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Wave driven magnetic reconnection in the Taylor problem

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An improved Laplace transform theory is developed in order to investigate the initial response of a stable slab plasma equilibrium enclosed by conducting walls to a suddenly applied wall perturbation in the so-called Taylor problem. The novel feature of this theory is that it does not employ asymptotic matching. If the wall perturbation is switched on slowly compared to the Alfvén time then the plasma response eventually asymptotes to that predicted by conventional asymptotic matching theory. However, at early times there is a compressible Alfvén wave driven contribution to the magnetic reconnection rate which is not captured by asymptotic matching theory, and leads to a significant increase in the reconnection rate. If the wall perturbation is switched on rapidly compared to the Alfvén time then strongly localized compressible Alfvén wave-pulses are generated which bounce backward and forward between the walls many times. Each instance these wave-pulses cross the resonant surface they generate a transient surge in the reconnection rate. The maximum pulse driven reconnection rate can be much larger than that predicted by conventional asymptotic matching theory. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1617983]

I. INTRODUCTION

This paper investigates a model resistive magnetohydrodynamical (MHD) problem which was first proposed by J. B. Taylor. In this problem, a stable slab plasma equilibrium is subject to a suddenly imposed, small amplitude boundary perturbation which is such as to drive magnetic reconnection at the center of the slab. This type of reconnection, which is not due to an intrinsic plasma instability, is generally termed ‘‘forced reconnection.’’ The so-called ‘‘Taylor problem’’ is of fundamental importance to the field of magnetic reconnection, and has therefore been the subject of extensive study.[1–6]

The standard analytical technique used to investigate the Taylor problem involves first taking the *Laplace transform* of the linearized MHD equations, and then solving the resulting equations via *asymptotic matching*.[1] The various stages in the matching process are as follows. First, the plasma is divided into two regions. The so-called ‘‘outer region’’ comprises most of the plasma, whereas the ‘‘inner region’’ is a narrow layer centered on the resonant surface (where the equilibrium magnetic field reverses sign). The outer region, throughout which plasma inertia, resistivity, and viscosity are neglected, is governed by the easily soluble equations of marginally stable, ideal-MHD. In the inner region, plasma inertia, resistivity, and viscosity are retained in the analysis, but the governing equations are considerably simplified by exploiting the narrowness of this region compared to the rest of the plasma. Asymptotic matching between the solutions obtained in the inner and outer regions yields an expression for the Laplace transformed reconnected magnetic flux. Finally, this expression is inverted to give the reconnected magnetic flux as a function of time.

The analytic solution obtained via asymptotic matching reveals that the *initial* response of the plasma to the wall perturbation is largely governed by plasma *inertia*. In the limit in which the perturbation is switched on very suddenly (at $t=0$), the reconnected flux varies as $\psi_0 \sim \eta t^2$, where η is the plasma resistivity at the resonant surface.[1] This particular result has been the subject of much dispute in the literature.[3–6]

One of the main difficulties encountered when discussing the initial response of the plasma to the wall perturbation lies in the fact that at very early times the asymptotic matching approach clearly breaks down. It should take at least a few Alfvén times for information regarding the suddenly applied wall perturbation to travel from the edge to the center of the plasma. During this time interval, inertia plays a significant role in the response throughout the *whole* plasma, i.e., there is no outer region in which the response is solely governed by marginally stable, ideal-MHD. Another way of putting this is that at very early times the inner region extends over all the plasma. In the conventional analysis of the Taylor problem, it is tacitly assumed that a few Alfvén times after the imposition of the wall perturbation the response relaxes to the marginally stable, ideal-MHD response throughout the bulk of the plasma, with deviations from this

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response being localized to a relatively thin layer centered on the resonant surface. Of course, once relaxation has occurred the usual asymptotic matching approach becomes valid. However, up to now, this important relaxation process has never been studied in any detail.

The aim of this paper is to investigate the *early time response* of the plasma to the wall perturbation *without* using asymptotic matching. We hope to characterize the early time response, and also to determine whether this response eventually asymptotes to that obtained via conventional asymptotic matching (as is generally assumed to be the case). We shall neglect plasma viscosity in our analysis, since this effect plays an negligible role in the initial plasma response.

II. PRELIMINARY ANALYSIS

A. Basic equations

Standard right-handed Cartesian coordinates (x,y,z) are adopted. It is assumed that there is no variation along the z-axis, i.e., $\partial/\partial z \equiv 0$. Consider a compressible plasma governed by equations of resistive MHD. Let the plasma density, ρ, and resistivity, η, both be uniform. It follows that

$$\mathbf{B}=\nabla\wedge\mathbf{A}, \tag{1}$$

$$\mu_0\mathbf{j}=\nabla\wedge\mathbf{B}, \tag{2}$$

$$-\frac{\partial\mathbf{A}}{\partial t}-\nabla\varphi+\mathbf{V}\wedge\mathbf{B}=\eta\mathbf{j}, \tag{3}$$

$$\rho\left[\frac{\partial\mathbf{V}}{\partial t}+(\mathbf{V}\cdot\nabla)\mathbf{V}\right]=-\nabla p+\mathbf{j}\wedge\mathbf{B}, \tag{4}$$

$$\frac{\partial p}{\partial t}+\mathbf{V}\cdot\nabla p=-\Gamma p\nabla\cdot\mathbf{V}, \tag{5}$$

where $\mathbf{A}$ is the vector potential, φ is the scalar potential, $\mathbf{B}$ is the magnetic field, $\mathbf{V}$ is the plasma velocity, p is the plasma pressure, $\mathbf{j}$ is the current density, and $\Gamma=5/3$ the ratio of specific heats.

Let $(x/a,y/a,z/a)\to(x,y,z)$, $t/(a/V_A)\to t$, $\mathbf{B}/B_0\to\mathbf{B}$, $\mathbf{A}/(B_0 a)\to\mathbf{A}$, $\mathbf{V}/V_A\to\mathbf{V}$, $\varphi/(B_0V_Aa)\to\varphi$, $p/(\rho V_A^2)\to p$, and $\mathbf{j}/(B_0/\mu_0 a)\to\mathbf{j}$, where $V_A=B_0/\sqrt{\mu_0\rho}$ is the Alfvén velocity, a is a convenient scale-length, and B_0 is a convenient scale magnetic field-strength.

Let $\mathbf{A}(x,y,t)=[0,0,\psi(x,y,t)]$, and $\mathbf{V}(x,y,t)=[u(x,y,t),v(x,y,t),0]$. It follows that

$$\frac{\partial u}{\partial t}=-u\frac{\partial u}{\partial x}-v\frac{\partial u}{\partial y}-\frac{\partial p}{\partial x}-\nabla^2\psi\frac{\partial\psi}{\partial x}, \tag{6}$$

$$\frac{\partial v}{\partial t}=-u\frac{\partial v}{\partial x}-v\frac{\partial v}{\partial y}-\frac{\partial p}{\partial y}-\nabla^2\psi\frac{\partial\psi}{\partial y}, \tag{7}$$

$$\frac{\partial\psi}{\partial t}=-u\frac{\partial\psi}{\partial x}-v\frac{\partial\psi}{\partial y}+\eta\nabla^2\psi, \tag{8}$$

$$\frac{\partial p}{\partial t}=-u\frac{\partial p}{\partial x}-v\frac{\partial p}{\partial y}-\Gamma p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right), \tag{9}$$

where $\nabla^2\equiv\partial^2/\partial x^2+\partial^2/\partial y^2$.

B. Plasma equilibrium

Suppose that the plasma is bounded by perfectly conducting walls located at $x=\pm1$, and is periodic in the y-direction with periodicity length L. The initial plasma equilibrium satisfies

$$u^{(0)}(x)=0, \tag{10}$$

$$v^{(0)}(x)=0, \tag{11}$$

$$\psi^{(0)}(x)=-\frac{x^2}{2}, \tag{12}$$

$$p^{(0)}(x)=p_0-\frac{x^2}{2}, \tag{13}$$

where p_0 is the central pressure. In unnormalized units, B_0 is the equilibrium magnetic field-strength at $x=a$, and a is half the distance between the conducting walls. Note that the above plasma equilibrium is completely stable to tearing modes.

C. Boundary conditions

Suppose that the conducting wall at $x=1$ is subject to a *small* (compared with unity) displacement $\Xi(t)\cos(ky)$ in the x-direction, where $k=2\pi/L$. An equal and opposite displacement is applied to the wall at $x=-1$. The appropriate *no-slip* boundary conditions at the walls are

$$u(1,y,t)=-u(-1,y,t)=\frac{\partial\Xi(t)}{\partial t}\cos(ky), \tag{14}$$

$$v(1,y,t)=v(-1,y,t)=0, \tag{15}$$

$$\psi(1,y,t)=\psi(-1,y,t)=-\tfrac{1}{2}+\Xi(t)\cos(ky), \tag{16}$$

$$p(1,y,t)=p(-1,y,t)=p_0-\tfrac{1}{2}+\Xi(t)\cos(ky). \tag{17}$$

Let

$$\Xi(t)=\Xi_0[1-e^{-t/\tau}-(t/\tau)e^{-t/\tau}] \tag{18}$$

for $t\geqslant0$, with $\Xi(t)=0$ for $t<0$. Note that both $\Xi(t)$ and $d\Xi(t)/dt$ are continuous at $t=0$.

III. LAPLACE TRANSFORM THEORY

A. Analysis

Let us write $u(x,y,t)=[\partial\xi(x,t)/\partial t]\cos(ky)$. Here, $\xi(x,t)$ is the plasma displacement in the x-direction. The boundary conditions on $\xi(x,t)$ are simply

$$\xi(\pm1,t)=\pm\Xi(t). \tag{19}$$

Moreover, it follows from symmetry that $\xi(-x,t)=-\xi(x,t)$, and hence that

$$\xi(0,t)=0. \tag{20}$$

The linearized and Laplace transformed versions of Eqs. (6)–(9) can be reduced to

$$\frac{\partial}{\partial x}\left[\left(x^2+\frac{g^2\Gamma P}{g^2+\Gamma Pk^2}\right)\frac{\partial\bar{\xi}}{\partial x}\right]-(k^2x^2+g^2)\bar{\xi}=0, \tag{21}$$

provided that η is *negligible*. Here, $P=p_0-x^2/2$ is the equilibrium pressure, and

$$\overline{\xi}(x,g)=\int_0^\infty \xi(x,t)e^{-gt}dt, \tag{22}$$

the Laplace transformed plasma displacement. The boundary conditions on $\overline{\xi}(x,g)$ are

$$\overline{\xi}(\pm 1,g)=\pm\int_0^\infty \Xi(t)e^{-gt}dt=\pm\frac{\Xi_0}{g(1+g\tau)^2}, \tag{23}$$

and

$$\overline{\xi}(0,g)=0. \tag{24}$$

Note that the neglect of resistivity during the derivation of Eq. (21) is justified provided that $t\ll\tau_1$, where[1]

$$\tau_1=\frac{1}{\eta^{1/3}k^{2/3}}. \tag{25}$$

In the following, we shall parameterize the plasma response to the wall perturbation in terms of the quantity,

$$J(t)=2\frac{\partial\xi(0,t)}{\partial x}, \tag{26}$$

which represents (minus) the perturbed current density at the resonant surface $(x=0)$. Now, it easily follows from Ohm's law (including resistivity) that

$$\frac{d\psi_0(t)}{dt}=\eta J(t), \tag{27}$$

where $\psi_0(t)$ is the reconnected magnetic flux. Thus, $J(t)$ is a measure of both the rate of magnetic reconnection and the amplitude of the current sheet driven at the resonant surface.

Let $Y(x,g)$ be a solution of Eq. (21) which satisfies

$$Y(0,g)=0, \tag{28}$$

$$Y(1,g)=1. \tag{29}$$

It follows that

$$J(t)=\frac{\Xi_0}{2\pi i}\int_C\frac{2Y'(0,g)e^{gt}}{g(1+g\tau)^2}dg, \tag{30}$$

where $Y'=\partial Y/\partial x$, and C represents the Bromwich contour.

B. Asymptotic matching response

The integrand on the right-hand side of Eq. (30) possesses obvious poles at $g=0$ and $g=-1/\tau$. Let us calculate the plasma response due to these poles. Provided that $\tau\gg 1$, both poles are characterized by $g\ll 1$. In this limit, Eq. (21) can be solved via asymptotic matching. The outer region corresponds to $|x|\gg|g|$. In this region, Eq. (21) reduces to

$$\frac{\partial}{\partial x}\left(x^2\frac{\partial\overline{\xi}}{\partial x}\right)-k^2x^2\overline{\xi}\simeq 0. \tag{31}$$

The solution to the above equation which satisfies the boundary condition (29) takes the form

$$Y\simeq\frac{\sinh kx}{x\sinh k}. \tag{32}$$

The inner region corresponds to $|x|\lesssim|g|$. In this region, Eq. (21) reduces to

$$\frac{\partial}{\partial x}\left[\left(x^2+\frac{g^2}{k^2}\right)\frac{\partial\overline{\xi}}{\partial x}\right]\simeq 0. \tag{33}$$

The solution to the above equation which satisfies the boundary condition (28), and matches to the outer solution, is written

$$Y\simeq\frac{2k}{\pi\sinh k}\tan^{-1}\left(\frac{kx}{g}\right). \tag{34}$$

Thus, it follows that

$$2Y'(0,g)=\frac{2kE_{sw}}{\pi g}, \tag{35}$$

where $E_{sw}=2k/\sinh k$. Finally, direct inversion of Eq. (30) yields

$$J(t)=\frac{2kE_{sw}\Xi_0}{\pi}[t+(t+2\tau)e^{-t/\tau}-2\tau]. \tag{36}$$

Note that the above expression is identical to that obtained from conventional asymptotic matching theory.[1,6]

C. Wave response

The integrand on the right-hand side of Eq. (30) also possesses poles which correspond to those of the function $Y'(0,g)$. These additional poles can be written $g=\pm i\omega_n$, for $n=1,2,3,\ldots$. Here, the ω_n (which are real) are the eigenvalues of the eigenequation,

$$\frac{d}{dx}\left[\left(x^2-\frac{\omega_n^2\Gamma P}{\Gamma Pk^2-\omega_n^2}\right)\frac{dY_n}{dx}\right]-(k^2x^2-\omega_n^2)Y_n=0, \tag{37}$$

where the eigenfunctions $Y_n(x)$ satisfy the boundary conditions $Y_n(0)=Y_n(1)=0$. Of course, the Y_n represent the natural Alfvénic modes of oscillation of the plasma, whereas the ω_n are the associated oscillation frequencies. The plasma response emanating from these new poles can be thought of as due to compressible Alfvén waves excited by the suddenly imposed wall perturbation.

Figure 1 shows the first ten ω_n values calculated as functions of k for $p_0=1$. For comparison, the curves $\omega_n=\sqrt{\Gamma p_0(n^2\pi^2+k^2)}$ for $n=1,10$ are also plotted. It can be seen that there is a fairly close correspondence between the calculated ω_n values and the curves, which indicates that the natural Alfvénic modes of oscillation of the plasma have frequencies which satisfy the approximate dispersion relation $\omega_n\simeq\sqrt{\Gamma p_0(n^2\pi^2+k^2)}$ for $n=1,2,3,\ldots$. The dependence on Γp_0 demonstrates that these modes are related to *compressible* (rather than shear) Alfvén waves. Note that for small k values the ω_n are *equally spaced*, whereas this is not the case for large k values.

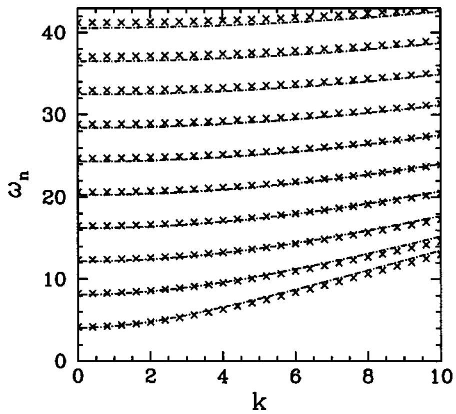

FIG. 1. The crosses show the first ten plasma eigenfrequencies, ω_n, calculated at various different k values for $p_0=1$. The curves show the functional relationships $\omega_n=\sqrt{\Gamma p_0(n^2\pi^2+k^2)}$ for $n=1,10$.

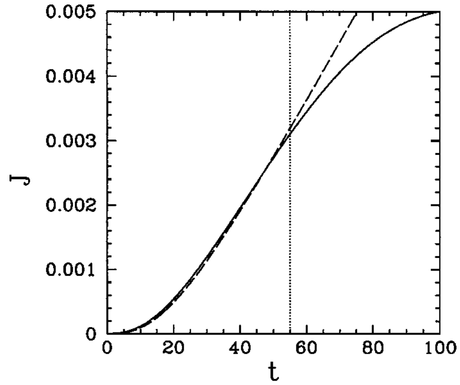

FIG. 2. The magnetic reconnection rate, J, as a function of time, t, for a calculation performed using $\Xi_0=10^{-4}$, $L=8$, $\eta=10^{-5}$, $p_0=1$, and $\tau=10$. The solid curve shows the numerical solution generated by the FLASH code. The long-dashed curve shows the solution produced by conventional asymptotic matching theory. The vertical dotted line indicates when $t=\tau_1$.

IV. NUMERICAL RESULTS

A. Introduction

Equations (6)–(9), plus the initial equilibrium (10)–(13), and the boundary conditions (14)–(18), have been implemented numerically within the massively parallel, adaptive mesh refinement (AMR) architecture of the FLASH code.[7] The particular integration scheme employed is a fully explicit, second-order (in both time and space), finite-volume, cell-centered method with limited gradient reconstruction.[8,9]

B. Code diagnostics

The z-component of Ohm's law is written

$$\frac{\partial\psi}{\partial t}+\mathbf{V}\cdot\nabla\psi=-\eta j, \tag{38}$$

where $j\equiv-\nabla^2\psi$ is the current density in the z-direction. Now, by definition, $\nabla\psi=0$ at the magnetic O- and X-points. Since there is zero equilibrium plasma flow, and the wall perturbation is nonpropagating, the positions of the O- and X-points are fixed and easily identifiable in our simulations. The reconnected flux is defined

$$\psi_0(t)=\tfrac{1}{2}[\psi(X\text{-point})-\psi(O\text{-point})]. \tag{39}$$

Our reconnection rate diagnostic takes the form

$$J(t)=\tfrac{1}{2}[j(O\text{-point})-j(X\text{-point})]. \tag{40}$$

This definition is equivalent to Eq. (26). It follows from Eq. (38), and the fact that $\nabla\psi=0$ at the O- and X-points, that $J=\eta^{-1}d\psi_0/dt$. Thus, $J(t)$ measures both the reconnection rate and the current density in the reconnecting region.

C. Results

In the following, we shall compare and contrast $J(t)$ curves produced by the FLASH code and by two different types of Laplace transform calculation. The conventional asymptotic matching calculation gives the $J(t)$ curve specified in Eq. (36). The improved calculation presented in this paper generates the $J(t)$ curve obtained by numerically inverting Eq. (30). The plasma response obtained from this improved calculation can be thought of as a combination of the standard asymptotic matching response discussed in Sec. III B and the wave response discussed in Sec. III C.

Figure 2 shows $J(t)$ curves generated by the FLASH code and conventional asymptotic matching theory for a case where the wall perturbation is switched on *comparatively slowly* (i.e., $\tau\gg\tau_A$). As expected, there is good agreement between the numerical and analytic curves when $t\ll\tau_1$. However, as $t\to\tau_1$ the two curves diverge because resistivity starts to play a role in the layer dynamics. (Recall, from Sec. III, that the asymptotic matching theory employed in this paper neglects the effect of resistivity on the layer dynamics.)

Figure 3 shows $J(t)$ curves at very early times for the same calculation as that presented in Fig. 2. It can be seen that the numerical curve lies somewhat above the asymptotic matching curve. Now, it is clear from Fig. 2 that the relative difference between the two curves decreases as time progresses, and eventually becomes negligible. Nevertheless, at early times (i.e., $t\lesssim 10\tau_A$) there is a significant discrep-

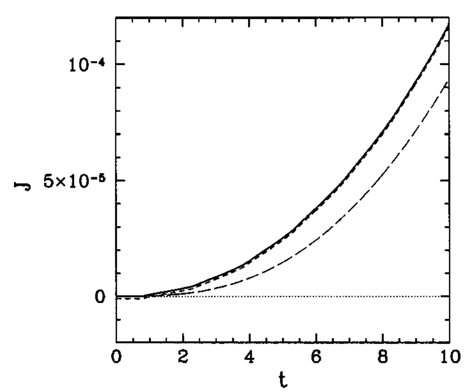

FIG. 3. The magnetic reconnection rate, J, as a function of time, t, for a calculation performed using $\Xi_0=10^{-4}$, $L=8$, $\eta=10^{-5}$, $p_0=1$, and $\tau=10$. The solid curve shows the numerical solution generated by the FLASH code. The long-dashed curve shows the solution produced by conventional asymptotic matching theory. The short-dashed curve (which has been shifted downward slightly to make it more visible) shows the solution generated by the improved Laplace transform calculation.

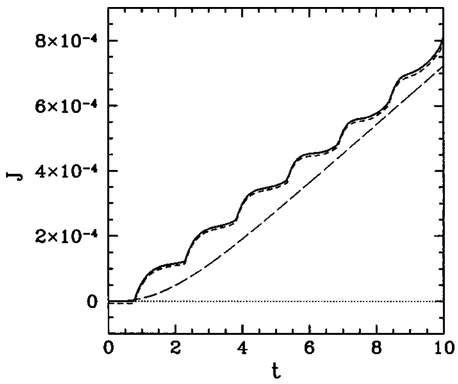

FIG. 4. The magnetic reconnection rate, *J*, as a function of time, *t*, for a calculation performed using $\Xi_0 = 10^{-4}$, $L=8$, $\eta = 10^{-5}$, $p_0 = 1$, and $\tau = 1$. The solid curve shows the numerical solution generated by the FLASH code. The long-dashed curve shows the solution produced by conventional asymptotic matching theory. The short-dashed curve (which has been shifted downward slightly to make it more visible) shows the solution generated by the improved Laplace transform calculation.

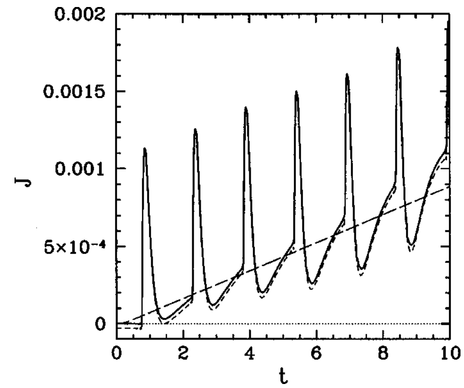

FIG. 5. The magnetic reconnection rate, *J*, as a function of time, *t*, for a calculation performed using $\Xi_0 = 10^{-4}$, $L=8$, $\eta = 10^{-5}$, $p_0 = 1$, and $\tau = 0.1$. The solid curve shows the numerical solution generated by the FLASH code. The long-dashed curve shows the solution produced by conventional asymptotic matching theory. The short-dashed curve (which has been shifted downward slightly to make it more visible) shows the solution generated by the improved Laplace transform calculation.

ancy between the numerical and asymptotic matching curves. Figure 3 also shows the $J(t)$ curve produced by the improved Laplace transform theory presented in Sec. III A. Note that this curve agrees exactly with the numerical curve generated by the FLASH code. We conclude that the discrepancy between the numerical and asymptotic matching $J(t)$ curves, which is evident in Fig. 3, is due to the *wave response* discussed in Sec. III C. This response is not captured by conventional asymptotic matching theory, and clearly leads to a significant increase in the reconnection rate at early times.

Figure 4 shows $J(t)$ curves evaluated at very early times for a case where the wall perturbation is switched on *moderately rapidly* (i.e., $\tau \sim \tau_A$). The numerical curve generated by the FLASH code again lies above the asymptotic matching curve. Moreover, the former curve exhibits pulse-like features which are entirely absent from the latter. However, the $J(t)$ curve produced by the improved Laplace transform theory again agrees exactly with the numerical curve.

Figure 5 shows $J(t)$ curves evaluated at very early times for a case where the wall perturbation is switched on *very rapidly* (i.e., $\tau \ll \tau_A$). The dominant feature of the numerical curve generated by the FLASH code is a set of evenly spaced spikes. It can be seen that this feature is reproduced exactly by the improved Laplace transform theory, but not at all by conventional asymptotic matching theory. Figure 6 shows $J(t)$ evaluated at later times for the same case. The numerical data are generated by the University of Iowa MHD code.[10] (Incidentally, the U. Iowa code is in good agreement with the FLASH code.) It can be seen that the spikes in the reconnection rate persist, although the average reconnection rate remains roughly in agreement with that predicted by conventional asymptotic matching theory (as long as $t < \tau_1$).

Figure 7 shows density plots of $u(x,y,t)$ for the same calculation as that presented in Fig. 5. It can be seen that the sudden switch-on of the wall perturbation generates two strongly localized *pulses* which propagate toward the resonant surface at the center of the plasma, pass through one another, and reflect off the walls. The two pulses subsequently bounce backward and forward between the walls many times. The arrival times of the pulses at the resonant surface correlate very well with the strong spikes in the reconnection rate shown in Fig. 5. Note that the pulses are essentially *compressional* Alfvén waves. [This is easily demonstrated by increasing the central plasma pressure, p_0, which has the effect of increasing the propagation speed of compressional Alfvén waves, and hence of decreasing the spacing between the spikes in the numerical $J(t)$ curve.] We conclude that the strong spikes in the numerical $J(t)$ curve shown in Fig. 5 represent magnetic reconnection driven by compressional Alfvén pulses which are excited by the sudden onset of the wall perturbation. These pulses are *not* captured by conventional asymptotic matching theory. Note that the maximum pulse driven reconnection rate can be *much larger* than that predicted by conventional asymptotic matching theory.

It is clear from the numerical $J(t)$ curve shown in Fig. 5 that there is a delay of about 1.5 Alfvén times between the switch-on of the wall perturbation and the onset of driven

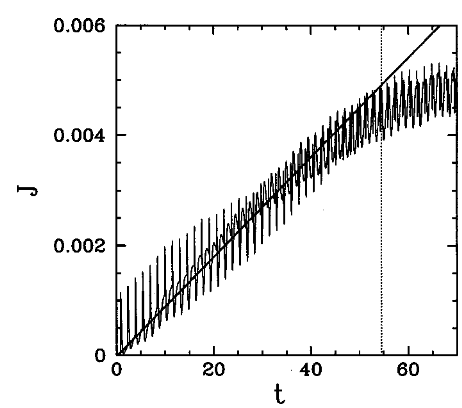

FIG. 6. The magnetic reconnection rate, *J*, as a function of time, *t*, for a calculation performed using $\Xi_0 = 10^{-4}$, $L=8$, $\eta = 10^{-5}$, $p_0 = 1$, and $\tau = 0.1$. The spiky curve shows data generated by the University of Iowa MHD code. The smooth curve shows the solution produced by conventional asymptotic matching theory. The vertical dotted line indicates when $t = \tau_1$.

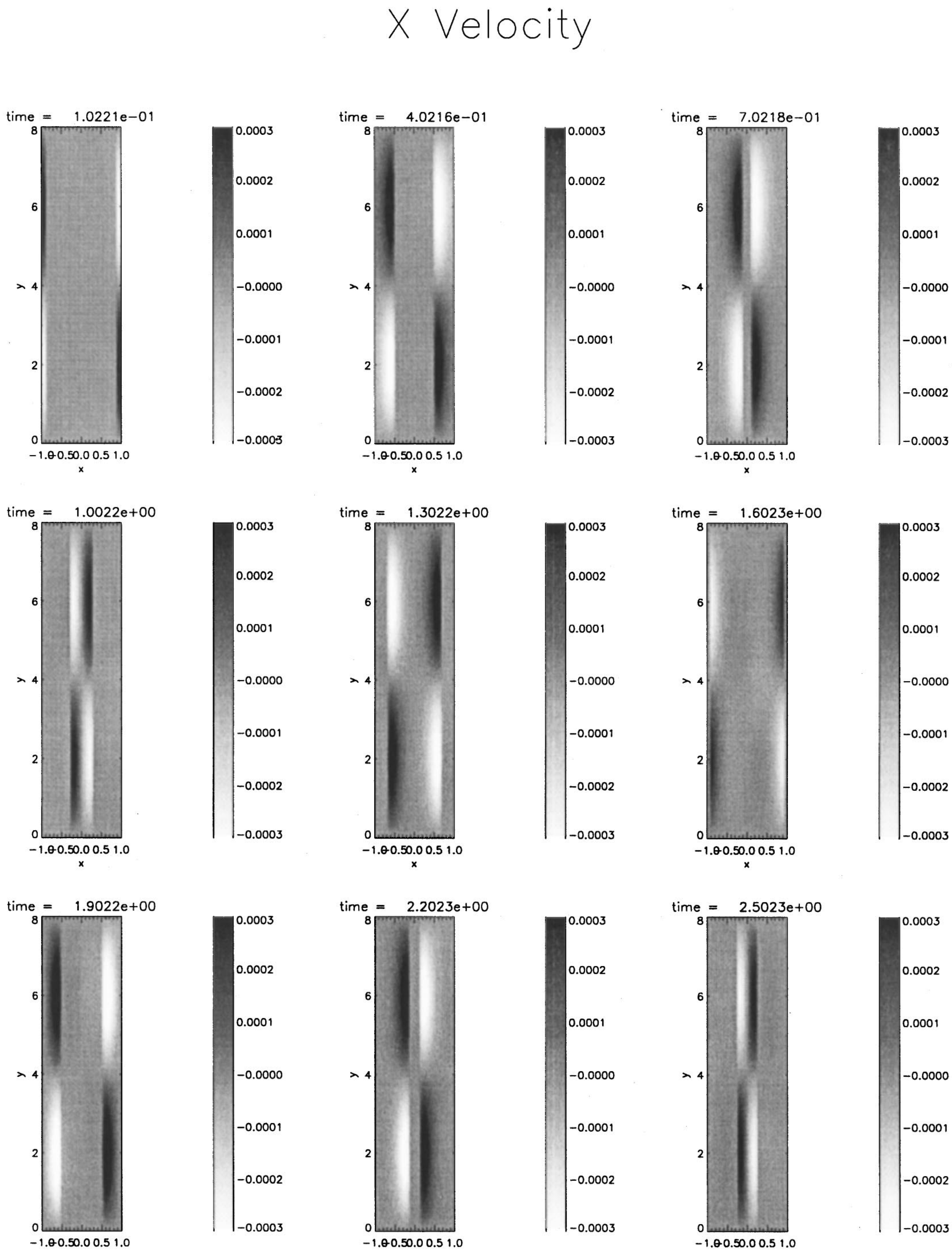

FIG. 7. Density plots of the x-velocity, $u(x,y,t)$, evaluated at various times for a calculation performed using $\Xi_0=10^{-4}$, $L=8$, $\eta=10^{-5}$, $p_0=1$, and $\tau=0.1$. Data generated by the FLASH code.

reconnection at the resonant surface. This delay simply corresponds to the travel time of a compressional Alfvén wave between the wall and the resonant surface. As the central pressure p_0 increases, we expect the time delay between the pulse switch-on and the onset of driven reconnection to decrease, since the propagation speed of compressional Alfvén waves varies as $\sqrt{p_0}$. Let us investigate the incompressible limit $p_0\rightarrow\infty$. Figure 8 shows $J(t)$ curves evaluated at very early times for a case where the wall perturbation is switched on very rapidly and $p_0\rightarrow\infty$. There is no numerical $J(t)$ curve, since the FLASH code cannot operate in the incompressible limit. Fortunately, however, there is no such restriction on the improved Laplace transform theory presented in this paper. The $J(t)$ curve generated by the improved theory exhibits no time delay between the perturbation switch-on and the onset of magnetic reconnection. This is as expected, since information regarding the wall perturbation is carried by *compressional* Alfvén waves, which travel infinitely fast in the incompressible limit. Note, however, that the $J(t)$ curve generated by the improved theory still lies significantly

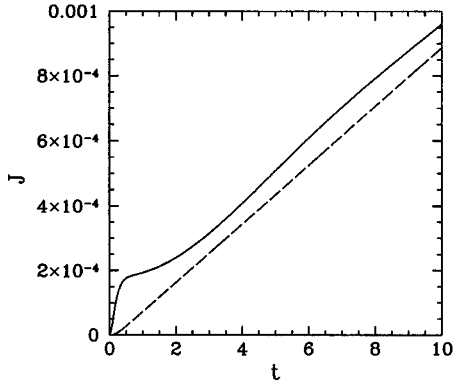

FIG. 8. The magnetic reconnection rate, J, as a function of time, t, for an incompressible calculation performed using $\Xi_0=10^{-4}$, $L=8$, $\eta=10^{-5}$, and $\tau=0.1$. The long-dashed curve shows the solution produced by conventional asymptotic matching theory. The solid curve shows the solution generated by the improved Laplace transform calculation.

above that produced by conventional asymptotic matching theory. This demonstrates that, even in the incompressible limit, wave driven magnetic reconnection (presumably mediated by infinitely fast compressional Alfvén waves) is important at early times. Note that the wave driven reconnection exhibits none of the pulse-like behavior shown in previous figures. This is true no matter how rapidly the wall perturbation is switched on.

Up to now, we have only considered cases where $L\gg a$ (i.e., $k\ll1$). In the opposite limit, $L\lesssim a$ (i.e., $k\gtrsim1$), compressional Alfvén pulses generated by the fast switch-on of the wall perturbation tend to disperse fairly rapidly, and consequently lose coherence after a few passes through the plasma. In marked contrast, the pulses shown in Figs. 5 and 7 remain coherent for very many transit times. The explanation is as follows. As is shown in Fig. 1, the compressional Alfvén eigenmodes of the plasma have *equally spaced* eigenfrequencies when $k\ll1$, but not when $k\gtrsim1$. Now, it is possible to construct a narrow nondispersive pulse from a superposition of eigenmodes with equally spaced eigenfrequencies. On the other hand, narrow pulses disperse rapidly when the eigenfrequencies are not equally spaced.

V. SUMMARY

We have developed an improved Laplace transform theory for investigating the initial response of a stable slab plasma equilibrium to a suddenly applied wall perturbation in the so-called Taylor problem. The novel feature of this new theory is that it does not employ asymptotic matching.

When the wall perturbation is switched on slowly compared to the Alfvén time, we find that the plasma response eventually asymptotes to that predicted by conventional asymptotic matching theory. However, at early times (i.e., $t\lesssim10\tau_A$), there is a compressible Alfvén wave driven contribution to the magnetic reconnection rate which is not captured by asymptotic matching theory, and leads to a significant increase in the reconnection rate.

When the wall perturbation is switched on rapidly compared to the Alfvén time, strongly localized compressible Alfvén wave-pulses are generated which bounce backward and forward between the walls many times. Each time these wave-pulses cross the resonant surface they generate a transient surge in the reconnection rate. Indeed, the maximum pulse driven reconnection rate can be much larger than that predicted by conventional asymptotic matching theory. Note that the pulses only remain coherent over many transits across the plasma when the wavelength of the wall perturbation greatly exceeds the wall separation.

In the incompressible limit, the pulses are absent, but there is still a significant wave driven contribution to the initial reconnection rate which is not accounted for by asymptotic matching theory.

The improved Laplace transform theory has been successfully benchmarked against numerical results from the FLASH code.

Note that the deviations from standard asymptotic matching theory reported here only affect the initial stages of the driven reconnection and have no influence on the final value of the reconnected flux nor the time taken to achieve full reconnection (which is much longer than any time scale considered in this paper).

Pulse-like driven magnetic reconnection has been reported previously in stressed X-point configurations.[11–13] Note, however, that in such configurations the pulses always stall at the X-point (which is equivalent to our resonant surface), and are only able to propagate past the X-point through the agency of resistivity. The problem considered in this paper is somewhat different, since our pulses pass through the resonant surface with a finite speed even in the limit $\eta\to0$.

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