{b}=\bar{\nabla}S, \tag{1}$$

for which

$$\bar{\nabla}^2 S=0. \tag{2}$$

Since the boundary condition far away from the wall is $\bar{\nabla}S = 0$, one can, in principle, determine S if the perpendicular gradient of S at the wall is given, i.e., if $b_\perp(y,z)$ is given. Therefore, if $b_\perp(y,z)$ is given, the parallel component of $\bar{b}$ on the outer side of the wall $\bar{b}_{\parallel 0}(y,z)$ can, in principle, be determined. Symbolically, one may write

$$\bar{b}_{\parallel 0}(y,z)=\overline{Op}_0\{b_\perp(y,z)\}. \tag{3}$$

As discussed above, we only need consider perturbations which develop slowly compared to the Alfvén time. There-

[a] Electronic mail: jensen@gav.gat.com
[b] IFS/The University of Texas at Austin, Austin, Texas.

fore, the perturbed configurations considered must also be MHD equilibria. Then, also on the inner side of the wall, one can find the parallel component of the field if the perpendicular component is given. Thus in analogy to Eq. (3) we may write

$$\overline{b_{\parallel i}}(y,z)=\overline{Op_i}\{b_\perp(y,z)\}. \tag{4}$$

Since $\overline{b}$ is divergence free and the wall is considered thin, $b_\perp(y,z)$ is the same on both sides of the wall.

If one then specifies $b_\perp(y,z)$, one can find the total current density (A/m) required to flow in the wall,

$$\overline{J}=(1/\mu_0)(\overline{b_{\parallel i}}-\overline{b_{\parallel 0}})\times\hat{n}=[\overline{Op_i}\{b_\perp\}-\overline{Op_0}\{b_\perp\}]\times\hat{n}, \tag{5}$$

where $\hat{n}$ is an outward pointing unit vector, perpendicular to the wall.

The resistive wall current density is

$$\overline{J_R}=\frac{\delta}{\eta}\,\hat{n}\times\overline{E}\times\hat{n}, \tag{6}$$

where η and δ are the resistivity of the wall material and its thickness, while $\overline{E}$ is the electric field. Taking the curl of Eq. (6) and utilizing the induction law one gets

$$\overline{\nabla}\times\overline{J_R}\cdot\hat{n}=\frac{\delta}{\eta}\,\overline{\nabla}\times\overline{E}\cdot\hat{n}=-\gamma\,\frac{\delta}{\eta}\,b_\perp(y,z). \tag{7}$$

The assumption is made here that the time dependence of the perturbation quantities is $e^{\gamma t}$.

It is assumed that none of the feedback loops link the hole of the toroidally shaped wall (or the torus itself). This is of practical importance since it means that a strong coupling between the feedback loops and the solenoid which drives the plasma current is avoided. It also means that the sum $[I(y,z)]$of feedback loop currents which link a given point (y,z) on the wall is well-defined. The wall current density due to the feedback loop current is then

$$\overline{J_L}=\overline{\nabla}I(y,z)\times\hat{n}. \tag{8}$$

The feedback loop currents are determined by a ''feedback prescription'' from measurement of $b_\perp(y,z)$; symbolically one may write

$$I(y,z)=FOp\{b_\perp(y,z)\}. \tag{9}$$

From Eqs. (3) to (5) and Eqs. (7) to (9), one now gets

$$\overline{\nabla}\cdot[\overline{Op_0}\{b_\perp(y,z)\}-\overline{Op_i}\{b_\perp(y,z)\}]$$
$$+\overline{\nabla}^2 FOp\{b_\perp(y,z)\}+\gamma\,\frac{\delta}{\eta}\,b_\perp(y,z)=0. \tag{10}$$

The operators $\overline{Op_i},\overline{Op_0},FOp$ are assumed linear; then Eq. (10) is a two-dimensional, scalar eigenvalue problem for which γ is the eigenvalue. If the real parts of all the eigenvalues are negative, the system is stable. Thus the task is to find a feedback prescription, i.e., FOp, which will make the real parts of all eigenvalues negative. Note that the formalism given allows analysis of quite general systems; the feedback loops may, for example, be overlapping, and FOp need not be local and it may involve a delay between I and $b_\perp$.

III. EXAMPLES

As an initial, unperturbed equilibrium we choose, for the sake of simplicity, one with two ignorable coordinates, namely y and z. For the perturbation we assume also that z is ignorable while it is assumed periodic in y with the period L. It is believed that even such simple cases, in respect to feedback stabilization, are representative of realistic cases.

The magnetic field of the initial equilibrium is described by the functions B_z and ψ through

$$\overline{B}=B_z\hat{z}+\overline{\nabla}\psi\times\hat{z}. \tag{11}$$

Ignoring plasma pressure, the equilibrium condition $(\overline{\nabla}\times\overline{B})\times\overline{B}=0$ is found to be

$$\nabla^2\psi+B_zB_z'=0,\quad \overline{\nabla}B_z=B_z'\overline{\nabla}\psi. \tag{12}$$

The perturbation of the magnetic field is described by the functions b_z and φ through

$$\overline{b}=b_z\hat{z}+\overline{\nabla}\varphi\times\hat{z}. \tag{13}$$

The periodicity assumption means that

$$b_z(x,y)=b_z(x,y+L),\quad \varphi(x,y)=\varphi(x,y+L). \tag{14}$$

The condition that the perturbed equilibrium is also an equilibrium is

$$\overline{\nabla}\times(\overline{B}+\overline{b})\times(\overline{B}+\overline{b})=0, \tag{15}$$

which to first order in the perturbation yields

$$\nabla^2\varphi+(B_zB_z')'\varphi=0. \tag{16}$$

The ideal MHD constraint yields that φ must vanish where $\overline{\nabla}\psi$ vanishes.

A particularly simple case to consider is one where $\psi(x)=\psi(-x)$ and $(B_zB_z')'=\beta^2$ is a constant in the plasma region. For this case the equilibrium is somewhat pathological in that it has a singular current at the singular surface; this does not affect the solutions considered since φ vanishes there. The solution to Eq. (16) can then be expressed as

$$\varphi(x,y)=\sum_\nu \varphi_\nu e^{2\pi i\nu y/L}\sin\left[x\sqrt{\beta^2-\left(\frac{2\pi\nu}{L}\right)^2}\right]. \tag{17}$$

One notices that ν/L is the ''poloidal'' wave number. We assume the resistive wall is located at $x=\pm a$ and need only consider $x>0$.

Using the concepts of Sec. II, one sees from Eq. (17) that

$$b_\perp=\frac{\partial\varphi}{\partial y}$$
$$=\sum_\nu \varphi_\nu\,\frac{2\pi i\nu}{L}\,e^{2\pi i\nu y/L}\sin\left[a\sqrt{\beta^2-\left(\frac{2\pi\nu}{L}\right)^2}\right]. \tag{18}$$

For $x>a$, we have $\overline{\nabla}\times\overline{b}=0$, that φ must vanish at $x\to\infty$ as well as that it must match the solution (18) at $x=a$. Using these conditions one finds

$$\overline{b_{\parallel 0}}\cdot\hat{y}=\sum_\nu \varphi_\nu\left|\frac{2\pi\nu}{L}\right|e^{2\pi i\nu y/L}\sin\left[a\sqrt{\beta^2-\left(\frac{2\pi\nu}{L}\right)^2}\right], \tag{19}$$

and from Eq. (17) one finds

$$\overline{b_{\|i}}\cdot\hat{y}=-\sum_{\nu}\ \varphi_{\nu}e^{2\pi i\nu y/L}\sqrt{\beta^2-\left(\frac{2\pi\nu}{L}\right)^2}\times\cos\left[a\sqrt{\beta^2-\left(\frac{2\pi\nu}{L}\right)^2}\right]. \tag{20}$$

We now consider, as an illustration, a feedback system with M feedback loops per period. We introduce

$$S_m(y)\equiv\begin{cases}0, & \text{for}\quad 0<y<\dfrac{m}{M}L,\\ 1, & \text{for}\ \dfrac{m}{M}<y<\dfrac{m+1}{M}L,\\ 0, & \text{for}\ \dfrac{m+1}{M}L<y<L.\end{cases} \tag{21}$$

Sensor loops measure the flux through each feedback loop,

$$f_m=\int_0^L S_m(y)b_{\perp}(y)dy. \tag{22}$$

We consider a simple feedback prescription [Eq. (9)], namely

$$I(y)=Q\sum_{m=0}^{M-1} f_m S_m(y), \tag{23}$$

where Q is a feedback gain. One can now express the eigenvalue problem (10) using Eqs. (18), (19), (20), and (23). Each term in the equation is multiplied by $e^{-2\pi i\mu y/L}$ and integrated over y from 0 to L. The resulting matrix eigenvalue equation then becomes

$$\left\{\frac{\gamma\delta}{\eta}+\left|\frac{2\pi\mu}{L}\right|+\sqrt{\beta^2-\left(\frac{2\pi\mu}{L}\right)^2}\,\frac{\cos\left[a\sqrt{\beta^2-\left(\frac{2\pi\mu}{L}\right)^2}\right]}{\sin\left[a\sqrt{\beta^2-\left(\frac{2\pi\mu}{L}\right)^2}\right]}+Q\,\frac{M}{L}\,2\left(1-\cos\frac{2\pi\mu}{M}\right)\right\}\varphi_{\mu}+\sum_N\ QM\left(1+\frac{NM}{\mu}\right)\left(1-\cos\frac{2\pi\mu}{M}\right)\varphi_{\mu+NM}=0, \tag{24}$$

where $N=\pm1,\pm2,\ldots$. When M is sufficiently large one may ignore the off-diagonal elements of Eq. (24) so that the eigenvalues become

$$\left(\frac{\gamma\delta}{\eta}\right)_{\mu}=-\left|\frac{2\pi\mu}{L}\right|-\sqrt{\beta^2-\left(\frac{2\pi\mu}{L}\right)^2}\,\frac{\cos\left[a\sqrt{\beta^2-\left(\frac{2\pi\mu}{L}\right)^2}\right]}{\sin\left[a\sqrt{\beta^2-\left(\frac{2\pi\mu}{L}\right)^2}\right]}-Q\,\frac{M}{L}\,2\left(1-\cos\frac{2\pi\mu}{M}\right). \tag{25}$$

For the case of no feedback, i.e., $Q=0$, Eq. (25) is well known. One recognizes that the most unstable mode is for $\mu=1$. For the case relevant to tokamaks $(2\pi/L)^2\ll\beta^2$ we have stability for $a\beta<\pi/2$, and one finds instability, even for the case of a conducting wall for $a\beta>\pi$; therefore the region of interest for the topic of this paper is $\pi/2<a\beta<\pi$.

For the case of feedback, $Q\neq0$, we make the assumption that the feedback amplifiers are ideal, i.e., Q is real and constant (independent of frequency). One sees from Eq. (25) that a positive Q is always stabilizing (except for $\mu=M, 2M,\ldots$, which are assumed stable without feedback). This suggests that the feedback scheme considered is practical even with relatively few feedback loops per period. One way of determining the needed number of feedback loops per period is to solve the matrix eigenvalue problem (24) for various values of M. Presently we do not know how many loops per period may be needed.

An example with a geometrical structure which may be considered closer to that of tokamaks is discussed in the Appendix. In that example, helical symmetry is common for both the initial and the perturbed equilibrium. The results obtained are similar to those obtained above.

IV. ESTIMATE OF POWER NEEDS OF FEEDBACK AMPLIFIERS

The amplifiers used in the feedback systems are considered linear. Real amplifiers are only linear for amplitudes below a certain maximum. A key factor for the cost of an amplifier is its output power at this maximum amplitude. Therefore, the cost of the feedback amplifiers depends on the fluctuation amplitude of the feedback stabilized equilibrium. The fluctuation amplitude cannot be determined from considerations of stability alone. A lower limit for the fluctuation amplitude may be obtained from thermodynamic arguments. Instead, we here take an empirical approach. Fluctuation amplitudes from fusion experiments are typically in the range of a few gauss. We use this amplitude $\widetilde{B}$ as an empirical input for the power estimate.

We consider a mode with the wave number k, and let each feedback loop have the area l^2. The effective feedback current density needed to counteract a mode of amplitude $\widetilde{B}$ is

$$J_{\text{eff}}\sim\widetilde{B}/\mu_0. \tag{26}$$

The current in each loop I is related to J_{eff} through

$$J_{\text{eff}}\sim|\overline{\boldsymbol{\nabla} I}|\sim kI. \tag{27}$$

The growth time of the mode is

$$\tau\sim\frac{\mu_0\delta}{k\eta}, \tag{28}$$

where δ and η are the thickness of the wall and the resistivity of its material. The inductive voltage needed in a loop is then

$$V\sim l^2\widetilde{B}/\tau. \tag{29}$$

One can then estimate the power needed per unit area of the wall, using Eqs. (26)–(29),

$$P \sim \frac{VI}{l^2} \sim \frac{\widetilde{B}^2 \eta}{\mu_0^2 \delta}. \tag{30}$$

If, for this estimate, one uses $\widetilde{B} = 5\times 10^{-4}$ V s/m^2, $\eta = 10^{-6}$ V m/A (appropriate for stainless steel), and $\delta = 5\times 10^{-3}$ m, one gets $P \sim 30$ W/m^2. This suggests that the cost of amplifiers will be small relative to other costs of fusion experimental devices.

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APPENDIX: EXAMPLE OF HELICAL SYMMETRY

Similar results can be obtained using configurations which appear closer to those of tokamaks. Using cylindrical coordinates r, θ, z we introduce the ''helical vector''

$$\overline{\mu} = \hat{z} + \frac{nr}{mR}\,\hat{\theta}. \tag{A1}$$

Here n and m are simulating toroidal and poloidal mode numbers while R simulates the major radius of the tokamak. Helical symmetry is defined by $\overline{\mu}\cdot\overline{\nabla} = 0$. We consider initial, unperturbed equilibria which possess both helical symmetry and axisymmetry, i.e., $\partial/\partial\theta = 0$, and perturbed equilibria which remains helically symmetric but periodic in θ with the period $2\pi/m$.

For this case it is convenient to represent the unperturbed field by the functions F and ψ through

$$\overline{B} = \frac{1}{\mu^2}\,[F\overline{\mu} + \overline{\nabla}\psi \times \overline{\mu}]. \tag{A2}$$

Using tokamak ordering,

$$(\overline{\nabla}\psi/F)^2 \ll 1, \quad \left(\frac{nr}{mR}\right)^2 \ll 1, \tag{A3}$$

one finds readily that $(\overline{\nabla}\times\overline{B})\times\overline{B} = 0$ yields

$$\overline{\nabla}F = F'\,\overline{\nabla}\psi, \quad \overline{\nabla^2}\psi + FF' - 2\,\frac{n}{mR}\,F = 0. \tag{A4}$$

The perturbation of the field is also assumed helically symmetric so that it may be represented by f and φ through [assume Eq. (A3) valid]

$$\overline{b} = f\overline{\mu} + \overline{\nabla}\varphi \times \overline{\mu}. \tag{A5}$$

We assume that f and φ are periodic in θ with the period $2\pi/m$. The condition that the perturbed configuration is an equilibrium (ignoring pressure),

$$\overline{\nabla}\times(\overline{B} + \overline{b})\times(\overline{B} + \overline{b}) = 0, \tag{A6}$$

yields to first order,

$$f = F'\varphi, \quad \overline{\nabla^2}\varphi + \left[FF' - 2\,\frac{n}{mR}\,F\right]'\varphi = 0. \tag{A7}$$

Note that because of the ordering assumption (A3) the partial derivative with respect to z becomes small so that the Laplacian becomes

$$\overline{\nabla^2} = \frac{1}{r}\,\frac{\partial}{\partial r}\,r\,\frac{\partial}{\partial r} + \frac{1}{r^2}\,\frac{\partial^2}{\partial\theta^2}. \tag{A8}$$

In ideal MHD, φ must vanish at the singular surface r_s where $\overline{\nabla}\psi$ vanishes.

In order to consider a simple case, we assume

$$\left[FF' - \frac{2n}{mR}\,F\right]' = \begin{cases} \beta^2, & r_s < r < a, \\ 0, & a < r < \infty. \end{cases} \tag{A9}$$

One sees then from Eq. (A7) that the solution for $r_s < r < a$ can be written as

$$\varphi(r,\theta) = \sum_{\nu} \varphi_\nu e^{im\nu\theta}[\alpha_\nu J_{\nu m}(r\beta) + \beta_\nu Y_{\nu m}(r\beta)], \tag{A10}$$

where the ratio α_ν/β_ν is adjusted so that $\varphi(r_s,\theta) = 0$, i.e., Eq. (A10) becomes similar to Eq. (17). For $a < r < \infty$, the solution to Eqs. (A7) and (A9) is of the form

$$\varphi(r,\theta) = \sum_{\nu} \rho_\nu e^{im\nu\theta}\left(\frac{r}{r_s}\right)^{-m\nu}. \tag{A11}$$

Here the ρ_ν's are

$$\rho_\nu = \alpha_\nu J_{\nu m}(a\beta) + \beta_\nu Y_{\nu m}(a\beta), \tag{A12}$$

to match the solutions at $r = a$. It is now clear that one can proceed in the same fashion as for the $\partial/\partial z = 0$ case discussed above and obtain similar results.

[1]E. A. Lazarus, J. B. Lister, and G. H. Nelson, Nucl. Fusion **30**, 111 (1990).
[2]A. W. Morris, T. C. Hender, J. Hugill, P. S. Haynes, P. C. Johnson, B. Lloyd, D. C. Robinson, and C. Sylvester, Phys. Rev. Lett. **64**, 1254 (1990).
[3]B. Alper, Phys. Fluids B **2**, 1338 (1990).
[4]A. B. Mikhailovskii and B. N. Kuvshinov, Plasma Phys. Rep. **2**, 172 (1996) (translated from Fiz. Plazmy).
[5]C. M. Bishop, Plasma Phys. Controlled Fusion **31**, 1179 (1989).
[6]R. Fitzpatrick and T. H. Jensen, Phys. Plasmas **3**, 2641 (1996).