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Feedback stabilization of the resistive shell mode in a tokamak fusion reactor

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Stabilization of the ‘resistive shell mode’ is vital to the success of the ‘advanced tokamak’ concept. The most promising reactor relevant approach is to apply external feedback using, for instance, the previously proposed ‘fake rotating shell’ scheme [R. Fitzpatrick and T. H. Jensen, *Phys. Plasmas* **3**, 2641 (1996)]. This scheme, like other simple feedback schemes, only works if the feedback controlled conductors are located inside the ‘critical radius’ at which a perfectly conducting shell is just able to stabilize the ideal external kink mode. In general, this is not possible in a reactor, since engineering constraints demand that any feedback controlled conductors be placed *outside* the neutron shielding blanket (i.e., relatively far from the edge of the plasma). It is demonstrated that the fake rotating shell feedback scheme can be modified so that it works even when the feedback controlled conductors are located well beyond the critical radius. The gain, bandwidth, current, and total power requirements of such a feedback system for a reactor sized plasma are estimated to be less than 100, a few Hz, a few tens of kA, and a few MW, respectively. These requirements could easily be met using existing technology. It is concluded that feedback stabilization of the resistive shell mode is possible in a tokamak fusion reactor. © 1997 American Institute of Physics. [S1070-664X(97)03307-7]

I. INTRODUCTION

The ‘advanced tokamak’ concept¹ aims to simultaneously maximize the plasma beta,² the energy confinement time, and the fraction of the current due to the noninductive bootstrap effect.³ The eventual aim is, of course, to design an attractive fusion power plant which can operate in steady state, at high fusion power density, with low recirculating power.⁴ The beta limits in advanced tokamak designs are invariably set by low mode number external kink modes. As is well known, external kink modes can be stabilized by placing a perfectly conducting shell sufficiently close to the edge of the plasma.⁵ In fact, acceptable beta limits are only obtained in advanced tokamak designs if the effect of a close fitting ideal shell is taken into account in the magnetohydrodynamical (MHD) stability calculations.⁶ In reality, of course, a stabilizing shell possesses a small, but finite, resistivity. According to conventional theory, a close fitting resistive shell can convert the fast growing external kink mode into a mode which grows on the comparatively long L/R time⁷ of the shell, but it cannot completely stabilize the mode.⁸ The residual slowly growing mode is termed a ‘resistive shell mode.’ The growth time of this mode is generally much shorter than the typical pulse length of a tokamak discharge. The resistive shell mode can, therefore, give rise to the premature termination of the discharge.⁹ It follows that the full advantages of the advanced tokamak concept are only achievable if the resistive shell mode is somehow stabilized.

There are two main approaches to suppressing the resistive shell mode. The first involves spinning the plasma toroidally using unbalanced neutral beam injection (NBI). It turns out that the resistive shell mode is stabilized above a certain critical rotation rate.¹⁰ There is a broad consensus of opinion^{10–12} that, under most circumstances, the critical ro-

tation velocity in tokamaks is around a few percent of the Alfvén velocity.¹³ Such a large rotation velocity is often achieved in present day tokamaks, and does, indeed, appear to give rise to the stabilization of the resistive shell mode.^{14,15} However, it is highly unlikely that this level of toroidal rotation could ever be maintained in a tokamak reactor by realistic levels of NBI power, because of the large plasma volume required by a reactor, plus the fact that in a big device NBI must be performed with high energy (and, therefore, low momentum) neutral particles. For instance, on the International Tokamak Experimental Reactor (ITER)¹⁶ (major radius 8 m, toroidal magnetic field strength 6 T, plasma number density 10^{20} m^{-3}) realistic levels of NBI (i.e., 50 MW of 1 MeV negative ion beam injection) are estimated¹⁷ to drive a central toroidal rotation velocity which is only a fraction of a percent (i.e., 0.4%) of the Alfvén velocity: This is almost certainly insufficient to stabilize the resistive shell mode.

The second method of suppressing the resistive shell mode is to employ a network of feedback controlled conductors surrounding the plasma. In principle, this approach is more likely to work in a reactor than the alternative approach of spinning the plasma toroidally. Two feedback schemes have been proposed: the ‘intelligent shell’¹⁸ [which was originally developed to stabilize the resistive shell mode in reversed field pinches (RFPs)], and the ‘fake rotating shell.’¹⁹ Both schemes lead to fairly modest current, power, and bandwidth requirements for the feedback amplifiers. However, the latter scheme has the advantage that, unlike the former, it does not require extremely high gain feedback circuits (which, in practice, are often problematic). Both schemes only work if the feedback controlled conductors are situated *inside* the radius at which a complete perfectly conducting shell causes the ideal external kink mode to become

marginally stable. Henceforth, this radius is referred to as the ‘‘critical radius.’’

The major issues which must be addressed when designing a feedback system for a tokamak reactor are threefold: space, maintainability, and survivability. By definition, there is a large flux of neutrons emanating from a reactor plasma. This necessitates surrounding the plasma by a thick neutron absorbing blanket. Certain critical tokamak components (in particular, any super-conducting components) may require additional shielding. The net result is that there is very little free space in the immediate vicinity of the plasma. It is, of course, vitally important that any feedback system in a reactor is capable of being repaired without the need to dismantle a large part of the device (in particular, any parts which have been strongly irradiated by neutrons). Finally, it is difficult to see how feedback controlled conductors which are required to carry substantial currents could survive for long in a high neutron flux environment, since conventional low resistance materials (e.g., copper, silver, aluminium) degrade rapidly when irradiated by neutrons. Thus space, maintainability, and survivability constraints virtually dictate that the closest a network of current carrying, feedback controlled conductors can be placed to the plasma in a tokamak reactor is just outside the neutron shielding blanket. Unfortunately, this implies that under most circumstances the network is situated *well outside* the critical radius for stabilizing the external kink mode. At first sight, this would seem to imply that it is impossible to reliably feedback stabilize the resistive shell mode in a tokamak reactor. The aim of this paper is to demonstrate that this conclusion is false. It turns out that the fake rotating shell feedback scheme can be modified so that it works even when all of the feedback controlled conductors are situated well outside the critical radius.

II. THE FAKE ROTATING SHELL FEEDBACK SCHEME

A. Introduction

It is convenient, at this stage, to review the salient features of the fake rotating shell feedback scheme. This scheme is described in more detail in Ref. 19.

B. Basic definitions

Consider a large aspect ratio, low beta, tokamak plasma²⁰ whose magnetic flux surfaces map out (almost) concentric circles in the poloidal plane. Such a plasma is well approximated as a periodic cylinder. Standard cylindrical polar coordinates (r, θ, z) are adopted. The system is assumed to be periodic in the z direction, with periodicity length $2\pi R_0$, where R_0 is the simulated major radius of the device. It is convenient to define a simulated toroidal angle $\phi = z/R_0$.

Suppose that the plasma is surrounded by a thin conducting shell of radius $r_w > a$ (where a is the minor radius of the plasma). The radially integrated perturbed current pattern flowing in the shell is specified by a current stream function $J_w(\theta, \phi)$ via

$$\delta \mathbf{I}_w = \nabla J_w \wedge \hat{\mathbf{r}}. \quad (1)$$

The perturbed magnetic field can be written in term of a perturbed poloidal flux function $\psi(r, \theta, \phi)$ using

$$\delta \mathbf{B} = \nabla \psi \wedge \hat{\mathbf{z}}. \quad (2)$$

The perturbed magnetic flux in the shell is defined

$$\Psi_w(\theta, \phi) \equiv \psi(r_w, \theta, \phi). \quad (3)$$

Consider an m/n mode for which a general perturbed quantity takes the form

$$A(r, \theta, \phi) = \hat{A}(r) e^{i(m\theta - n\phi)}. \quad (4)$$

The poloidal flux eigenfunction $\hat{\psi}(r)$ satisfies the ‘‘cylindrical tearing mode equation’’⁵

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{d\hat{\psi}}{dr} \right) - \frac{m^2}{r^2} \hat{\psi} + \frac{\mu_0 j'_\phi}{B_\phi(nq/m - 1)} \hat{\psi} = 0, \quad (5)$$

where $B_\phi(r)$ is the equilibrium poloidal magnetic field strength, $j_\phi(r)$ is the equilibrium toroidal plasma current [with $j_\phi(r > a) = 0$], ' denotes d/dr , and

$$q(r) = \frac{rB_\phi}{R_0 B_\theta} \quad (6)$$

is termed the ‘‘safety factor.’’ Here, B_ϕ is the (approximately constant) equilibrium toroidal magnetic field strength. It is assumed that there is no ‘‘rational surface’’ lying within the plasma [i.e., there is no solution to $q(r < a) = m/n$]. The eigenfunction $\hat{\psi}(r)$ satisfies physical boundary conditions at $r=0$ and $r=\infty$. In general, $\hat{\psi}(r)$ is continuous across the conducting shell whereas $d\hat{\psi}(r)/dr$ is discontinuous. The ‘‘shell stability index’’ for the m/n mode is defined

$$\Delta_w = \left[r \frac{d\hat{\psi}}{dr} / \hat{\psi} \right]_{r=r_w^-}^{r=r_w^+}. \quad (7)$$

This quantity is uniquely determined by the plasma equilibrium, the mode numbers m and n , and the radius of the shell. It is easily demonstrated that the m/n external kink mode is unstable (in the absence of the shell) whenever $\Delta_w > 0$.¹² Finally, Ampère’s law integrated across the shell yields

$$im\mu_0 \hat{J}_w = \Delta_w \hat{\Psi}_w. \quad (8)$$

C. The resistive shell mode

Suppose that the shell located at radius r_w is a conventional shell of uniform thickness δ_w and uniform electrical conductivity σ_w . The L/R time of the shell is written

$$\tau_w = \mu_0 r_w \sigma_w \delta_w. \quad (9)$$

Asymptotic matching of the poloidal flux eigenfunction $\hat{\psi}(r)$ inside and outside the shell yields the well-known dispersion relation

$$\gamma \tau_w = \Delta_w. \quad (10)$$

This expression is valid as long as the thickness of the shell is much less than its radius. Thus, the resistive shell mode grows on the L/R time of the shell whenever the shell stability index Δ_w is positive, and the mode is stable whenever the shell stability index is negative. In the vacuum region

outside the plasma the most general solution of Eq. (5) is some linear combination of r^m and r^{-m} functions. Using this fact, it is easily demonstrated that

$$\Delta_w = \frac{2m}{(r_c/r_w)^{2m} - 1}, \quad (11)$$

where r_c is termed the ‘‘critical radius,’’ and is uniquely determined by the plasma equilibrium and the poloidal and toroidal mode numbers. The above expression is only valid in the case where the m/n external kink mode is *unstable* in the absence of a shell. Clearly, if the radius of the shell r_w is less than the critical radius r_c then the shell stability index Δ_w is $O(1)$ and positive, so the resistive shell mode grows on the L/R time of the shell. As $r_w \rightarrow r_c$, the shell stability index becomes increasingly large and the growth rate of the resistive shell mode gradually accelerates. The mode eventually makes a transition to the much faster growing ideal external kink mode. For $r_w > r_c$, the shell is unable to stabilize the ideal external kink mode, which consequently grows on some very fast time scale determined by plasma inertia. Thus a resistive shell is only capable of moderating the growth of the external kink mode if it is situated *inside* the critical radius.

Suppose, now, that the shell contains vacuum gaps. In a wide range of circumstances, the conventional dispersion relation (10) generalizes to²¹

$$\gamma\tau_w = \frac{\Delta_w}{1 - \Delta_w/\Delta_c}, \quad (12)$$

where $\tau_w = (1-f)\tilde{\tau}_w$ is the effective L/R time of the shell, and $\tilde{\tau}_w = \mu_0 r_w \sigma_w \delta_w$ is the L/R time of the shell in the absence of gaps. Here,

$$\Delta_c = 2m \left(\frac{1}{f} - 1 \right) \quad (13)$$

is termed the ‘‘critical shell stability index,’’ and f is the area fraction of gaps in the shell. The transition from the slowly growing resistive shell mode to the rapidly growing ideal external kink mode now takes place as $\Delta_w \rightarrow \Delta_c$.

The dispersion relation (12) can be rewritten

$$\gamma\tilde{\tau}_w = \tilde{\Delta}_w, \quad (14)$$

where $\tilde{\Delta}_w$ is the m/n shell stability index calculated for a shell located at the ‘‘effective radius’’

$$\tilde{r}_w = r_w \left(1 + \frac{2m}{\Delta_c} \right)^{1/2m} = r_w \left(\frac{1}{1-f} \right)^{1/2m}. \quad (15)$$

According to the dispersion relation (14), a partial shell acts just like a complete shell whose effective radius \tilde{r}_w is somewhat larger than the actual radius of the shell r_w , and whose L/R time is that characteristic of the conducting segments of the shell.

D. The fake shell

Suppose that the shell in question is actually a ‘‘fake shell’’ made up of a network of resistors and inductors (see Fig. 1). Let $\Delta\theta$ and $\Delta\phi$ be the angular spacings of the network cells in the poloidal and toroidal directions, respec-

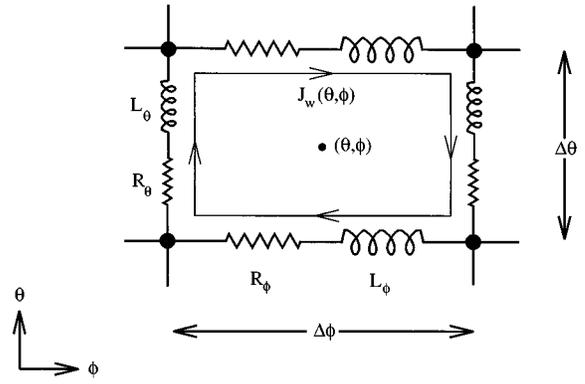


FIG. 1. Schematic diagram showing a single loop of a uniform two dimensional network of resistors and inductors surrounding the plasma. The loop is centered on the point (θ, ϕ) . The resistances of the poloidal and toroidal legs of the loop are R_θ and R_ϕ , respectively. The inductances of the poloidal and toroidal legs are L_θ and L_ϕ , respectively. The current which flows around the loop is $J_w(\theta, \phi)$.

tively. In the limit $m\Delta\theta \ll 2\pi$ and $n\Delta\phi \ll 2\pi$, where the network is sufficiently fine that there is little phase variation of the mode between neighboring cells, the currents flowing in the network can be approximated as a continuous distribution of the form given in Eq. (1). It is easily demonstrated that the current which flows around the network loop centered on (θ, ϕ) is approximately given by the local value of the current stream function $J_w(\theta, \phi)$. Consider, for the sake of simplicity, the limit in which the poloidal inductance and resistance of the network are negligible. This limit corresponds to $n^2 r_w \Delta\phi \ll m^2 R_\theta \Delta\theta$ provided that inductance and resistance per unit length of the network are approximately the same in the poloidal and toroidal directions. The circuit equation for an individual network loop yields

$$-\gamma\delta\hat{B}_r \text{ plasma } r_w R_\theta \Delta\theta \Delta\phi - \gamma L_\phi (m\Delta\theta)^2 \hat{J}_w = R_\phi (m\Delta\theta)^2 \hat{J}_w, \quad (16)$$

where γ is the growth rate of the mode, L_ϕ and R_ϕ are the inductance and resistance, respectively, of a toroidal leg of the loop, and

$$\delta\hat{B}_r \text{ plasma} = i \frac{m}{r_w} \left(1 + \frac{\Delta_w}{2m} \right) \hat{\Psi}_w \quad (17)$$

is the radial magnetic field at the loop due to the perturbed currents flowing in the plasma.

Equations (8), (16), and (17) can be combined to give the ‘‘dispersion relation’’ of the network,

$$\gamma\tau_w = \frac{\Delta_w}{1 - \Delta_w/\Delta_c}, \quad (18)$$

where

$$\tau_w = \frac{\mu_0 m^2 R_\theta \Delta\theta \Delta\phi}{R_\phi (m\Delta\theta)^2}, \quad (19)$$

and

$$\Delta_c = \frac{\mu_0 m^2 R_\theta \Delta\theta \Delta\phi}{\tilde{L}_\phi (m\Delta\theta)^2}. \quad (20)$$

Here,

$$\tilde{L}_\phi = L_\phi - \frac{\mu_0 R_0}{2m} \frac{\Delta\phi}{\Delta\theta} \quad (21)$$

is the inductance of a toroidal leg of an individual network loop due to the short wavelength (i.e., of order the size of the network loop) components of the magnetic field. Equation (18) has the same form as the dispersion relation of a conventional shell containing vacuum gaps (see Sec. II C).²¹ Here, τ_w is the effective L/R time of the shell, and Δ_c is related to the effective area fraction of gaps in the shell f via

$$f = \frac{1}{1 + \Delta_c/2m}. \quad (22)$$

In the absence of the fake shell, the external kink mode is marginally stable when $\Delta_w = 0$, and is unstable whenever $\Delta_w > 0$. In the presence of the shell, the resistive shell mode is unstable for $\Delta_c > \Delta_w > 0$. The modified marginal stability criterion for the external kink mode is $\Delta_w = \Delta_c$. For $\Delta_w > \Delta_c$ the external kink mode escapes through the holes in the network at some (extremely rapid) rate determined by the plasma inertia. Henceforth, Δ_c is termed the ‘‘critical stability index’’ of the shell.

E. The fake rotating shell

Suppose that each loop in the network is accompanied by a high impedance sensor loop of equal area which measures the rate at which magnetic flux escapes through the loop (see Fig. 2). The voltage generated in an individual sensor loop is

$$\hat{V}(t) = -\frac{d}{dt} (\delta B_r \text{plasma} r_w R_0 \Delta\theta \Delta\phi + L_\phi (m\Delta\theta)^2 \hat{J}_w). \quad (23)$$

This signal can be integrated to give

$$\hat{\mathcal{V}}(t) = \int_0^t \hat{V}(t) dt = -(\delta B_r \text{plasma} r_w R_0 \Delta\theta \Delta\phi + L_\phi (m\Delta\theta)^2 \hat{J}_w), \quad (24)$$

assuming that the mode amplitude is negligibly small at time $t=0$. Suppose that the integrated signal is amplified by a factor $1/\tau$ and then fed into the network, as shown in Fig. 2. The modified circuit equation for a network loop takes the form

$$\hat{V} - i \frac{m\Delta\theta}{\tau} \hat{\mathcal{V}} = R_\phi (m\Delta\theta)^2 \hat{J}_w. \quad (25)$$

Equations (8), (17), (23), (24), and (25) can be combined to give the modified dispersion relation of the network,

$$\gamma = i \frac{m\Delta\theta}{\tau} + \frac{\Delta_w/\tau_w}{1 - \Delta_w/\Delta_c}. \quad (26)$$

This is the dispersion relation of an incomplete shell of L/R time τ_w and critical stability index Δ_c , which rotates poloidally with the velocity

$$v_\theta = -\frac{r_w \Delta\theta}{\tau}. \quad (27)$$

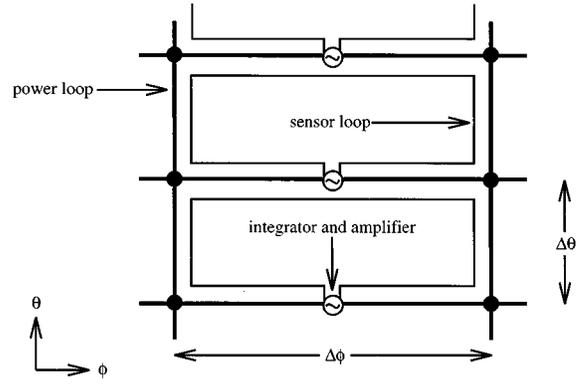


FIG. 2. Schematic diagram showing a few loops of a uniform two dimensional network of resistors and inductors, surrounding the plasma, which is subject to feedback. The low impedance loops containing the resistors and inductors are denoted the power loops (the resistors and inductors are not shown for the sake of clarity). The signals detected by the high impedance sensor loops are integrated, amplified, and then fed into the power loops, as shown.

Thus the feedback causes the network to act like a ‘‘fake rotating shell.’’ The effective rotation velocity is determined by the amplification factor $1/\tau$.

F. Stabilization of the resistive shell mode

Suppose that the plasma is surrounded by a complete nonrotating shell of radius r_v and L/R time τ_v . This, in turn, is surrounded by a fake rotating shell of radius r_w , effective L/R time τ_w , and critical stability index Δ_c . Thus $r_w > r_v > a$. The first shell represents the vacuum vessel in a conventional tokamak where the vessel is situated relatively close to the edge of the plasma. In the vacuum region outside the plasma the most general solution of Eq. (5) is some linear combination of r^m and r^{-m} functions. Using this fact, it is easily demonstrated that the dispersion relation for an m/n mode in the presence of the two shells takes the form

$$(\Delta_v - E_v)(\Delta_w - E_w) - (E_{vw})^2 = 0, \quad (28)$$

where

$$\Delta_v = \gamma \tau_v, \quad (29)$$

and [after rearranging Eq. (26)]

$$\Delta_w = \frac{(\gamma + i\Omega_w)\tau_w}{1 + (\gamma + i\Omega_w)\tau_w/\Delta_c}, \quad (30)$$

where

$$\Omega_w = -\frac{m\Delta\theta}{\tau} \quad (31)$$

is the effective angular rotation frequency of the fake shell. The remaining terms in Eq. (28) are given by

$$E_v = \frac{2m}{(r_c/r_v)^{2m} - 1} - \frac{2m}{(r_w/r_v)^{2m} - 1}, \quad (32a)$$

$$E_w = -\frac{2m(r_w/r_v)^{2m}}{(r_w/r_v)^{2m} - 1}, \quad (32b)$$

$$E_{vw} = \frac{2m(r_w/r_v)^m}{(r_w/r_v)^{2m} - 1}. \quad (32c)$$

Here, r_c is the critical radius beyond which a complete perfectly conducting shell is unable to stabilize the external kink mode. It is assumed that the vacuum vessel lies within this critical radius, so that $r_v < r_c$.

It is convenient (and fairly reasonable) to assume that the L/R time of the vacuum vessel is much longer than the effective L/R time of the fake shell. Thus $\tau_v \gg \tau_w$. With this ordering, plus $\Omega_w \tau_w \sim O(1)$, the resistive shell mode root [i.e., the root with $\gamma \tau_v \sim O(1)$] of Eqs. (28)–(32) can easily be shown to take the form

$$\begin{aligned} \gamma \tau_v \approx & \frac{2m}{(r_c/r_v)^{2m} - 1} \\ & \times \left(\frac{1 - \hat{\Omega}_w^2 [(r_c/\tilde{r}_w)^{2m} - 1] / [1 - (r_v/\tilde{r}_w)^{2m}]}{1 + \hat{\Omega}_w^2} \right) \\ & - i \frac{2m}{(\tilde{r}_w/r_v)^{2m} - 1} \frac{\hat{\Omega}_w}{1 + \hat{\Omega}_w^2}, \end{aligned} \quad (33)$$

where

$$\hat{\Omega}_w = \frac{\Omega_w \tilde{\tau}_w}{2m} [1 - (r_v/\tilde{r}_w)^{2m}]. \quad (34)$$

Here,

$$\tilde{r}_w = r_w \left(1 + \frac{2m}{\Delta c} \right)^{1/2m} = r_w \left(\frac{1}{1-f} \right)^{1/2m}, \quad (35a)$$

$$\tilde{\tau}_w = \tau_w \left(1 + \frac{2m}{\Delta c} \right) = \tau_w \left(\frac{1}{1-f} \right). \quad (35b)$$

As far as the plasma is concerned, the fake shell acts like an *incomplete* rotating shell, with the area fraction of gaps f given in Eq. (22) and the effective L/R time τ_w , which is located at the radius r_w . Alternatively, the fake shell acts like a *complete* rotating shell, with the effective L/R time $\tilde{\tau}_w$, which is located at the ‘‘effective radius’’ \tilde{r}_w (see Sec. II C).^{19,21} Note that the effective radius of the fake shell is always larger than the actual radius of the network of feedback controlled conductors.

It is clear from Eq. (33) that, in the absence of feedback (i.e., $\Omega_w = 0$), the resistive shell mode takes the form of a nonrotating mode growing on the L/R time of the vacuum vessel. Feedback causes the resistive shell mode to propagate in the direction of apparent rotation of the fake shell, but it also modifies the growth rate of the mode. If the effective radius of the fake shell \tilde{r}_w lies beyond the critical radius r_c of the m/n mode then feedback always causes an increase in the growth rate. However, if $r_c > \tilde{r}_w$ then feedback causes the growth rate to decrease. In the latter case, there is a critical value of the effective angular rotation frequency of the fake shell Ω_w beyond which the resistive shell mode is stabilized. The corresponding critical value of $g = \tau_w \Delta \theta / \tau$ in the feedback circuits is

$$g_c = 2 \frac{(r_w)^{2m}}{\sqrt{(r_c)^{2m} - (\tilde{r}_w)^{2m}} \sqrt{(\tilde{r}_w)^{2m} - (r_v)^{2m}}}. \quad (36)$$

The critical ‘‘voltage gain’’ $G = 1/|\gamma| \tau$ in the feedback circuits (i.e., the ratio of the voltage put out by an individual feedback amplifier to that which is generated by magnetic induction in the corresponding network loop) is given by

$$G_c = \frac{\tau_v}{m \Delta \theta \tau_w} \left(\frac{r_w}{r_v} \right)^{2m} \frac{(r_c)^{2m} - (r_v)^{2m}}{(r_c)^{2m} - (\tilde{r}_w)^{2m}}. \quad (37)$$

It is easily demonstrated¹⁹ that the critical current which must be supplied by an individual feedback amplifier is of order

$$I_c \sim \frac{r_w R_0 \Delta \theta \Delta \phi b_r}{\tau R_\phi} = g_c \frac{r_w \Delta \theta b_r}{\mu_0}, \quad (38)$$

where b_r is the perturbed radial magnetic field strength at radius r_w . The *total* power which must be supplied by the feedback amplifiers is

$$P_c \sim \frac{2\pi}{\Delta \theta} \frac{2\pi}{\Delta \phi} \frac{I_c^2 R_\phi}{2} = g_c^2 \frac{2\pi^2 r_w^2 R_0 b_r^2}{\mu_0 \tau_w}. \quad (39)$$

Note that the total power expended by the feedback system whilst stabilizing the resistive shell mode is independent of the number of cells in the network.

G. Summary

The fake rotating shell feedback scheme consists of a combination of passive and active elements. The main passive element is the stationary shell (i.e., the vacuum vessel). The active element is the network of feedback controlled conductors, surrounding the stationary shell, which constitute the fake rotating shell. The feedback scheme is capable of stabilizing the resistive shell mode provided that the effective radius of the fake rotating shell lies inside the critical radius.

III. A REACTOR RELEVANT FEEDBACK SCHEME

A. Introduction

In most tokamak reactor designs^{16,22} the plasma is surrounded by an extremely close fitting ‘‘first wall’’ (i.e., the plasma facing components), which is generally made of some highly conducting material. Outside the first wall is a thick neutron shielding blanket which is surrounded by the vacuum vessel, the toroidal field coils, and the poloidal field coils, respectively. The fake rotating shell feedback scheme can be implemented in a reactor with the first wall playing the role of the innermost passive stabilizing shell. It is assumed, for the sake of simplicity, that the vacuum vessel is sufficiently resistive that its L/R time is much less than that of the first wall. Thus the vacuum vessel can be neglected for the purposes of this investigation, since a resistive shell mode which grows on the L/R time of the first wall only excites weak eddy currents in the vessel. As is discussed in Sec. I, space, maintainability, and survivability constraints demand that the network of feedback controlled conductors be placed outside the neutron shielding blanket (i.e., rather a

long way from the edge of the plasma). Thus, the first wall (radius r_v , L/R time τ_v) certainly lies well within the critical radius r_c for the m/n mode, whereas the fake rotating shell (effective radius \tilde{r}_w , effective L/R time $\tilde{\tau}_w$) probably lies outside this critical radius. Under these circumstances, the feedback scheme described in Sec. II is incapable of stabilizing the resistive shell mode. Fortunately, it is possible to modify this scheme in such a manner that the resistive shell mode can be stabilized even when all of the feedback controlled conductors are situated outside the critical radius. The necessary modifications are described below.

B. Relocation of the sensor network

The active element of the fake rotating shell feedback scheme consists of two networks of conductors situated at the same radius, r_w . The first network is made up of high impedance conductors which carry virtually no current, and is responsible for detecting the mode. The signals generated by this network, after suitable integration and amplification, are fed into a second, low impedance network which carries substantial currents and can, thereby, modify the growth rate and propagation frequency of the resistive shell mode. Henceforth, the first network is termed the ‘‘sensor network’’ and the second network is termed the ‘‘power network.’’

Suppose, for the sake of argument, that the sensor network is relocated to some radius r_l which is different from that of the power network, r_w . It is assumed that the two networks still contain an equal number of cells in the toroidal and poloidal directions. Furthermore, these cells are located at the same angular positions, and remain interconnected according to the scheme set out in Fig. 2.

Equation (23) specifies the voltage generated in an individual cell of the sensor network when the radius of this network coincides with that of the power network. This expression can be rewritten

$$\hat{V}(t) = -\frac{d}{dt} (imR_0\Delta\theta\Delta\phi\hat{\Psi}_w + \tilde{L}_\phi(m\Delta\theta)^2\hat{J}_w), \quad (40)$$

where use has been made of Eqs. (8), (16), and (21). The first term on the right-hand side is due to m/n helical magnetic fields generated by the m/n currents flowing in the plasma and the power network. The second term is due to short wavelength (i.e., of order the size of the network cells) magnetic fields generated by currents circulating in the power network. The latter fields are very strongly localized in the vicinity of the network. In fact, the short wavelength fields decay effectively to zero on a radial length scale $d_w = r_w\Delta\theta/2\pi \ll r_w$ (assuming that $r_w\Delta\theta \ll R_0\Delta\phi$). Thus if $|r_l - r_w| \geq d_w$ (as is assumed to be the case), then the signal detected by an individual cell of the relocated sensor network is given by

$$\hat{V}_l(t) \approx -\frac{d}{dt} (imR_0\Delta\theta\Delta\phi\hat{\Psi}_l), \quad (41)$$

where $\hat{\Psi}_l \equiv \hat{\psi}(r_l)$ is the value of the m/n flux eigenfunction at the sensor network radius. This signal can be integrated to give

$$\hat{\mathcal{V}}_l(t) = \int_0^t \hat{V}_l(t) dt = -imR_0\Delta\theta\Delta\phi\hat{\Psi}_l(t), \quad (42)$$

assuming that the mode amplitude is negligibly small at time $t=0$. If the integrated signal is amplified by a factor $1/\tau$ and then fed into the power network, in an analogous manner to that shown in Fig. 2, then the circuit equation for an individual loop of the power network takes the form [see Eq. (25)]

$$\hat{V} - i\frac{m\Delta\theta}{\tau}\hat{\mathcal{V}}_l = R_\phi(m\Delta\theta)^2\hat{J}_w. \quad (43)$$

The corresponding dispersion relation for the power network is written

$$\gamma = i\frac{m\Delta\theta}{\tau} \frac{\hat{\Psi}_l/\hat{\Psi}_w}{1 - \Delta_w/\Delta_c} + \frac{\Delta_w/\tau_w}{1 - \Delta_w/\Delta_c}. \quad (44)$$

Suppose that the sensor network is located somewhere between the passive shell and the power network, so that $r_v < r_l < r_w$. The m/n flux eigenfunction in this region is given by

$$\hat{\psi}(r) = \hat{\Psi}_v \frac{(r/r_w)^m - (r/r_w)^{-m}}{(r_v/r_w)^m - (r_v/r_w)^{-m}} + \hat{\Psi}_w \frac{(r/r_v)^m - (r/r_v)^{-m}}{(r_w/r_v)^m - (r_w/r_v)^{-m}}, \quad (45)$$

where $\hat{\Psi}_v \equiv \hat{\psi}(r_v)$ is the value of the flux eigenfunction at the radius of the passive shell. It follows that

$$\frac{\hat{\Psi}_l}{\hat{\Psi}_w} = \alpha_v \frac{\hat{\Psi}_v}{\hat{\Psi}_w} + \alpha_w, \quad (46)$$

where

$$\alpha_v = \left(\frac{r_v}{r_l}\right)^m \frac{(r_w)^{2m} - (r_l)^{2m}}{(r_w)^{2m} - (r_v)^{2m}}, \quad (47a)$$

$$\alpha_w = \left(\frac{r_w}{r_l}\right)^m \frac{(r_l)^{2m} - (r_v)^{2m}}{(r_w)^{2m} - (r_v)^{2m}}. \quad (47b)$$

The dispersion relation for an m/n mode in the presence of the passive shell and the power network can be written²³

$$\Delta_v \hat{\Psi}_v = E_v \hat{\Psi}_v + E_{vw} \hat{\Psi}_w, \quad (48a)$$

$$\Delta_w \hat{\Psi}_w = E_w \hat{\Psi}_w + E_{vw} \hat{\Psi}_v, \quad (48b)$$

where Δ_v is given in Eq. (29), and [after rearranging Eq. (44)]

$$\Delta_w = \frac{\gamma\tau_w + i\Omega_w\tau_w(1 + \kappa\hat{\Psi}_v/\hat{\Psi}_w)}{1 + \gamma\tau_w/\Delta_c}, \quad (49)$$

with

$$\Omega_w = -\alpha_w \frac{m\Delta\theta}{\tau}, \quad (50)$$

and

$$\kappa = \frac{\alpha_v}{\alpha_w} = \left(\frac{r_v}{r_w}\right)^m \frac{(r_w)^{2m} - (r_l)^{2m}}{(r_l)^{2m} - (r_v)^{2m}}. \quad (51)$$

Equations (48) and (49) can be combined to give the quadratic dispersion relation

$$\gamma^2(1 - E_w/\Delta_c)\tau_w\tau_v - \gamma(E_w\tau_v + E_v(1 - E_{w\infty}/\Delta_c)\tau_w - i\Omega_w\tau_w\tau_v) + [E_w E_{v\infty} - i\Omega_w\tau_w(E_v - \kappa E_{vw})] = 0, \quad (52)$$

where

$$E_{v\infty} = E_v - \frac{(E_{vw})^2}{E_w} = \frac{2m}{(r_c/r_v)^{2m} - 1}, \quad (53a)$$

$$E_{w\infty} = E_w - \frac{(E_{vw})^2}{E_v} = \frac{2m}{(r_c/r_w)^{2m} - 1}. \quad (53b)$$

It is again assumed that $\tau_v \gg \tau_w$ and $\Omega_w\tau_w \sim O(1)$. The resistive shell mode root of the dispersion relation (52) [i.e., the root with $\gamma\tau_v \sim O(1)$] takes the form

$$\gamma\tau_v \approx \frac{E_w E_{v\infty} - i\Omega_w\tau_w(E_v - \kappa E_{vw})}{E_w - i\Omega_w\tau_w}, \quad (54)$$

or

$$\gamma\tau_v \approx \frac{E_{v\infty}[1 + (\Omega_w\tau_w/E_w)^2(E_w/E_{w\infty} - \kappa E_{vw}/E_{v\infty})]}{1 + (\Omega_w\tau_w/E_w)^2} - i\left(\frac{\Omega_w\tau_w}{E_w}\right)\frac{E_{vw}(E_{vw}/E_w - \kappa)}{1 + (\Omega_w\tau_w/E_w)^2}. \quad (55)$$

The above equation reduces to

$$\begin{aligned} \gamma\tau_v &\approx \frac{2m}{(r_c/r_v)^{2m} - 1} \\ &\times \left(\frac{1 - \hat{\Omega}_w^2[(r_c/r_l)^{2m} - 1]/[1 - (r_v/r_l)^{2m}]}{1 + \hat{\Omega}_w^2} \right) \\ &- i \frac{2m}{(r_l/r_v)^{2m} - 1} \frac{\hat{\Omega}_w}{1 + \hat{\Omega}_w^2}, \end{aligned} \quad (56)$$

where

$$\hat{\Omega}_w = \frac{\Omega_w\tau_w}{2m} [1 - (r_v/r_w)^{2m}]. \quad (57)$$

It is clear from a comparison of Eqs. (33) and (56) that when the sensor network is displaced from the power network then the effective radius of the fake rotating shell is determined *solely* by the radius of the sensor network. In fact, the effective radius becomes equal to the radius of the sensor network.

The resistive shell mode can be stabilized provided that the sensor network is located inside the critical radius r_c of the m/n mode (i.e., provided that $r_l < r_c$). It is easily demonstrated from Eq. (56) that the critical value of $g = \tau_w\Delta\theta/\tau$ which must be exceeded in order to stabilize the resistive shell mode is given by

$$g_c = 2 \left(\frac{r_l}{r_w}\right)^m \frac{(r_w)^{2m}}{\sqrt{(r_c)^{2m} - (r_l)^{2m}} \sqrt{(r_l)^{2m} - (r_v)^{2m}}}. \quad (58)$$

The critical voltage gain $G = (r_w/r_l)^m/|\gamma|\tau$ in the feedback circuits (i.e., the ratio of the voltage put out by an individual

feedback amplifier to that which is generated by magnetic induction in the corresponding loop of the power network) is

$$G_c = \frac{\tau_v}{m\Delta\theta\tau_w} \left(\frac{r_w}{r_v}\right)^{2m} \frac{(r_c)^{2m} - (r_v)^{2m}}{(r_c)^{2m} - (r_l)^{2m}}. \quad (59)$$

The corresponding critical current which must be supplied to the power network by an individual feedback amplifier is of order

$$I_c \sim g_c \frac{r_l\Delta\theta b_r}{\mu_0}, \quad (60)$$

where b_r is the perturbed radial magnetic field strength at the sensor network. The total power which must be supplied by the feedback amplifiers is

$$P_c \sim g_c^2 \frac{2\pi^2 r_l^2 R_0 b_r^2}{\mu_0\tau_w}. \quad (61)$$

It is clear from Eqs. (58)–(60) that the optimum location for the sensor network is somewhere between the radius of the passive shell r_v and the critical radius r_c . As the radius of the sensor network approaches either that of the passive shell or the critical radius, the current and power which must be supplied by each feedback amplifier in order to stabilize the resistive shell mode both tend to infinity. Note that it is possible to stabilize the resistive shell mode even when the power network (radius r_w) is located well beyond the critical radius, provided that the sensor network is situated inside this radius. Nevertheless, it is advantageous for the power network to be placed as close as practically possible to the plasma, since the critical voltage gain, current, and power in the feedback circuits increase like r_w^{2m} , r_w^m , and r_w^{2m} , respectively, as $r_w \rightarrow \infty$.

The second root of the dispersion relation (52) takes the form

$$\gamma\tau_w \approx \frac{E_w}{1 - E_w/\Delta_c} - i \frac{\Omega_w\tau_w}{1 - E_w/\Delta_c}. \quad (62)$$

Since $E_w < 0$, this root is clearly stable.

C. Summary

In the fake rotating shell concept the combined action of the sensor network and the power network create the illusion of a rotating shell surrounding the plasma. In the original concept (see Sec. II) both networks are located at the same radius, r_w . It turns out that in this case the effective radius of the fake rotating shell, \tilde{r}_w , is somewhat larger than r_w [see Eq. (35a)]. This occurs because the sensor network is able to detect the short wavelength magnetic fields generated by the currents circulating in the power network [i.e., the second term on the right-hand side of Eq. (40)]. If, however, the sensor network is slightly displaced from the power network, so that the short wavelength fields decay before reaching the sensor network, then the effective radius of the fake rotating shell becomes equal to the (approximately) common radius of the two networks, r_w . The fake rotating shell is only able to stabilize the resistive shell mode when its effective radius is less than the critical radius, r_c .

It is clear from the analysis of Sec. III B that the effective radius of the fake rotating shell is determined solely by the position of the sensor network. In fact, if the sensor network is placed between the passive stabilizing shell and the power network then the effective radius of the fake rotating shell becomes the same as that of the sensor network. The sensor network is basically a set of flux loops which carry very little current. As such, it can be constructed from thin wires made of some high resistance (compared to copper), low activation material (e.g., vanadium). Since the network carries little current, it is not subject to large electromagnetic stresses and, therefore, does not require a bulky mechanical support structure. Thus although it is difficult to conceive of putting a network of current carrying conductors (such as the power network) inside the neutron absorbing blanket (because the conductors have to be fabricated out of some low resistance, high activation material such as copper, and because the conductors are subject to substantial electromagnetic stresses and, therefore, require a bulky mechanical support structure) it is possible to conceive of placing a high impedance detector network (such as the sensor network) inside the blanket. In fact, the sensor network is likely to take up so little space that it may be possible to place many networks inside the blanket, so that if the main sensor network fails during the lifetime of the reactor then a replacement is immediately available, and there is no need to disassemble the reactor. Thus a reactor relevant version of the fake rotating shell feedback scheme can be constructed by placing the sensor network inside the neutron shielding blanket, with the power network outside the blanket. This scheme is capable of stabilizing the resistive shell mode provided that the radius of the sensor network lies inside the critical radius. In principle, the power network can be placed arbitrarily far away from the plasma. However, the power required to stabilize the resistive shell mode increases strongly as the radius of the power network increases, so it is clearly advantageous to place this network as close as practically possible to the plasma.

IV. THE VIRTUAL FAKE ROTATING SHELL

A. Introduction

The reactor relevant feedback scheme described in the previous section consists of three elements; a close fitting passive shell, radius r_v , a high impedance sensor network, radius r_l , and a low impedance power network, radius r_w . It is assumed that $r_v < r_l < r_w$. The passive shell and the sensor network are both located inside the neutron shielding blanket, whereas the power network is located outside the blanket. It is possible to envisage placing the sensor network inside the blanket because this network carries very little current and can, therefore, be fabricated out of a low activation metal, such as vanadium, without the need for extensive mechanical support. Nevertheless, from an engineering standpoint, it is far simpler to place the sensor network outside the blanket, just like the power network.

Recall, from the previous section, that the combined action of the sensor network and the power network create the illusion of a rotating shell surrounding the plasma. The ef-

fective radius of this fake rotating shell is the same as the radius of the sensor network. It is possible to stabilize the m/n resistive shell mode provided that the effective radius lies inside the critical radius for the m/n mode, r_c . This is likely to be the case if the sensor network is situated inside the neutron shielding blanket, but not otherwise. Thus the feedback scheme outlined in the previous section is unlikely to work when the sensor network is located outside the blanket. Fortunately, it is possible to modify this scheme in such a manner that the resistive shell mode can be stabilized even when both the sensor network and the power network are situated beyond the critical radius. The necessary modifications are described below.

B. Multiple sensor networks

It is clear from the previous analysis that the effective radius of the fake rotating shell is determined by the radius at which the helical magnetic fields diffusing through the passive shell are *detected*. Thus it is plausible that if the fields at some radius r_e are *reconstructed*, using signals derived from a sensor network located well outside this radius, and fed into the power network in the usual manner, then the effective radius of the fake rotating shell becomes r_e . This being the case, it is possible to produce a fake rotating shell whose effective radius lies well inside that of either the sensor network or the power network: this type of shell is termed a “virtual fake rotating shell.” The possibility of a virtual shell (i.e., a shell whose effective radius lies well inside the radii of the associated feedback controlled conductors or sensor loops) was first suggested by Bishop and Bevir.¹⁸

According to Eq. (45), it is possible to uniquely reconstruct the m/n flux eigenfunction $\hat{\psi}(r)$ throughout some current free region given the value of the eigenfunction at *two* different radii in this region. This suggests that it may be possible to construct a virtual fake rotating shell using signals derived from *two* sensor networks located at two different radii.

Consider a modified feedback scheme consisting of a power network (radius r_w , effective L/R time τ_w , critical stability index Δ_c) connected to two sensor networks (radii r_l and r_l'). The signals detected by the two sensor networks are integrated, amplified by factors $1/\tau$ and $1/\tau'$, respectively, and then fed into the power network. By analogy with Eq. (44), the dispersion relation of the power network becomes

$$\gamma = i \frac{m \Delta \theta}{\tau} \frac{\hat{\Psi}_l / \hat{\Psi}_w}{1 - \Delta_w / \Delta_c} + i \frac{m \Delta \theta}{\tau'} \frac{\hat{\Psi}_{l'} / \hat{\Psi}_w}{1 - \Delta_w / \Delta_c} + \frac{\Delta_w / \tau_w}{1 - \Delta_w / \Delta_c}, \quad (63)$$

where $\hat{\Psi}_{l'} \equiv \hat{\psi}(r_{l'})$ is the value of the m/n flux eigenfunction at the radius of the second sensor network. Suppose that the first sensor network lies between the passive shell and the power network, so that $r_v < r_l < r_w$, but that the second sensor network lies beyond the power network, so that $r_{l'} > r_w$. The m/n flux eigenfunction in the region $r > r_w$ is given by

$$\hat{\psi}(r) = \hat{\Psi}_w \left(\frac{r}{r_w} \right)^{-m}. \quad (64)$$

Thus

$$\frac{\hat{\Psi}_{l'}}{\hat{\Psi}_w} = \alpha_{w'}, \quad (65)$$

where

$$\alpha_{w'} = \left(\frac{r_w}{r_{l'}} \right)^m. \quad (66)$$

Equation (63) can be rearranged to give

$$\Delta_w = \frac{\gamma\tau_w + i\Omega_w\tau_w(1 + \kappa'\hat{\Psi}_v/\hat{\Psi}_w)}{1 + \gamma\tau_w/\Delta_c}, \quad (67)$$

with

$$\Omega_w = -(\alpha_w + \alpha_{w'}\tau/\tau') \frac{m\Delta\theta}{\tau}, \quad (68)$$

and

$$\kappa' = \frac{\alpha_v}{\alpha_w + \alpha_{w'}\tau/\tau'} \equiv \left(\frac{r_v}{r_w} \right)^m \frac{(r_w)^{2m} - (r_e)^{2m}}{(r_e)^{2m} - (r_v)^{2m}}. \quad (69)$$

Here, by analogy with Eq. (51), r_e is the effective radius of the fake rotating shell. The dispersion relation for the resistive shell mode is given by Eq. (56), with r_l replaced by r_e . Thus

$$\begin{aligned} \gamma\tau_v &\approx \frac{2m}{(r_c/r_v)^{2m} - 1} \\ &\times \left(\frac{1 - \hat{\Omega}_w^2[(r_c/r_e)^{2m} - 1]/[1 - (r_v/r_e)^{2m}]}{1 + \hat{\Omega}_w^2} \right) \\ &- i \frac{2m}{(r_e/r_v)^{2m} - 1} \frac{\hat{\Omega}_w}{1 + \hat{\Omega}_w^2}, \end{aligned} \quad (70)$$

where

$$\hat{\Omega}_w = \frac{\Omega_w\tau_w}{2m} [1 - (r_v/r_w)^{2m}]. \quad (71)$$

It is easily demonstrated that

$$r_e = r_l \left(\frac{1 - \lambda(r_w/r_l)^{2m}}{1 - \lambda} \right)^{1/2m}, \quad (72)$$

where

$$\lambda = -\frac{\tau}{\tau'} \left(\frac{r_l}{r_{l'}} \right)^m, \quad (73)$$

and also that

$$\Omega_w = -\frac{m\Delta\theta}{\tau} \left(\frac{r_w}{r_l} \right)^m \frac{(r_e)^{2m} - (r_v)^{2m}}{(r_w)^{2m} - (r_v)^{2m}} \frac{(r_w)^{2m} - (r_l)^{2m}}{(r_w)^{2m} - (r_e)^{2m}}. \quad (74)$$

Note that the effective radius of the fake rotating shell is *less* than that of the inner sensor network (and the power network) as long as $0 < \lambda < (r_l/r_w)^{2m}$. This implies that $1/\tau'$

< 0 ; i.e., the signals from the outer sensor network must be inverted, as well as integrated and amplified, before they are fed into the power network.

According to Eqs. (70), (71), and (74), the feedback scheme has a stabilizing effect on the resistive shell mode provided that the effective radius of the fake rotating shell lies between the critical radius and the radius of the passive shell; i.e., provided that $r_v < r_e < r_c$. It follows that stabilization of the resistive shell mode is only possible when the parameter λ lies in the range

$$\frac{(r_l)^{2m} - (r_c)^{2m}}{(r_w)^{2m} - (r_c)^{2m}} < \lambda < \frac{(r_l)^{2m} - (r_v)^{2m}}{(r_w)^{2m} - (r_v)^{2m}}. \quad (75)$$

Here, it is assumed that $r_c < r_w$. If $r_c > r_w$ then the lower limit becomes $-\infty$.

Equation (70) specifies the growth rate of the m/n resistive shell mode for the case where the m/n external kink mode is unstable in the absence of an external shell. In the opposite case, where the m/n external kink mode is stable, the factor $(r_c)^{2m}$ is replaced by some negative number. Clearly, the feedback scheme has a stabilizing effect on intrinsically stable modes provided that the effective radius of the fake rotating shell, r_e , lies outside the radius of the passive shell, r_v , and has a destabilizing effect otherwise. Note, from Eqs. (72) and (73), that the effective radius *increases* monotonically with increasing poloidal mode number. Thus if $r_e \geq r_v$ for $m = 1$ then this inequality is bound to be satisfied for all $m > 1$, and, hence, there is no danger of the feedback scheme inadvertently destabilizing some modes. The condition which must be satisfied reduces to

$$\frac{\tau}{-\tau'} \leq \left(\frac{r_{l'}}{r_l} \right) \frac{(r_l)^2 - (r_v)^2}{(r_w)^2 - (r_v)^2}. \quad (76)$$

The critical value of $g_l = \tau_w\Delta\theta/\tau$ which must be exceeded in order to stabilize the resistive shell mode is given by

$$\begin{aligned} (g_l)_c &= 2 \left(\frac{r_l}{r_w} \right)^m \frac{(r_w)^{2m}}{\sqrt{(r_c)^{2m} - (r_e)^{2m}} \sqrt{(r_e)^{2m} - (r_v)^{2m}}} \\ &\times \frac{(r_w)^{2m} - (r_e)^{2m}}{(r_w)^{2m} - (r_l)^{2m}}. \end{aligned} \quad (77)$$

Likewise, the critical value of $g_{l'} = -\tau_w\Delta\theta/\tau'$ takes the form

$$\begin{aligned} (g_{l'})_c &= 2 \left(\frac{r_{l'}}{r_w} \right)^m \\ &\times \frac{(r_w)^{2m}}{\sqrt{(r_c)^{2m} - (r_e)^{2m}} \sqrt{(r_e)^{2m} - (r_v)^{2m}}} \\ &\times \frac{(r_l)^{2m} - (r_e)^{2m}}{(r_w)^{2m} - (r_l)^{2m}}. \end{aligned} \quad (78)$$

The critical voltage gain $G = (r_w/r_l)^m(1 - \lambda)/|\gamma|\tau$ in the feedback circuits (i.e., the ratio of the voltage put out by an individual feedback amplifier to that which is generated by magnetic induction in the corresponding loop of the power network) is

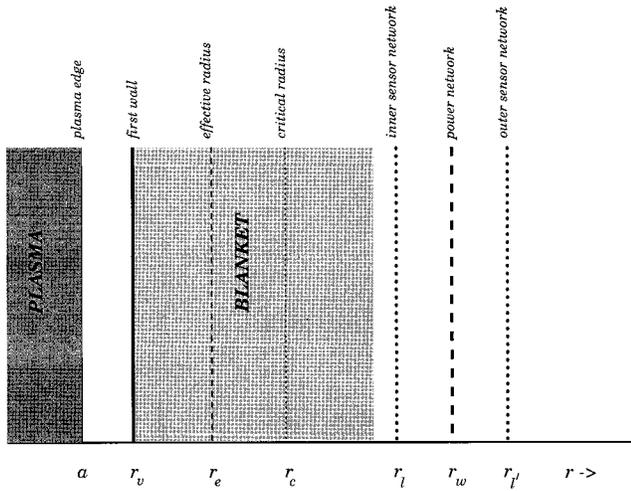


FIG. 3. Schematic diagram showing the proposed scheme for stabilizing the resistive shell mode in a tokamak reactor.

$$G_c = \frac{\tau_v}{m\Delta\theta\tau_w} \left(\frac{r_w}{r_v}\right)^{2m} \frac{(r_c)^{2m} - (r_v)^{2m}}{(r_c)^{2m} - (r_e)^{2m}}. \quad (79)$$

The corresponding critical current which is driven in the power network is given by

$$I_c \sim \frac{R_0\Delta\theta\Delta\phi}{R_\phi} \left(\frac{r_l b_r(r_l)}{\tau} + \frac{r_l' b_r(r_l')}{\tau'} \right) = g_c \frac{r_w \Delta\theta b_r(r_w)}{\mu_0}, \quad (80)$$

where

$$g_c = 2 \frac{(r_w)^{2m}}{\sqrt{(r_c)^{2m} - (r_e)^{2m}} \sqrt{(r_e)^{2m} - (r_v)^{2m}}}. \quad (81)$$

Here, the variation of the radial magnetic field strength in the immediate vicinity of the power network is approximated as $b_r(r) \propto r^{-(m+1)}$. The total power expended by the feedback amplifiers is given by

$$P_c \sim \frac{2\pi}{\Delta\theta} \frac{2\pi}{\Delta\phi} \frac{I_c^2 R_\phi}{2} = g_c^2 \frac{2\pi^2 r_w^2 R_0 b_r^2(r_w)}{\mu_0 \tau_w}. \quad (82)$$

C. Summary

A power network straddled by two sensor networks is capable of mimicking the action of a rotating shell whose radius lies well inside those of either the power or sensor networks. This scheme could be used to actively stabilize the resistive shell mode in a reactor relevant configuration in which all of the feedback controlled conductors and their associated detector arrays are situated outside the neutron shielding blanket.

V. SUMMARY AND DISCUSSION

The proposed scheme for stabilizing the resistive shell mode in a tokamak fusion reactor is sketched in Fig. 3. The plasma (minor radius a) is surrounded by a tight fitting, highly conducting first wall (radius r_v , L/R time τ_v). The first wall is surrounded by a thick neutron shielding blanket.

The vacuum vessel (which lies outside the blanket) is assumed to be sufficiently resistive that it has no influence on the stability of the resistive shell mode. The blanket is surrounded by three networks of conductors: two high impedance sensor networks (radii r_l and r_l' , with $r_l < r_l'$), with a low impedance power network (radius r_w , effective L/R time τ_w) sandwiched between them. Note that, from an engineering standpoint, the feedback scheme is far easier to implement in a reactor if all three networks can be placed outside the blanket. The signals detected by each cell in the two sensor networks are integrated, amplified (by factors $1/\tau$ and $1/\tau'$ for the networks situated at radii r_l and r_l' , respectively), and then fed into the corresponding cell of the power network, in an analogous manner to that shown in Fig. 2. The net result is that the networks mimic the action of a rotating shell of L/R time τ_w located at some radius r_e which lies inside the radii of all three networks. The effective radius of the fake rotating shell is determined by the ratio of the two amplification factors, $1/\tau$ and $1/\tau'$. The effective rotation frequency of the shell depends on the magnitude of the amplification (the larger the amplification, the faster the shell appears to rotate). Stabilization of the m/n resistive shell mode is possible provided that the effective radius of the fake shell lies outside that of the first wall, but inside the critical radius r_c at which a perfectly conducting shell is just able to stabilize the m/n ideal external kink mode. Stabilization is achieved once the effective rotation frequency of the fake shell exceeds a value of order $1/\tau_w$. Note that if $r_e < r_v$ or $r_e > r_c$ then the feedback scheme has a destabilizing effect on the mode. For a reactor possessing a thick blanket the critical radius is likely to lie well inside the radii of the three networks. However, contrary to conventional wisdom, this does not prevent the feedback scheme from being able to stabilize the resistive shell mode (provided that $r_e < r_c$). For a mode which is intrinsically stable in the absence of an external shell the feedback scheme has a stabilizing effect when $r_e > r_v$ and a destabilizing effect when $r_e < r_v$. Clearly, it is vitally important to ensure that the effective radius of the fake shell never becomes less than that of the first wall, otherwise the feedback scheme is likely to have a detrimental effect on plasma stability.

Suppose that the two sensor networks are equidistant from the power network, so that $r_l = r_w - d$ and $r_l' = r_w + d$. The radial spacing d determines the minimum possible number of network cells in the poloidal direction. Recall, from Sec. III B, that the sensor networks must be located sufficiently far away from the power network that they are unable to detect the short wavelength magnetic fields generated by currents circulating in latter network. This implies that

$$\frac{\Delta\theta}{2\pi} \lesssim \frac{d}{r_w}. \quad (83)$$

Furthermore, if the feedback scheme is required to stabilize resistive shell modes whose maximum toroidal mode number is N then the minimum possible number of network cells in the toroidal direction is given by $2N+1$.¹⁹

It is convenient to fix the effective radius of the fake rotating shell for $m=1$ modes to be same as the radius of the

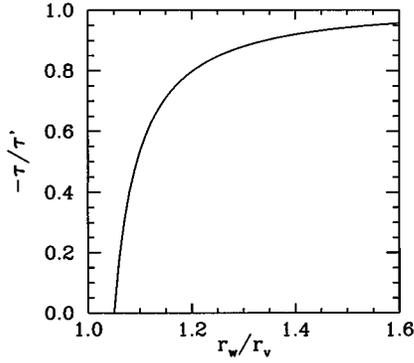


FIG. 4. The ratio of amplification factors $-(1/\tau')/(1/\tau)$ evaluated as a function of the radius of the power network according to Eq. (84). The network spacing is $d=0.05r_v$.

passive stabilizing shell, r_v . This criterion determines the ratio of the amplification factors via

$$-\frac{1/\tau'}{1/\tau} = \left(\frac{r_l}{r_l}\right) \frac{(r_l)^2 - (r_v)^2}{(r_w)^2 - (r_v)^2}. \quad (84)$$

Note that in this configuration the feedback scheme has no effect on the growth rate of the $m=1$ resistive shell mode. However, this is not a problem since tokamaks are conventionally operated with sufficiently high values of the edge safety factor q_a that they are stable to (predominantly) $m=1$ external kink modes. The fact that the effective radius r_e increases monotonically with increasing poloidal mode number ensures that $r_e > r_v$ for all $m > 1$. Thus there is no danger of the feedback scheme inadvertently destabilizing any $m > 1$ external kink modes.

Figure 4 shows the ratio of amplification factors calculated, using Eq. (84), as a function of the radius of the power network for $d=0.05r_v$. The ratio is zero (i.e., there is zero feedback from the outermost sensor network) when the inner sensor network touches the passive shell, and asymptotes to unity (at which point the feedback signals from the two sensor networks are equal and opposite) as the radius of the power network tends to infinity. At fixed r_w the ratio increases monotonically as the network spacing d is decreased.

The effective radius of the fake rotating shell for $m > 1$ is calculated using

$$r_e = r_l \left(\frac{1 - \lambda (r_w/r_l)^{2m}}{1 - \lambda} \right)^{1/2m}, \quad (85)$$

where $\lambda = -(\tau/\tau')(r_l/r_l)^m$. Figure 5 shows the effective radius evaluated as a function of the radius of the power network for poloidal mode numbers in the range 2 to 5 using the values of $-\tau/\tau'$ shown in Fig. 4. It can be seen that the feedback scheme allows the effective spacing between the fake rotating shell and the passive shell to be reduced by more than a factor of 2 with respect to the spacing between the power network and the passive shell. At fixed r_w the effective radius increases monotonically with increasing m and decreasing d .

The critical voltage gain in the feedback circuits which must be exceeded before the resistive shell mode is stabilized takes the form

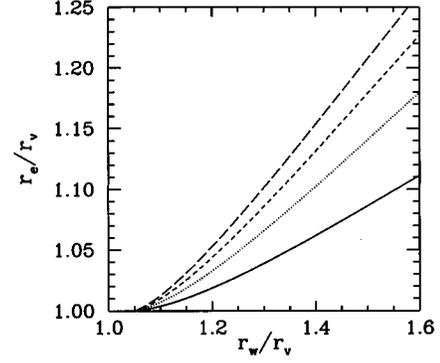


FIG. 5. The effective radius r_e of the fake rotating shell calculated as a function of the radius of the power network, using Eq. (85), for $d=0.05r_v$. The solid line corresponds to $m=2$, the dotted line to $m=3$, the short dashed line to $m=4$, and the long dashed line to $m=5$. Note that $r_e = r_v$ for $m=1$.

$$G_c = g \frac{\tau_v}{m \Delta \theta \tau_w}, \quad (86)$$

where

$$g = \left(\frac{r_w}{r_v}\right)^{2m} \frac{(r_c)^{2m} - (r_v)^{2m}}{(r_c)^{2m} - (r_e)^{2m}}. \quad (87)$$

The corresponding critical current which flows in the power network is

$$I_c \sim h \frac{r_v \Delta \theta b_r(r_v)}{\mu_0}, \quad (88)$$

where

$$h = \frac{2r_v^m r_w^m}{\sqrt{(r_c)^{2m} - (r_e)^{2m}} \sqrt{(r_e)^{2m} - (r_v)^{2m}}}. \quad (89)$$

The above formulas are obtained using the rather crude approximation that the perturbed radial magnetic field varies as $b_r(r) \propto r^{-(m+1)}$ outside the passive shell. The critical total power expended by the feedback amplifiers is given by

$$P_c \sim h^2 \frac{2\pi^2 r_v^2 R_0 b_r^2(r_v)}{\mu_0 \tau_w}. \quad (90)$$

Equations (86)–(90) are valid for all three implementations of the fake rotating shell feedback scheme discussed in this paper. In other words, these equations are *generic* to the fake rotating shell concept. The principal difference between the three implementations is the manner in which the effective radius of the fake rotating shell is determined. In the original implementation (see Sec. II), where the sensor network and the power network are situated at the same radius, the effective radius lies *outside* the common radius of the networks because of short wavelength inductive coupling between the networks. In the second implementation (see Sec. III), where the sensor network is situated inside the power network, the effective radius is the same as that of the sensor network. Finally, in the third implementation (see Sec. IV), in which two sensor networks straddle the power network, the effective radius lies *inside* the radii of all three networks.

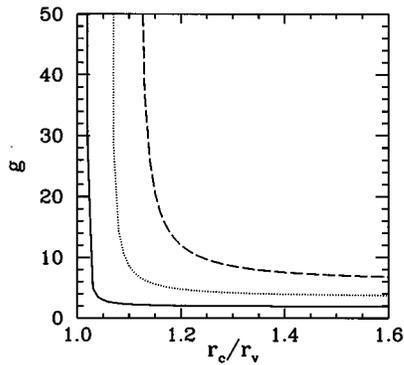


FIG. 6. The parameter g calculated as a function of the critical radius, using Eq. (87), for $m=3$ and $d=0.05r_v$. The solid line corresponds to $r_w = 1.2r_v$, the dotted line to $r_w = 1.4r_v$, and the dashed line to $r_w = 1.6r_v$. Note that $r_c = r_v$ for $m=1$.

The third implementation is clearly the most reactor relevant, and is, therefore, the one adopted in this paper.

Note that the total power expended by the feedback system in order to stabilize the resistive shell mode, which is given in Eq. (90), is of the same order of magnitude as the power which would be dissipated by the mode, via ohmic heating, if the plasma were surrounded by a single conventional shell of radius r_w and L/R time τ_w .

Figures 6 and 7 show the parameters g and h evaluated as a function of the critical radius r_c , for $m=3$, using the effective radii shown in Fig. 5. It can be seen, for instance, that if the power network is located relatively far from the plasma at $r_w = 1.6r_v$ then the feedback scheme is capable of stabilizing all $m=3$ external modes whose critical radii lie beyond $1.3r_v$, provided that the parameter g (which is directly related to the gain in the feedback circuits) exceeds a critical value of about 28. The corresponding critical value of the parameter h is about 4.2. These critical values can be used to calculate the current and power requirements of the feedback system using Eqs. (88) and (90), respectively. The parameter g increases strongly with increasing poloidal mode number m and increases weakly with decreasing network spacing d . The parameter h increases strongly with increasing poloidal mode number m and weakly with de-

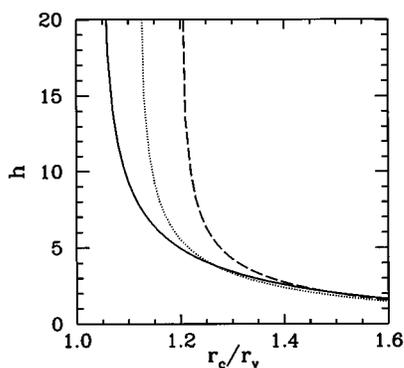


FIG. 7. The parameter h calculated as a function of the critical radius, using Eq. (89), for $m=3$ and $d=0.05r_v$. The solid line corresponds to $r_w = 1.2r_v$, the dotted line to $r_w = 1.4r_v$, and the dashed line to $r_w = 1.6r_v$. Note that $r_c = r_v$ for $m=1$.

creasing network spacing d at comparatively low values of the critical radius r_c , but varies in the opposite direction at higher values of r_c .

The major radius, average minor radius, and on-axis toroidal magnetic field strength of the proposed ITER¹⁶ tokamak are $R_0 = 8.1$ m, $a = 3$ m, and $B_\phi = 5.7$ T, respectively. The first wall possesses an average minor radius of $r_v \sim 1.1a$ and an L/R time of $\tau_v \sim 1$ s.²⁴ Consider a fake rotating shell feedback system which is required to stabilize the $m=3$ resistive shell mode in ITER at perturbed magnetic field levels of up to 1% of the edge poloidal magnetic field strength: i.e., $b_r(r_v) \leq 6 \times 10^{-3}$ T (assuming that $q_a \sim 3.5$). Suppose that the sensor and power networks are located *outside* the vacuum vessel, so that $r_w = 1.6a$, and that the radial spacing between the networks is $d = 0.05a$. According to Eq. (83) the minimum number of network cells in the poloidal direction is 32. Eight cells in the toroidal direction are sufficient to enable the system to stabilize instabilities with toroidal mode numbers in the range $n \leq 3$. According to Eqs. (84) and (85), the effective radius of the fake rotating shell for an $m=3$ mode is $r_e = 1.24a$. Suppose that the feedback system is required to stabilize the $m=3$ resistive shell mode whenever its critical radius lies beyond $1.3a$. This implies that the $m=3$ kink stability boundaries of the plasma are equivalent to those calculated in the presence of a perfectly conducting shell located at radius $1.3a$ (this is sufficiently close to the plasma to significantly increase the β limit¹⁴). Using Eqs. (87) and (89), the critical values of the parameters g and h are calculated to be 23 and 7, respectively. It follows from Eq. (88) that the peak current which flows in the power network is around 20 kA. Assuming that this network is constructed out of 5 cm diameter copper wires (resistivity $1.7 \times 10^{-8} \Omega\text{m}$ ²⁵), it follows from Eq. (19) that its effective L/R time is $\tau_w \sim 0.75$ s. According to Eq. (86), the critical voltage gain in the feedback circuits is around 50. Finally, Eq. (90) gives a peak power expended by the feedback amplifiers of about 3 MW. Note that the above estimates are slightly inaccurate since the ordering $\tau_w \ll \tau_v$ (which was adopted purely for the sake of simplicity) is not very well satisfied in this case. (It turns out that the power expended by the feedback amplifiers, as a function of τ_w , attains its minimum value at $\tau_w \approx \tau_v$, so the situation under consideration is near optimal.) The vacuum vessel (whose L/R time in ITER is smaller than, but still comparable with, that of the first wall) is also likely to give rise to modifications of the above results. Nevertheless, the above calculations can probably be relied on to give, at least, an order of magnitude estimate of the current and power requirements of a resistive shell mode feedback system on ITER. The current and power levels mentioned above fall well within the acceptable bounds. Note that the feedback amplifiers only have to respond to magnetic fields which are able to diffuse through the first wall. Thus, the bandwidth of the amplifiers need only be $\Delta f \sim 1/\tau_v$.

In conclusion, it appears likely that a feedback system which mimics the effect of a relatively close fitting ideal shell can be installed on a tokamak reactor, despite the fact that all of the feedback controlled conductors and sensor loops are located relatively far from the plasma (i.e., at least

outside the blanket, and possibly outside the vacuum vessel). The gain, bandwidth, current, and total power requirements of the system are extremely modest (i.e., less than 100, a few Hz, a few tens of kA, and a few MW, respectively), and could easily be achieved using existing technology. Thus an “advanced tokamak” reactor appears eminently feasible.

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¹C. Kessel, J. Manickam, G. Rewoldt, and W. M. Tang, *Phys. Rev. Lett.* **72**, 1212 (1994).

²The conventional definition of this parameter is $\beta = 2\mu_0 \langle p \rangle / \langle B^2 \rangle$, where $\langle \dots \rangle$ denotes a volume average, p is the plasma pressure, and B is the magnetic field strength.

³R. J. Bickerton, J. W. Connor, and J. B. Taylor, *Nature Phys. Sci.* **229**, 110 (1979).

⁴R. J. Goldston, *Phys. Plasmas* **3**, 1794 (1996).

⁵J. A. Wesson, *Nucl. Fusion* **18**, 87 (1978).

⁶J. Manickam, M. S. Chance, S. C. Jardin, C. Kessel, D. Monticello, N. Pomphrey, A. Reiman, C. Wang, and L. E. Zakharov, *Phys. Plasmas* **1**, 1601 (1994).

⁷The L/R time of a toroidal shell is defined as the ratio of inductance to resistance calculated for a mode which excites low mode number helical current patterns in the shell.

⁸D. Pfirsch and H. Tasso, *Nucl. Fusion* **11**, 259 (1971).

⁹B. Alper, M. K. Bevir, H. A. B. Bodin, C. A. Bunting, P. G. Carolan, J. Cunnane, D. E. Evans, C. G. Gimblett, R. J. Hayden, T. C. Hender, A. Lazaros, R. W. Moses, A. A. Newton, P. G. Noonan, R. Paccagnella, A. Patel, H. Y. W. Tsui, and P. D. Wilcock, *Plasma Phys. Controlled Nucl. Fusion* **31**, 205 (1989).

¹⁰A. Bondeson and D. J. Ward, *Phys. Rev. Lett.* **72**, 2709 (1994).

¹¹R. Betti and J. P. Freidberg, *Phys. Rev. Lett.* **74**, 2949 (1995).

¹²R. Fitzpatrick and A. Y. Aydemir, *Nucl. Fusion* **36**, 11 (1996).

¹³The (toroidal) Alfvén velocity is defined $v_A = B_\phi / \sqrt{\mu_0 \rho}$, where B_ϕ is the toroidal magnetic field strength, and ρ is the plasma mass density.

¹⁴E. J. Strait, T. S. Taylor, A. D. Turnbull, J. R. Ferron, L. L. Lao, B. Rice, O. Sauter, S. J. Thompson, and D. Wróblewski, *Phys. Rev. Lett.* **74**, 2483 (1995).

¹⁵M. Okabayashi, N. Pomphrey, J. Manickam, D. Ward, R. Bell, R. Hatcher, R. Kaita, S. Kaye, H. Kugel, B. LeBlanc, F. Levington, D. W. Roberts, S. Sesnic, Y. Sun, and H. Takahashi, *Nucl. Fusion* **36**, 1167 (1996).

¹⁶P.-H. Rebut, *Fusion Technol.* **27**, 3 (1995).

¹⁷W. M. Nevins (private communication, 1996).

¹⁸C. M. Bishop, *Plasma Phys. Controlled Nucl. Fusion* **31**, 1179 (1989).

¹⁹R. Fitzpatrick and T. H. Jensen, *Phys. Plasmas* **3**, 2641 (1996).

²⁰The standard large aspect ratio, low beta tokamak ordering is $R_0/a \gg 1$ and $\beta \sim (a/R_0)^2$, where R_0 and a are the major and minor radii of the plasma, respectively.

²¹R. Fitzpatrick, in *Plasma Physics and Controlled Nuclear Fusion Research 1994*, Proceedings of 15th International Conference, Seville, 1994 (International Atomic Energy Agency, Vienna, 1996), Vol. 3, p. 245.

²²R. W. Conn, *et al.*, in *Plasma Physics and Controlled Nuclear Fusion Research 1990*, Proceedings of 13th International Conference, Washington, D.C., 1990 (International Atomic Energy Agency, Vienna, 1991), Vol. 3, p. 659.

²³R. Fitzpatrick, R. J. Hastie, T. J. Martin, and C. M. Roach, *Nucl. Fusion* **33**, 1533 (1993).

²⁴M. N. Rosenbluth (private communication, 1995).

²⁵*Handbook of Chemistry and Physics*, edited by D. R. Lide (Chemical Rubber Company, Boca Raton 1995).