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Richard Fitzpatrick

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Interaction of scrape-off layer currents with magnetohydrodynamical instabilities in tokamak plasmas

Richard Fitzpatrick
Institute for Fusion Studies, Department of Physics, University of Texas at Austin, Austin, Texas 78712

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A simple theoretical model is developed which describes how current eddies are excited in the scrape-off layer (SOL) of a large-aspect-ratio, low-β, circular cross-section tokamak by time-varying magnetohydrodynamical instabilities originating from within the plasma. This model is used to study the interaction of SOL currents with tearing modes and resistive wall modes in a typical tokamak plasma. SOL currents are found to be fairly effective at braking the rotation of tearing modes, and to have a significant destabilizing effect on resistive wall modes. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2747624]

I. INTRODUCTION

A tokamak is a device designed to confine a hot plasma on a set of closed, toroidally nested, magnetic flux surfaces.[1] The last closed flux surface (LCFS) is defined as that outside which all flux surfaces intersect solid boundaries. In general, the main plasma in a tokamak is confined within the LCFS, but is surrounded by a relatively cold plasma, known as the *scrape-off layer* (SOL), which extends slightly beyond the LCFS (see Fig. 1).

Tokamak confinement is often limited by magnetohydrodynamical (MHD) instabilities, such as tearing modes and resistive wall modes (RWMs), which are driven by radial current and pressure gradients within the main plasma. Such instabilities are commonly observed in combination with strong electric currents flowing in the SOL (see Refs. 2–5 and, in particular, Refs. 6 and 7). Neither the origin of these currents is fully understood, nor their likely effect on the various types of MHD instability with which they are usually associated. The aim of this paper is to shed some light on this phenomenon by constructing a simple theoretical model in which it is assumed that the SOL currents arise via *passive inductive coupling* to MHD instabilities emanating from inside the plasma. All calculations are performed using the standard large-aspect-ratio, low-β, circular flux-surface tokamak ordering.

II. PHYSICS OF SOL CURRENTS

A. Plasma equilibrium

Let us adopt the conventional cylindrical polar coordinates (r,θ,z). Consider a cylindrical plasma equilibrium characterized by the magnetic field $\mathbf{B}=(0,B_\theta(r),B_\phi)$, where $B_\phi \gg B_\theta$. Let the system be periodic in the z direction, with periodicity length $2\pi R_0$, where R_0 is the simulated major radius. It is helpful to define a simulated toroidal angle $\phi = z/R_0$. Finally, the safety-factor profile is written as $q(r) = rB_\phi/R_0B_\theta(r)$.

B. The SOL

Suppose that the plasma is limited by a solid, coaxial, axisymmetric, electrically conducting ring whose inner surface is a uniform distance a (where $a \ll R_0$) from the center of the plasma (see Fig. 1). Let the limiter extend over the range of poloidal angles $-\Delta\theta \leqslant \theta \leqslant \Delta\theta$ (where $\Delta\theta \ll 2\pi$). The SOL is a thin layer of relatively cold plasma, extending from $r=a$ to $r=a+\delta_p$ (where $\delta_p \ll a$), which is sandwiched between the relatively hot main plasma (extending over $r<a$), and the surrounding vacuum (extending over $r>a+\delta_p$). Unlike the main plasma, which is confined on *closed* magnetic flux surfaces, the SOL plasma is confined on flux surfaces which *intersect* the limiter. Actually, the setup sketched in Fig. 1 can be thought of as a very simple representation of a conventional magnetic divertor scheme, with the two sides of the limiter in contact with the SOL playing the roles of the two divertor plates.

C. SOL current eddies

We can write the perturbed magnetic field associated with an MHD instability originating inside the main plasma in the conventional form, $\mathbf{b}=\nabla\psi\times\hat{\mathbf{z}}$. Here, $\psi(r,\theta,\phi)$ is the perturbed magnetic flux function. We can also identify

$$\Psi(\theta,\phi) \equiv \psi(a,\theta,\phi) \tag{1}$$

as the perturbed magnetic flux penetrating the SOL. (It is assumed, for the sake of simplicity, that ψ does not have a strong radial variation across the SOL.) Likewise, the quantity

$$\Delta\Psi(\theta,\phi) \equiv \left[r\frac{\partial\psi(r,\theta,\phi)}{\partial r} \right]_{r=a_-}^{r=a_+} \tag{2}$$

is a convenient stream function for the radially integrated perturbed electric current flowing in the SOL.[8]

It seems reasonable to treat the comparatively cold plasma at the edge of a tokamak discharge as essentially *force free*; i.e., possessing no appreciable equilibrium plasma current. Now, in a force-free plasma, the perturbed current must flow *parallel* to the equilibrium magnetic field, otherwise unbalanced electromagnetic forces are generated. Sup-

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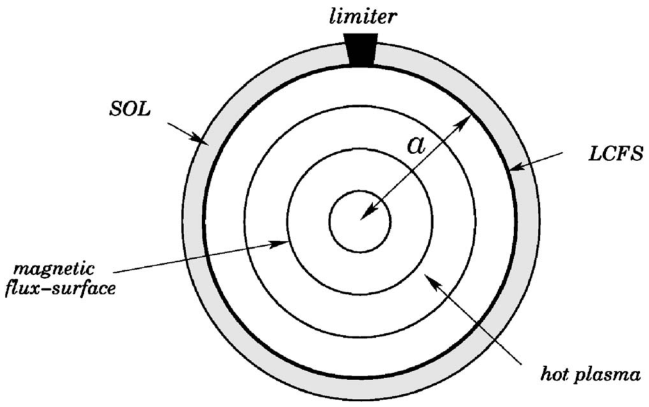

FIG. 1. A schematic diagram showing the poloidal cross section of a tokamak with an axisymmetric, poloidally localized limiter, and the associated SOL.

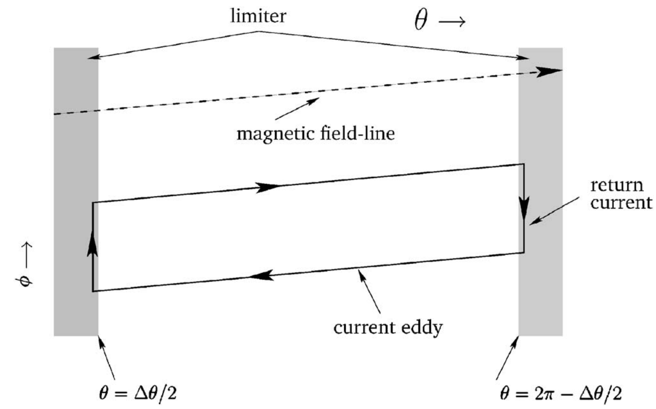

FIG. 2. A schematic diagram of an SOL current eddy.

pose that all perturbed quantities possess a common $e^{-in\phi}$ dependence in the toroidal direction, where the positive integer n is the toroidal mode number of the MHD instability. It follows that

$$\mathbf{B}\cdot\nabla \equiv \frac{B_\theta}{r}\left(\frac{\partial}{\partial\theta} - inq\right). \tag{3}$$

Now, the current stream function in the SOL must satisfy $(\mathbf{B}\cdot\nabla)\Delta\Psi=0$, in order to ensure that the perturbed current flows parallel to the equilibrium magnetic field. Hence,

$$\Delta\Psi(\theta,\phi) \sim e^{in(q_a\theta-\phi)} \tag{4}$$

in the SOL, where $q_a=q(a)$. (It is assumed, for the sake of simplicity, that there is no significant radial variation in the helical pitch of the equilibrium magnetic field lines across the SOL.) Note that nq_a is *noninteger*, since the SOL *does not*, in general, lie on a rational magnetic flux surface (i.e., a flux surface for which $nq=m$, where m is a positive integer).

Equation (4) implies that if $\Delta\Psi$ is nonzero then it must be *discontinuous* at some particular poloidal angle (because nq_a is noninteger and $\Delta\Psi$ certainly cannot be multivalued in θ). Now, a poloidal discontinuity in the current stream function $\Delta\Psi$ indicates the presence of strong *return currents*, flowing across magnetic field lines, which close the SOL current eddies. But, as we have already seen, cross-field currents *cannot* flow in the force-free SOL plasma, since they would generate unbalanced electromagnetic forces. Recall, however, that the SOL plasma does not extend over all poloidal angles, but is interrupted by the solid limiter for angles in the range $-\Delta\theta \leqslant \theta \leqslant \Delta\theta$. Moreover, cross-field currents *can* flow within the limiter, because the associated electromagnetic forces can be balanced there by *mechanical stresses*.

It follows, from the above discussion, that it is possible for SOL current eddies, closed by cross-field return currents flowing within the limiter, to be excited by MHD activity originating inside the plasma. A typical SOL current eddy is sketched in Fig. 2. Note that it is *not* possible for similar current eddies to be excited in the edge plasma immediately inside the LCFS, since there is no way of balancing the forces generated by the associated return currents.

D. Plasma sheath effects

Plasma sheaths form at the boundaries between the SOL plasma and the limiter.[9] Moreover, it is well known that the electric current density flowing across such a sheath cannot exceed the *ion saturation current density*,[10]

$$j_{\rm sat} = \frac{en_e}{2\sqrt{2}}\left(\frac{T_i+T_e}{m_i}\right)^{1/2}, \tag{5}$$

when the current flowing from the plasma to the limiter is positive. There is no such restriction when the current is negative. Here, n_e, T_i, and T_e are the electron number density, ion temperature, and electron temperature, respectively, just above the limiter, and m_i is the ion mass. Since SOL current eddies must form complete circuits that cross plasma sheaths in both directions, the ion saturation current places an effective *upper limit* on the intensity of such eddies: i.e., the current density excited in the SOL plasma *cannot* exceed $j_{\rm sat}$.

III. SOL RESPONSE TO MHD ACTIVITY

A. Limiter response

Let the limiter have electrical conductivity σ_l and radial thickness δ_l (where $\delta_l \ll a$). The conventional thin-shell time constant of the limiter is thus $\tau_l=\mu_0 a\,\delta_l\sigma_l$ (Ref. 8). Suppose that all perturbed quantities vary in time as $e^{\gamma t}$. The response of the limiter to the inductive electric field generated by a growing MHD mode is then given by[8]

$$\Delta\Psi = \gamma\tau_l\Psi. \tag{6}$$

This equation holds over the whole poloidal and toroidal extent of the limiter.

B. SOL plasma response

Let the SOL plasma have electrical conductivity σ_p and radial thickness δ_p. The conventional thin-shell time constant of the SOL plasma is thus $\tau_p=\mu_0 a\,\delta_p\sigma_p$. We expect the response of the SOL plasma to be similar to that of the limiter, except in one crucially important respect. Namely, that only currents which flow *parallel* to equilibrium magnetic field, and return through the limiter, can be excited in the force-free SOL plasma. Hence, we can model the response of the SOL plasma by writing

$$\Delta\Psi = \gamma\tau_p \Psi_p e^{in(q_a\theta-\phi)} \tag{7}$$

and

$$\Psi_p = \frac{1}{4\pi^2}\int_p \Psi e^{-in(q_a\theta-\phi)} d\theta d\phi. \tag{8}$$

Here, the integral is taken over the whole SOL plasma.

Actually, since a typical SOL current eddy flows through the SOL plasma a distance of order $2\pi\, R_0\, q_a$, and flows through the limiter a distance of order $2\pi\, R_0/n$, it is more accurate to replace τ_p in Eq. (7) by $\tau_{p'}$, where

$$\tau_{p'}^{-1} = \tau_p^{-1} + \tau_l^{-1} n q_a. \tag{9}$$

Thus, the modified SOL plasma response is

$$\Delta\Psi = \gamma\tau_{p'} \Psi_p e^{in(q_a\theta-\phi)}. \tag{10}$$

This equation holds over the whole poloidal and toroidal extent of the SOL plasma.

C. SOL response matrix

We can write

$$\Psi(\theta,\phi) = \sum_{m'} \Psi^{m',n} e^{i(m'\theta-n\phi)}, \tag{11}$$

$$\Delta\Psi(\theta,\phi) = \sum_{m'} \Delta\Psi^{m',n} e^{i(m'\theta-n\phi)}, \tag{12}$$

where the m' are integers. Of course, Eq. (12) can be inverted to give

$$\Delta\Psi^{m,n} = \frac{1}{4\pi^2}\oint\oint \Delta\Psi e^{-i(m\theta-n\phi)} d\theta d\phi, \tag{13}$$

where the current stream function $\Delta\Psi$ is given by Eq. (6) in the limiter, and Eq. (10) in the SOL plasma. It follows that

$$\Delta\Psi^{m,n} = \gamma \sum_{m'} \tau^{m,m'} \Psi^{m',n}. \tag{14}$$

The SOL response matrix, $\tau^{m,m'}$, takes the form

$$\begin{aligned}\tau^{m,m'} = {} & \tau_l f \operatorname{sinc}[(m-m')f\pi] + \tau_{p'} \\ & \times (1-f)^2 e^{-i(m-m')\pi} \operatorname{sinc}[(m-nq_a)(1-f)\pi] \\ & \times \operatorname{sinc}[(m'-nq_a)(1-f)\pi],\end{aligned} \tag{15}$$

where $\operatorname{sinc}(x) \equiv \sin x/x$ and $f=\Delta\phi/2\pi$ is the area fraction of the SOL flux surface occupied by the limiter.

D. Effective SOL time constant

It is clear from Eq. (15) that an m, n MHD instability inside the plasma can excite a pattern of currents in the SOL which couples the mode to MHD modes with *different* poloidal mode numbers. In this paper, however, we are primarily interested in the back reaction of the SOL currents on the original mode. Hence, we shall neglect poloidal coupling via SOL currents, and simply write

$$\Delta\Psi^{m,n} = \gamma\tau_a \Psi^{m,n}, \tag{16}$$

where

$$\tau_a \equiv \tau^{m,m} = \tau_l f + \tau_{p'}(1-f)^2 \operatorname{sinc}^2[(m-nq_a)(1-f)\pi]. \tag{17}$$

According to the above expression, the response of the SOL to a helical MHD instability is similar to that of a *thin resistive wall* of effective time constant τ_a, located at the edge of the plasma.[8] Note, however, that, unlike a conventional wall, the *effective time constant* of the SOL, τ_a, *varies strongly* with the mode numbers, m and n, of the instability, and with the edge safety factor, q_a. Indeed, the time constant is relatively long, when the *helical pitch* of the equilibrium magnetic field lines in the SOL closely matches that of the instability, and relatively short otherwise.

E. Ion saturation current

It is convenient to convert the ion saturation current density, introduced in Sec. II D, into an (approximate) total current circulating in an SOL eddy; i.e.,

$$J_{\rm sat} \simeq \frac{\pi R_0 \delta_p j_{\rm sat}}{n}. \tag{18}$$

We shall refer to $J_{\rm sat}$ as the *ion saturation current*. Plasma sheath physics effectively prevents the net current circulating in the SOL, which can be estimated as $\pi R_0 \Delta\Psi^{m,n}/(na\mu_0)$, from exceeding $J_{\rm sat}$.

IV. THE SOL IN A TYPICAL DIII-D PLASMA

The DIII-D tokamak[11] has a major radius $R_0=1.69$ m, mean minor radius $a=0.54$ m, typical on-axis toroidal magnetic field strength $B_\phi=2.1$ T, typical on-axis electron number density $n_0=6\times10^{19}$ m^{-3}, and typical momentum confinement time scale $\tau_M=60$ ms (Refs. 12 and 13). Moreover, the typical electron number density, electron temperature, ion temperature, and cross flux-surface thermal transport coefficient in the SOL of a DIII-D discharge are $n_e=10^{19}$ m^{-3}, $T_e=50$ eV, $T_i=100$ eV, and $\chi=1$ m^2 s^{-1}, respectively.[14] Let us assume, for the sake of simplicity, that the SOL plasma is a pure hydrogen plasma.

Now, according to standard theory, we can estimate the radial width of the SOL by balancing parallel and perpendicular thermal transport.[9] Hence, we obtain

$$\delta_p \sim \sqrt{\frac{\chi L}{0.5 c_s}}, \tag{19}$$

where $c_s=\sqrt{(T_i+T_e)/m_i}$ is the sound speed, and $L=2\pi q_a R_0$ the length of a magnetic field line in the SOL plasma. Assuming that $q_a\sim3$, we get

$$\delta_p \sim 0.02 \text{ m}. \tag{20}$$

Now, the parallel Spitzer resistivity of the SOL plasma is[15]

$$\eta_\parallel \sim 3 \times 10^{-6}\ \Omega\,\mathrm{m}. \tag{21}$$

Hence, the thin-shell time constant of the SOL plasma becomes

$$\tau_p = \mu_0 a \delta_p \eta_\parallel^{-1} \sim 6 \times 10^{-3}\ \mathrm{s}. \tag{22}$$

Assuming, as seems reasonable, that the SOL return currents flow mostly through centimeter-thick stainless steel ($\eta \sim 7 \times 10^{-7}\ \Omega$ m), it follows, from Eq. (9), that the time scale $\tau_{p'}$ is largely determined by the resistance of the SOL plasma, rather than that of the limiter; i.e., $\tau_{p'} \sim \tau_p$. (Of course, the SOL currents must flow across the graphite tiles covering the limiter in order to return through the stainless steel. However, given that the resistivity of graphite, $8 \times 10^{-6}\ \Omega$ m, is not much greater than that of the SOL plasma, and that the current path length through the tiles is much less than that through the plasma, it seems reasonable to assume that the resistivity of the tiles does not contribute significantly to the effective resistivity of the SOL.)

Under the assumption that the poloidal extent of the limiter is small (i.e, $f \to 0$), it follows, from Eq. (17), that the effective time constant of the SOL in a cylindrical plasma with similar parameters to a typical DIII-D plasma, when interacting with an m, n mode, is given by

$$\tau_a \sim \tau_p \mathrm{sinc}^2[(m - nq_a)\pi], \tag{23}$$

where $\tau_p \sim 6$ ms.

In DIII-D, the electron number density at the limiter—in reality, at the divertor plates—is similar to that in the main SOL plasma; i.e., $n_e \sim 10^{19}\ \mathrm{m}^{-3}$ (Ref. 16). Furthermore, the typical ion and electron temperatures at the divertor plates are $T_i \sim 100$ eV and $T_e \sim 50$ eV, respectively, giving an estimated ion saturation current (for an $n=1$ mode) of $J_{\mathrm{sat}} \sim 8$ kA (Ref. 16).

V. SOL INTERACTION WITH TEARING MODES

A. Introduction

Tearing modes are slowly growing, rotating MHD instabilities whose rational surfaces lie on closed magnetic flux surfaces located within the main plasma. In the following, we shall investigate the interaction of various different tearing modes with the SOL in a cylindrical plasma with similar parameters to a typical DIII-D plasma.

B. Theory

All perturbed quantities are assumed to vary with θ and ϕ like $\exp[i(m\theta - n\phi)]$, where the integers m and n are the poloidal and toroidal mode numbers, respectively, of the tearing mode. In addition, all perturbed quantities are assumed to vary in time like $\exp(in\Omega t)$ (i.e., $\gamma = in\Omega$), where Ω is plasma toroidal rotation frequency at the rational surface. Here, we are making use of the so-called *no-slip constraint*, according to which a tearing mode corotates with the plasma at its own rational surface.[17] Incidentally, we are also assuming that the amplitude of the mode is constant, or very slowly varying, and that the plasma only rotates in the toroidal direction.[17]

Let b_r and b_{r0} be the radial magnetic field perturbations in the SOL in the presence, and in the absence, of SOL currents, respectively. These two quantities are related via[17]

$$b_r = \frac{b_{r0}}{1 + i\Omega/\Omega_c}, \tag{24}$$

where

$$\Omega_c = \frac{2m}{n\tau_a} \tag{25}$$

is the critical value of Ω above which currents induced in the SOL start to significantly *shield* the magnetic perturbation associated with the tearing mode from the vacuum region outside the SOL. Thus, there is no shielding if $\Omega \ll \Omega_c$, partial shielding if $\Omega \sim \Omega_c$, and strong shielding if $\Omega \gg \Omega_c$.

The net toroidal electromagnetic braking torque exerted on the plasma by the induced SOL currents is[17]

$$T_\phi = \frac{2Vn}{m} \frac{\Omega/\Omega_c}{1 + (\Omega/\Omega_c)^2} \frac{b_{r0}^2}{\mu_0}, \tag{26}$$

where $V = 2\pi^2 a^2 R_0$ is the plasma volume. Note that this torque is exerted at the rational surface.[17] In a steady state, we can balance the above torque against a phenomenological viscous restoring torque to obtain the following torque balance equation:[17]

$$V\rho R_0^2 \left(\frac{\Omega_0 - \Omega}{\tau_M} \right) = T_\phi. \tag{27}$$

Here, ρ is the plasma mass density, Ω_0 the value of Ω in the absence of SOL currents, and τ_M the momentum confinement time. The above equation reduces to

$$4(1 - \hat{\Omega}) = \frac{\hat{\Omega}}{\alpha + \hat{\Omega}^2} \left(\frac{b_{r0}}{b_{rc}} \right)^2, \tag{28}$$

where $\hat{\Omega} = \Omega / \Omega_0$,

$$\alpha = \left(\frac{\Omega_c}{\Omega_0} \right)^2, \tag{29}$$

and

$$b_{rc} = \frac{\Omega_0 \tau_H}{4} \sqrt{\frac{\tau_a}{\tau_M}} B_\phi. \tag{30}$$

Here, b_{rc} is the critical value of b_{r0} above which the braking torque exerted on the plasma by the induced SOL currents significantly reduces the plasma rotation frequency at the rational surface (assuming that $\alpha \ll 1$). Also, $\tau_H = R_0 \sqrt{\mu_0 \rho} / B_\phi \sim 3 \times 10^{-7}$ s is the standard hydromagnetic time scale.

Equation (28) describes how the eddy currents induced in the SOL by a rotating tearing mode *brake* the plasma rotation frequency at the rational surface. As is easily demonstrated, when $1 \gg \alpha > 1/27$, this braking takes place *smoothly* and reversibly, as the amplitude of the tearing mode varies.[17] On the other hand, when $\alpha < 1/27$, there are two separate branches of solutions—a high and a low rotation frequency branch—and there are *bifurcations* between these

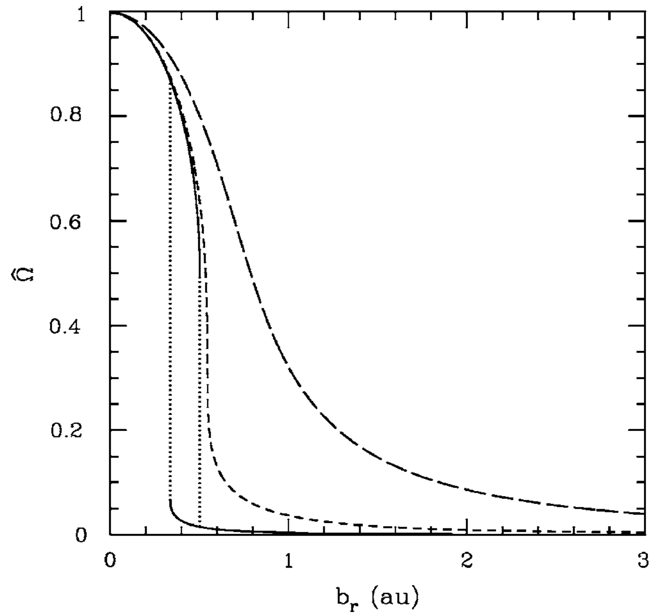

FIG. 3. Slowing-down curves for a tearing mode interacting with the SOL. The solid curves shows the case $\alpha=\alpha_c/10$, where $\alpha_c=1/27$. The bifurcations between the high and low rotation branches are indicated as dotted lines. The short-dashed curve shows the critical case $\alpha=\alpha_c$. The long-dashed curve shows the case $\alpha=10\alpha_c$.

two branches, as the amplitude of the tearing mode varies, accompanied by considerable hystereses.[17] The various different types of slowing-down curves are shown in Fig. 3.

Incidentally, the critical radial field in the SOL, taking the SOL currents into account (which is the field which would actually be measured in the SOL), is

$$b_r=\left(\frac{4\alpha}{1+4\alpha}\right)^{1/2}b_{rc}. \tag{31}$$

Moreover, the critical island width, W, associated with the tearing mode is

$$\frac{W}{a}\simeq 2\left[\frac{(q_a/q_s)^{m/2}}{n\epsilon_0}\frac{b_{rc}}{B_\phi}\right]^{1/2}, \tag{32}$$

where $\epsilon_0=a/R_0$ and $q_s=m/n$. Finally, the critical current circulating in the SOL is

$$J_a\sim\frac{2\pi R_0 b_{rc}}{n\mu_0}\left(\frac{1}{1+4\alpha}\right)^{1/2}. \tag{33}$$

C. The 2,1 mode

The most commonly occurring tearing mode in a typical tokamak plasma is the $m=2$, $n=1$ mode, resonant at the $q=2$ surface inside the plasma. Consider a discharge in which $q_a=3.5$. It follows, from Eq. (23), that the effective time constant of the SOL in a DIII-D-like plasma, when interacting with a 2,1 mode, is $\tau_a\sim 0.28$ ms. Hence, from Eq. (25), the critical toroidal plasma rotation frequency at the $q=2$ surface above which the SOL currents start to shield the mode from the vacuum region outside the SOL is $\Omega_c\sim 2$ kHz. Now, the typical value of Ω_0 for a 2,1 mode in a DIII-D discharge heated by unbalanced neutral beams is about 10 kHz (Ref. 18). Note that $\Omega_0/\Omega_c\sim 5$. Hence, we conclude that the SOL currents excited by a rotating 2,1 tearing mode in a DIII-D-like plasma are sufficient to *partially shield* the magnetic perturbation associated with the mode from the vacuum region surrounding the plasma.

Now, according to Eq. (29), $\alpha\sim 0.05$ for a 2,1 mode. Since $0.05>1/27$, it follows that the braking torque exerted on the plasma by the induced SOL currents causes the plasma toroidal rotation frequency at the $q=2$ surface to slow down smoothly, with no sudden jumps, as the amplitude of the tearing mode gradually increases (see the long-dashed curve in Fig. 3). From Eq. (30), the critical perturbed radial magnetic field in the SOL (in the absence of SOL currents), above which the induced SOL currents start to cause a significant reduction in the plasma rotation frequency at the $q=2$ surface, is $b_{rc}\sim 7$ G. The critical field which would actually be measured in the SOL is $b_r\sim 3$ G, and the critical 2,1 island width within the plasma is $W/a\sim 0.08$. Finally, the critical current circulating in the SOL is $J_a\sim 5.2$ kA. This is *similar* to our estimate for the ion saturation current. Hence, we conclude that the SOL currents associated with the slowing down of a 2,1 tearing mode are close to saturation.

D. The 3,1 mode

The next most common tearing mode, after the 2,1 mode, is the 3,1 mode, resonant at the $q=3$ surface inside the plasma. Consider, again, a discharge in which $q_a=3.5$. It follows, from Eq. (23), that the effective time constant of the SOL in a DIII-D-like plasma, when interacting with a 3,1 mode, is $\tau_a\sim 2.5$ ms. Note that this time constant is much larger than that obtained earlier for interaction with a 2,1 mode. This is because, when $q_a=3.5$, the helical pitch of a 3,1 mode much more closely matches the pitch of the equilibrium magnetic field lines in the SOL than that of a 2,1 mode. From Eq. (25), the critical toroidal plasma rotation frequency at the $q=3$ surface above which the SOL currents start to shield the mode from the vacuum region outside the SOL is $\Omega_c\sim 0.4$ kHz. Now, the typical value of Ω_0 for a 3,1 mode in a DIII-D discharge heated by unbalanced neutral beams is about 3.5 kHz (Ref. 18). Note that $\Omega_0/\Omega_c\gg 9$. Hence, we conclude that the SOL currents excited by a rotating 3,1 tearing mode in a DIII-D-like plasma are sufficient to *strongly shield* the magnetic perturbation associated with the mode from the vacuum region surrounding the plasma.

Now, according to Eq. (29), $\alpha\sim 0.012$ for a 3,1 mode. Since $0.012<1/27$, it follows that the braking torque exerted on the plasma by the induced SOL currents causes the plasma toroidal rotation frequency at the $q=3$ surface to slow down in a discontinuous manner, characterized by a very sudden reduction when the amplitude of the tearing mode exceeds a certain critical value (see the solid curve in Fig. 3). From Eq. (30), the critical perturbed radial magnetic field in the SOL (in the absence of SOL currents), above which the induced SOL currents cause a significant reduction in the plasma rotation frequency at the $q=3$ surface, is $b_{rc}\sim 3$ G. The critical field that would actually be measured in the SOL is $b_r\sim 0.7$ G, and the critical 3,1 island width within the plasma is $W/a\sim 0.05$. Finally, the critical current circulating in the SOL is $J_a \sim 2.8$ kA. This is, again, similar to our estimate for the ion saturation current. Hence, we conclude that the SOL currents associated with the slowing down of a 3,1 tearing mode are also close to saturation.

VI. SOL INTERACTION WITH RESISTIVE WALL MODES

A. Introduction

The resistive wall mode arises when an ideal kink mode—a mode whose rational surface lies *just outside* the plasma—is stable in the presence of a perfectly conducting wall, but unstable in the absence of such a wall.[19] In the following, we shall investigate the interaction of a RWM with the SOL in a cylindrical plasma with similar parameters to a typical DIII-D plasma.

B. Analysis

As before, all perturbed quantities are assumed to vary with θ and ϕ like $\exp[i(m\theta - n\phi)]$, where the integers m and n are the poloidal and toroidal mode numbers, respectively, of the RWM. In addition, all perturbed quantities are assumed to vary in time like $\exp(\gamma t)$. The stability of the RWM is usually determined using a plasma model that contains plasma inertia, plasma rotation, and some form of plasma dissipation. Reference 20 describes a simple plasma model, based on cylindrical geometry, in which the requisite plasma dissipation is provided by *neoclassical flow damping*.[21,22] According to this model, all of the information regarding the plasma response to the RWM is contained in a single *plasma response parameter*,

$$s(\gamma) = -\frac{1}{2}\left[1 + m^{-1}\frac{d\ln\psi}{d\ln r}\right]_{r=a}. \tag{34}$$

Here, a is the radius of the LCFS, and ψ the perturbed magnetic flux.

Let us neglect the SOL for the moment. If the plasma is surrounded by a concentric, thin resistive wall of radius r_w and thin shell time constant τ_w, then the RWM dispersion relation takes the form[20,23]

$$s = \frac{z}{1-z}, \tag{35}$$

where

$$z = c_w \frac{\gamma}{\gamma + \gamma_w}. \tag{36}$$

Here, $c_w = (r_w/a)^{2m}$ is the so-called *coupling constant*, which parametrizes the strength of the coupling between the plasma and the wall, whereas $\gamma_w = 2m/\tau_w$ is the typical growth rate of the RWM.

Although it is possible to estimate the wall parameters c and γ_w from cylindrical theory, these parameters can be determined much more precisely by fitting the above dispersion relation to data obtained from a finite-element code, such as VALEN, which can accurately model the DIII-D wall in three dimensions.[24] This procedure is described in Ref. 20. For the case of a 3,1 RMW, we find that $c_w = 0.14$ and $\gamma_w = 2.6 \times 10^2$ s^{-1}.

Let us now take the SOL into account. As we have seen, the SOL acts like a resistive wall located at the very edge of the plasma. Thus, the plasma is effectively surrounded by two walls—the SOL and the actual wall—with the SOL located between the actual wall and the plasma. Suppose that the coupling constant and characteristic growth rate for the SOL are c_a and γ_a, respectively. In the presence of two walls, the expression (36) generalizes to

$$z = c_a z_a + c_w z_w, \tag{37}$$

where

$$z_a = \frac{\gamma[c_a\gamma_w + \gamma(c_a - c_w)]}{c_a(\gamma+\gamma_a)(\gamma+\gamma_w) - c_w\gamma^2}, \tag{38}$$

$$z_w = \frac{c_a\gamma\gamma_a}{c_a(\gamma+\gamma_a)(\gamma+\gamma_w) - c_w\gamma^2}. \tag{39}$$

We can also derive the following simple relationship between the typical current circulating in the SOL, J_a, and the perturbed radial magnetic field in the SOL, b_a:

$$J_a \sim \frac{2\pi R_0}{n\mu_0}\frac{\gamma}{\gamma_a} b_a. \tag{40}$$

It remains to estimate c_a and γ_a. Now, according to cylindrical theory, $c_a = 1$. However, this is probably somewhat of an overestimate. It is certainly the case that, in realistic geometry, the plasma-wall coupling constants obtained from VALEN are generally *less* that those estimated from cylindrical theory. Nevertheless, we can be fairly certain that c_a is *significantly greater* than c_w. In other words, the SOL is much more strongly coupled to the plasma than the wall, given the SOL's relative proximity to the plasma. Also from cylindrical theory, $\gamma_a = 2m/\tau_a$. Now, the 3,1 RWM is only unstable when q_a is slightly less than 3, i.e., where $m - nq_a \ll 1$. Hence, it follows from Eq. (23) that $\tau_a \simeq \tau_p \sim 6$ ms. Thus, $\gamma_a \sim 9.7 \times 10^2$ s^{-1} for a 3,1 RWM in a DIII-D-like plasma.

Figure 4 shows the RWM stability boundary calculated for a DIII-D-like plasma using the plasma model described in Ref. 20, and the two-wall RWM dispersion relation given above (with $\gamma_a = 9.7 \times 10^2$ s^{-1}, and various values of c_a). The stability boundary is plotted in normalized plasma rotation frequency versus plasma stability space. Here, the plasma rotation is normalized with respect to $\tau_H \sim 3 \times 10^{-7}$ s, and the stability parameter $\bar{s}$ is defined such that the 3,1 ideal external-kink mode is stable for $\bar{s} < 0$, only stable if the actual wall is perfectly conducting when $0 < \bar{s} < 1$, and unstable even if the actual wall is perfectly conducting when $\bar{s} > 1$. Thus, the conventional no-wall and perfect-wall stability boundaries correspond to $\bar{s} = 0$ and $\bar{s} = 1$, respectively.

As can be seen from Fig. 4, in the absence of SOL currents, a normalized plasma rotation frequency of about 6×10^{-3} is sufficient to stabilize the RWM all the way to the perfect-wall stability boundary. Furthermore, Fig. 4 clearly shows that SOL currents have two main effects on RWM stability. Firstly, they *increase* the critical rotation frequency

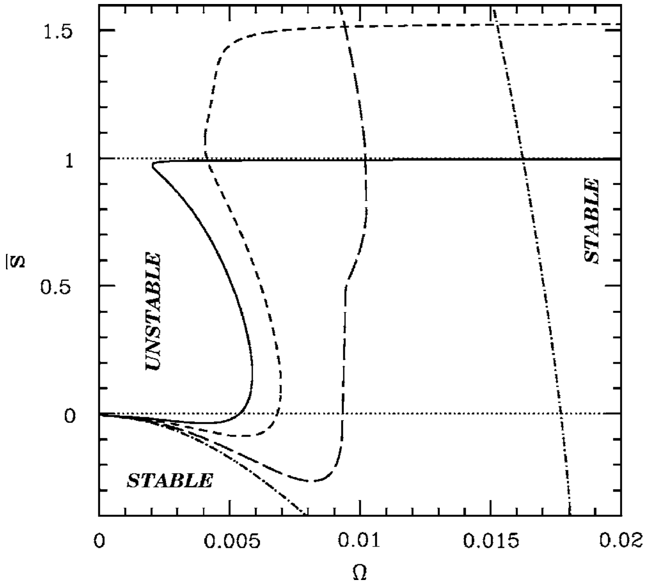

FIG. 4. RWM stability boundary for a DIII-D-like plasma plotted in normalized plasma rotation frequency vs plasma stability space. The solid curve shows the stability boundary calculated in the absence of SOL currents. The short-dashed, long-dashed, and dot-dashed curves show the stability boundaries calculated in the presence of SOL currents, assuming that the plasma-SOL coupling constant, c_a, takes the values 0.2, 0.3, and 0.4, respectively. The SOL growth rate is $\gamma_a = 9.7\times 10^2\ \mathrm{s}^{-1}$. The remaining plasma parameters are as specified in the caption of Fig. 3, in Ref. 20. The no-wall and perfect-wall stability boundaries lie at $\bar{s}=0$ and $\bar{s}=1$.

required to stabilize the RWM. Secondly, they allow the RWM to be stable *above* the perfect-wall stability boundary. Thus, when the plasma-SOL coupling constant, c_a, is relatively small, SOL currents have a largely beneficial effect on RWM stability, since they allow the RWM to be stabilized above the perfect-wall stability boundary at the cost of only slightly increasing the critical rotation frequency. However, as the coupling constant increases, SOL currents quickly begin to have a highly detrimental effect on RWM stability, in that they *strongly increase* the critical rotation frequency, and also start to destabilize the RWM *below* the no-wall stability boundary.

Let us now estimate the typical current circulating in the SOL in the presence of a RWM. Assuming that $\gamma \sim n\Omega$, where $\Omega\tau_H \sim 6\times 10^{-3}$, Eq. (40) yields $J_a(\mathrm{kA}) \sim 17 b_a(G)$. Hence, as soon as the radial field in the SOL exceeds about a gauss, the current circulating in the SOL is predicted to rise above the ion saturation current, J_{sat} (which we estimate to be about 8 kA). Of course, this does not happen. Instead, the current circulating in the SOL *saturates* at J_{sat}, and the effect of SOL currents on the stability of the RWM is considerably diminished compared to that predicted in Fig. 4.

VII. SUMMARY AND DISCUSSION

In this paper, we have shown that the presence of the solid conducting limiter, interrupting magnetic flux surfaces, gives the SOL plasma in a tokamak a degree of freedom not accorded to the plasma confined on the closed magnetic flux surfaces lying within the LCFS. This extra degree of freedom allows helical current eddies to flow *parallel* to the equilibrium magnetic field in the SOL plasma, even when the plasma *does not* lie on a rational flux surface. The return currents required to close these eddies flow through the limiter, where the $\mathbf{j}\times\mathbf{B}$ forces generated as they cross equilibrium magnetic field lines can be balanced by mechanical stresses.

We have also shown that the response of the SOL to a helical magnetic perturbation is similar to that of a resistive wall. However, unlike a conventional resistive wall, which responds to *all* helical perturbations with the *same* effective time constant, the SOL plasma responds to *different* helical perturbations with *different* effective time constants. In fact, the effective time constant of the SOL is relatively long, when the helical pitch of the perturbation closely matches that of the equilibrium magnetic field lines within the SOL, and relatively short otherwise.

Finally, we have shown that, as a consequence of plasma sheath physics, the net current that can circulate in the SOL cannot exceed the so-called *ion saturation current*.

Using the above three principles, a simple theoretical model of the response of the SOL to a time-varying helical magnetic perturbation in a large-aspect-ratio, low-β, circular cross-section, tokamak has been developed. We have employed this model to estimate the likely impact of SOL currents on various different types of MHD instabilities in a cylindrical plasma with similar parameters to a typical DIII-D plasma.

For *rotating tearing modes* resonant well inside the plasma, the interaction with SOL currents is relatively weak. This interaction gives rise to partial shielding of the magnetic perturbation associated with the mode from the vacuum region outside the SOL. The interaction also produces a braking of the plasma rotation at the tearing mode's rational surface. This braking causes a fairly smooth reduction in the plasma rotation as the amplitude of the mode gradually increases. On the other hand, for rotating tearing modes resonant close to the edge of the plasma, the interaction with SOL currents is relatively strong. This interaction gives rise to strong shielding of the magnetic perturbation associated with the mode from the vacuum region outside the SOL. The rotation braking generated by the SOL currents also causes a fairly abrupt reduction in the plasma rotation as the amplitude of the mode gradually increases. For both types of tearing mode, the plasma rotation is significantly reduced as soon as the amplitude of the perturbation at the edge of the plasma exceeds a few gauss. The net current excited in the SOL by rotating tearing modes is predicted to be similar to the ion saturation current.

The interaction of SOL currents with resistive wall modes is particularly strong, since RWMs are usually resonant just beyond the LCFS. This interaction has a highly deleterious effect on RWM stability, causing a significant increase in the critical plasma rotation frequency required to stabilize the mode. Note, however, that the net current excited in the SOL by a RWM is predicted to exceed the ion saturation current when the amplitude of the radial magnetic perturbation in the SOL rises above about a gauss. In this case, we expect the SOL current to saturate, leading to a considerable diminution of the effect of SOL currents on

RWM stability. Since, in reality, the ion saturation current ultimately depends on the plasma parameters at the divertor, this raises the interesting possibility that the stability of a tokamak, such as DIII-D, to the RWM may be significantly affected by divertor conditions.

Although we have only examined the interaction of conventional tearing modes and resistive wall modes with SOL currents, our analysis could easily be extended to deal with other types of MHD instability, such as neoclassical tearing modes and edge localized modes (ELMs). The interaction of ELMs with SOL currents is likely to be particularly strong, since ELMs are usually resonant just inside the LCFS.

Throughout this paper, we have implicitly assumed that the only currents flowing in the SOL are those driven by the inductive electric fields associated with time-varying MHD instabilities originating from within the plasma. Of course, there are other mechanisms thought to drive SOL currents. These include the thermoelectric potential,[25] Pfirsch-Schlüter effects,[26] bootstrap effects,[2] and the loop voltage.[2] However, all of these mechanisms drive an *axisymmetric* SOL current in an axisymmetric SOL. Thus, it is necessary for some toroidal asymmetry to exist, either in the SOL plasma or the limiter, before these mechanisms can generate nonaxisymmetric SOL currents capable of interacting with MHD modes.

We have also implicitly assumed that there is no toroidal plasma rotation in the SOL, and, hence, that the SOL responds to a time-varying helical magnetic perturbation, like a *stationary*, rather than a rotating, resistive wall. It is certainly plausible that the plasma rotation in the SOL is small, given the inevitable strong parallel transport of momentum to the limiter. Nevertheless, residual plasma rotation in the SOL could still have an important effect, since it would generate an impedance mismatch between the flowing SOL plasma and the stationary limiter, thereby suppressing SOL current eddies.

According to the simple model discussed in this paper, the effective time constant of the SOL scales as $\tau_a \sim L^{1/2} m^{1/4}$, whereas the ion saturation current scales as $J_{\rm sat} \sim L^{1/2}/m^{1/4}$. Here, L is the mean length of magnetic field lines in the SOL plasma, and m is the mass of the fueling ions. It follows that there is a slight isotope effect: i.e., all other things being equal, the SOL in a deuterium plasma is slightly more conductive than that in a hydrogen plasma. However, this effect is offset somewhat by the fact that the ion saturation current is slightly lower in a deuterium plasma. Note that, as the mean length of magnetic field lines in the SOL plasma increases, the effective time constant of the SOL and the ion saturation current both increase. This implies that SOLs with relatively long magnetic path lengths through the plasma are likely to interact significantly more strongly with MHD modes than those with relatively short path lengths. Hence, we would expect SOL effects to be more apparent in single-null divertor (SND) plasmas, compared to similar double-null divertor (DND) plasmas. Now, in an axisymmetric SND plasma, all magnetic field lines in the SOL plasma have the same path length. However, this is not the case for DND plasmas. Indeed, since the path length in the inboard SOL plasma is significantly greater than that in the outboard SOL plasma, due to toroidicity and plasma shaping, we might expect the inboard SOL in a DND plasma to be more strongly coupled to MHD modes than the outboard SOL. This effect is likely to significantly modify the interaction of the SOL with MHD modes in DND plasmas compared to SND plasmas. Thus, the above discussion suggests that a possible experimental signature of SOL interaction with MHD modes would be a significant difference between MHD behavior in otherwise similar SND and DND plasmas.

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